Edexcel GCE

Core Mathematics S1

Normal Distribution

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. From experience a high jumper knows that he can clear a height of at least 1.78 m once in 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10 attempts. 20% 70%

Assuming that the heights the high jumper can reach follow a Normal distribution,

(a) draw a sketch to illustrate the above information,

(3)

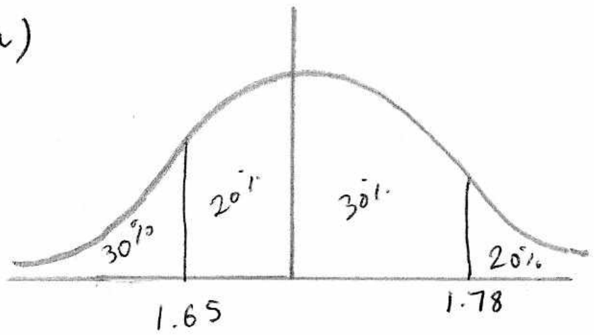
(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high jumper can reach,

(6)

(c) calculate the probability that he can jump at least 1.74 m.

(3)

1a)



b/ $p = 0.2$ $\quad z = 0.8416$ $\quad X = 1.78$ $\qquad p = 0.3$ $\quad z = -0.5244$ $\quad X = 1.65$

$$0.8416 = \frac{1.78 - \mu}{\sigma} \qquad -0.5244 = \frac{1.65 - \mu}{\sigma}$$

$$0.8416\sigma = 1.78 - \mu \qquad -0.5244\sigma = 1.65 - \mu$$

$$\sigma = \frac{1.78 - \mu}{0.8416} \qquad \sigma = \frac{1.65 - \mu}{-0.5244}$$

$$-0.5244(1.78 - \mu) = 0.8416(1.65 - \mu)$$

$$-0.933432 + 0.5244\mu = 1.38864 - 0.8416\mu$$

$$1.366\mu = 2.322072$$

$$\mu = 1.700 \ (3dp)$$

$$\sigma = \frac{1.78 - \text{"}1.700\text{"}}{0.8416}$$

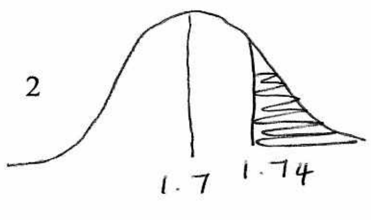
$$= 0.095 \ (3dp)$$

c/ $$z = \frac{1.74 - 1.700}{0.095}$$

$$= 0.42$$

$$p = 0.6628$$

$$1 - 0.6628 = 0.3372$$



2. The measure of intelligence, IQ, of a group of students is assumed to be Normally distributed with mean 100 and standard deviation 15.

μ σ

(*a*) Find the probability that a student selected at random has an IQ less than 91.

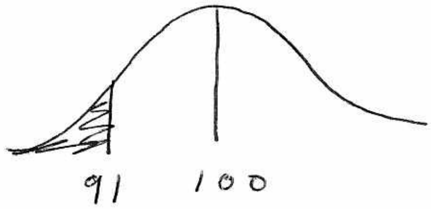
(4)

The probability that a randomly selected student as an IQ of at least 100 + *k* is 0.2090.

(*b*) Find, to the nearest integer, the value of *k*.

(6)

a/



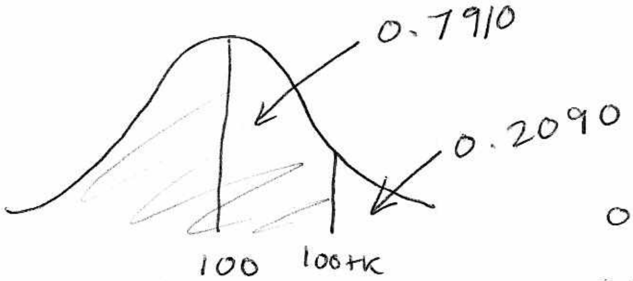
$$z = \frac{91 - 100}{15}$$

$$= -0.6$$

$$P(X < 91) = 1 - 0.7257$$

$$= \underline{0.2743}$$

b/



$$P = 0.7910$$

$$z = 0.81$$

$$0.81 = \frac{100 + k - 100}{15}$$

$$\underline{12.15 = k}$$

$$\underline{k = 12}$$

3. The random variable X has a normal distribution with mean 20 and standard deviation 4.

μ σ

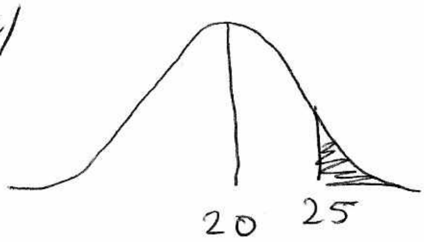
(*a*) Find $P(X > 25)$.

(3)

(*b*) Find the value of d such that $P(20 < X < d) = 0.4641$.

(4)

a/



$$z = \frac{25 - 20}{4}$$

$$= 1.25$$

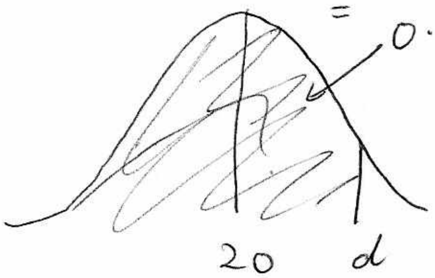
$$P(X > 25) = 1 - 0.8944$$

$$= 0.1056$$

b/

$0.5 + 0.4641$

$= 0.9641$



$p = 0.9641 \quad z = 1.8$

$$1.8 = \frac{d - 20}{4}$$

$$7.2 = d - 20$$

$$d = 27.2$$

4. The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

(*a*) Write down the median weight of the bags of popcorn. **(1)**

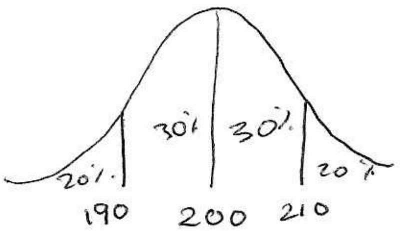
(*b*) Find the standard deviation of the weights of the bags of popcorn. **(5)**

A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g.

(*c*) Find the probability that a customer will complain. **(3)**

a/ 200g

b/



$p = 0.2 \quad z = 0.8416 \quad x = 210$

$0.8416 = \dfrac{210 - 200}{\sigma}$

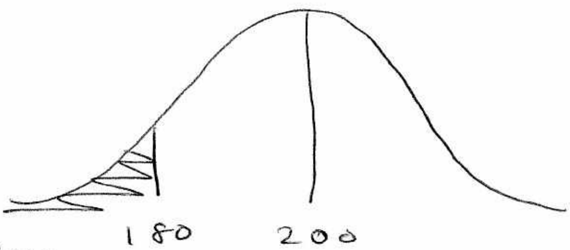
$0.8416\sigma = 10$

$\sigma = \dfrac{10}{0.8416}$

$\sigma = 11.9$ g (3sf)

c/ $z = \dfrac{180 - 200}{11.9}$

$z = -1.68$



$P(X < 180) = 1 - 0.9535$

$= 0.0465$

5. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

μ σ

(*a*) Find $P(X > 53)$.

(3)

(*b*) Find the weight that is exceeded by 99% of the bags.

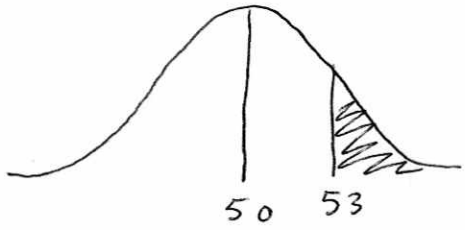
(5)

Three bags are selected at random.

(*c*) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)

a/



$$z = \frac{53 - 50}{2}$$

$$= 1.5$$

$$P(X > 53) = 1 - 0.9332$$

$$= \underline{0.0668}$$

b/

$p = 0.01 \quad z = 2.3263$

$$-2.3263 = \frac{x - 50}{2}$$

$$-4.6526 = x - 50$$

$$\cancel{x = 54.6526}$$

$$\underline{x = 45.3474}$$

b/ MML
MLM
LMM

3 combinations

$$3(0.0668 \times 0.0668 \times 0.9332)$$

$$= \underline{0.0125} \quad (3sf)$$

6. The random variable X has a normal distribution with mean 30 and standard deviation 5.

μ σ

(*a*) Find $P(X < 39)$.

(2)

(*b*) Find the value of d such that $P(X < d) = 0.1151$.

(4)

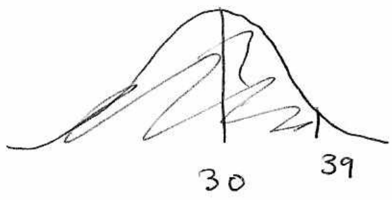
(*c*) Find the value of e such that $P(X > e) = 0.1151$.

(2)

(*d*) Find $P(d < X < e)$.

(2)

a/

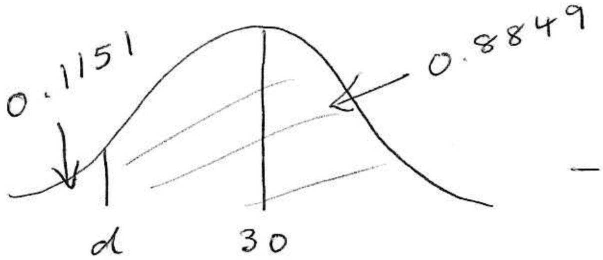


$$z = \frac{39 - 30}{5}$$

$$= 1.8$$

$$p = 0.9641$$

b/



$$z = -1.20$$

$$-1.20 = \frac{d - 30}{5}$$

$$-6 = d - 30$$

$$d = 24$$

c/ $e = 36$

d/ $1 - 0.1151 - 0.1151$

$$= 0.7698$$

7. The lifetimes of bulbs used in a lamp are normally distributed.

A company *X* sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

μ σ

(*a*) Find the probability of a bulb, from company *X*, having a lifetime of less than 830 hours. **(3)**

(*b*) In a box of 500 bulbs, from company *X*, find the expected number having a lifetime of less than 830 hours. **(2)**

A rival company *Y* sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

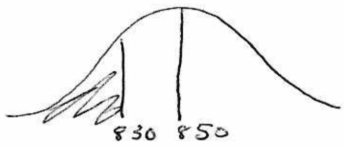
μ

(*c*) Find the standard deviation of the lifetimes of bulbs from company *Y*. **(4)**

Both companies sell the bulbs for the same price.

(*d*) State which company you would recommend. Give reasons for your answer. **(2)**

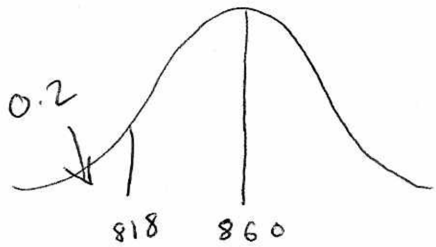
a/ $z = \dfrac{830 - 850}{50} = -0.4$



$P(X < 830) = 1 - 0.6554$
$= \underline{0.3446}$

b/ $0.3446 \times 500 = \underline{\underline{172}}$ (nearest whole number)

c/



$p = 0.2 \quad z = -0.8416$

$-0.8416 = \dfrac{818 - 860}{\sigma}$

$\sigma = \dfrac{-42}{-0.8416}$

$= \underline{\underline{49.9}}$ (3sf)

d/ Company Y → the lightbulbs have a longer mean lifetime and a similar spread (st. deviation). The light bulbs last longer.

8. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm μ and a standard deviation 5.2 σ cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 μ kg and standard deviation 7.1 σ kg.

Find the probability that a randomly chosen athlete

(*a*) is taller than 188 cm,

(3)

(*b*) weighs less than 97 kg.

(2)

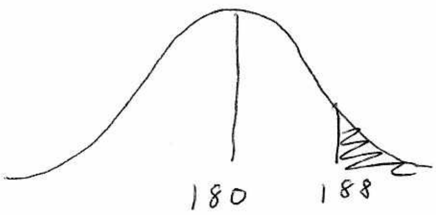
(*c*) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

(3)

(*d*) Comment on the assumption that height and weight are independent.

(1)

a/



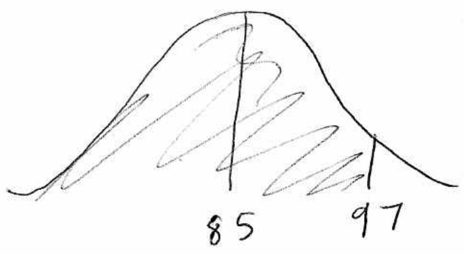
$$z = \frac{188 - 180}{5.2}$$

$$= 1.54$$

$$P(h > 188) = 1 - 0.9382$$

$$= \underline{0.0618}$$

b/



$$z = \frac{97 - 85}{7.1}$$

$$= 1.69$$

$$P(w < 97) = \underline{0.9545}$$

c/ $0.0618 \times 0.0455 = 0.00281$ (3sf)

d/ Height and weight are not independent. Taller people are more likely to weigh more.