

Write your name here

Surname

Other Names

Mathematics

2019 Paper 1 (Non-Calculator) Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may not be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

- 1 In a box there are blue pens, red pens and green pens.
The ratio of blue pens to red pens to green pens is 5:3:2

There are 18 more blue pens than red pens.
How many green pens are in the box?

$$B : R : G$$

$$5 : 3 : 2$$

Difference between blue and red = 2 parts

$$2 \text{ parts} = 18 \text{ pens}$$

$$1 \text{ part} = 9 \text{ pens}$$

$$\text{Green } 2 \times 9$$

.....18

(Total for question 1 is 3 marks)

- 2 Four builders working 6 hours a day can build a wall in two days.

How many days will it take two builders working 8 hours a day to build the same wall.

$$4 \times 6 \times 2 = 48 \text{ hours of work needed}$$

$$48 \div 2 = 24 \text{ hours for each builder}$$

$$24 \div 8 = 3 \text{ days}$$

.....3

.....days

- (b) State one assumption you made in your working out to part (a).

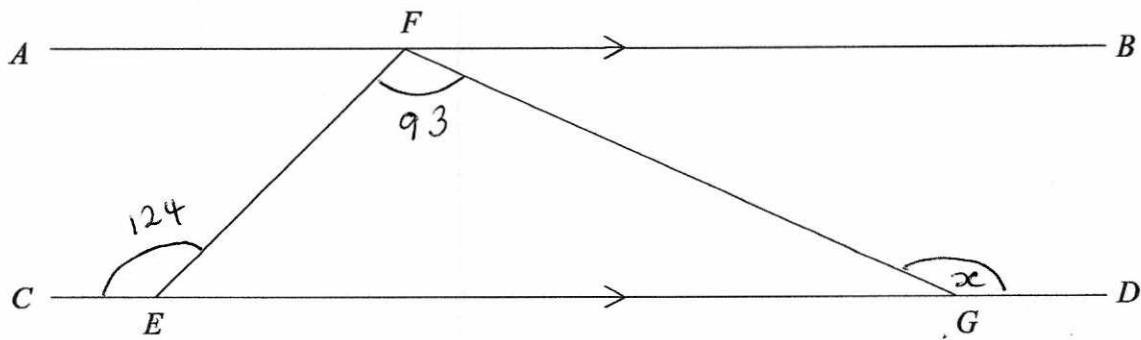
(2)

.....All of the builders work at the same rate

.....(1)

(Total for question 2 is 3 marks)

3



AB and CD are parallel.

Angle $CEF = 124^\circ$

Angle $EFG = 93^\circ$

Find the size of angle FGD .

You must show how you got your answer.

$$\begin{aligned} FEG &= 180 - 124 \\ &= 56^\circ \end{aligned}$$

Angles on a straight line
sum to 180°

$$\begin{aligned} FGE &= 180 - 56 - 93 \\ &= 124 - 93 \\ &= 31^\circ \end{aligned}$$

Angles in a triangle
sum to 180°

$$\begin{aligned} FGD &= 180 - 31 \\ &= 149^\circ \end{aligned}$$

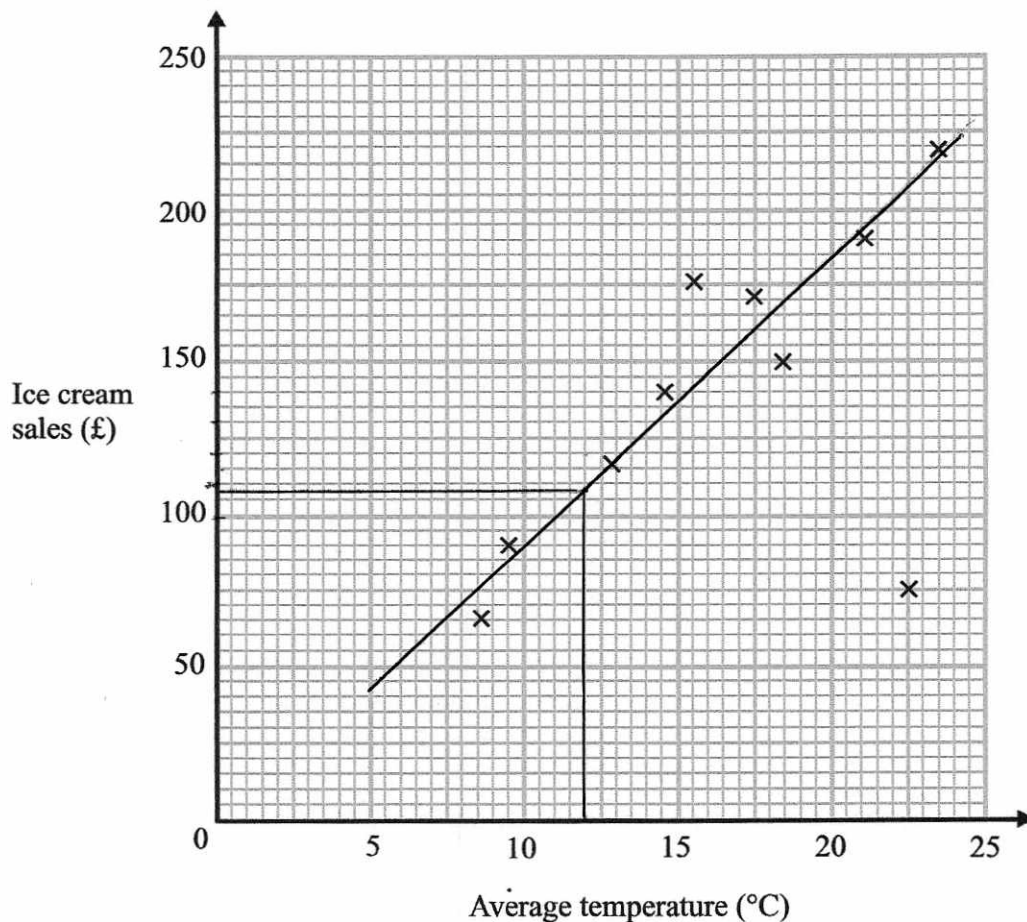
Angles on a straight line
add to 180°

.....149..... $^\circ$

(Total for question 3 is 3 marks)

- 4 The average daytime temperature for 10 days is recorded.
A shop also records its ice cream sales for each of the 10 days.

The scatter graph shows this information.



- (a) What type of correlation does the scatter graph show?

positive (1)

- (b) One of the points is an outlier. Write down the coordinates for this point.

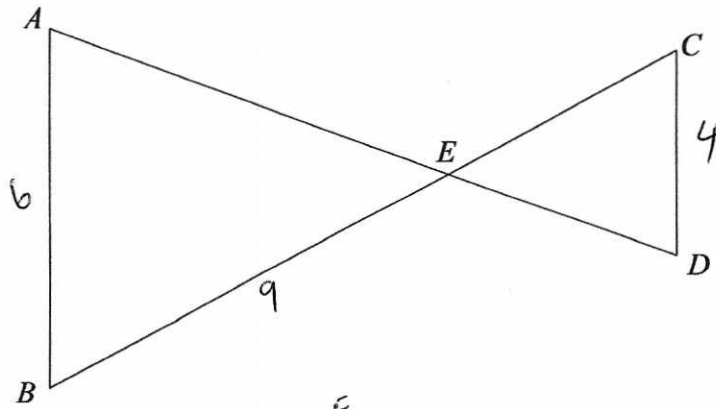
(22.5), (75) (1)

- (c) On another day the temperature was 12°. Estimate the ice cream sales on this day.

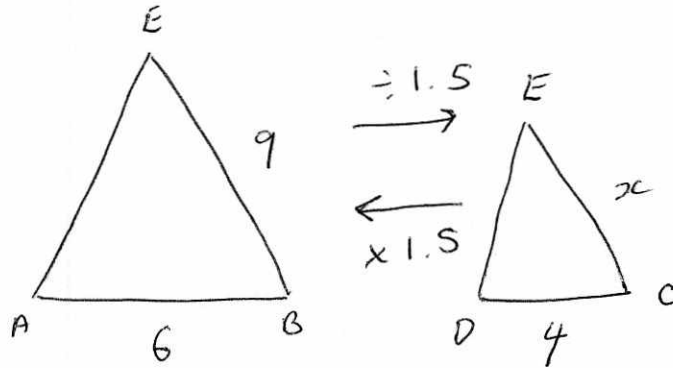
£ 107.50
£100 to £120⁽²⁾

(Total for question 4 is 4 marks)

5



AB and CD are parallel lines.
 AD and BC are straight lines
 $AB = 6$ cm,
 $CD = 4$ cm,
 $BE = 9$ cm,



(a) Find the length of CE

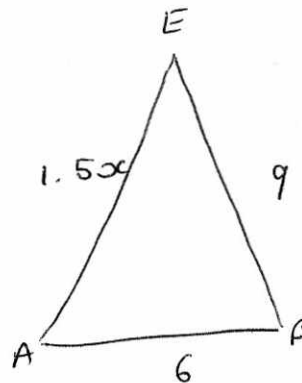
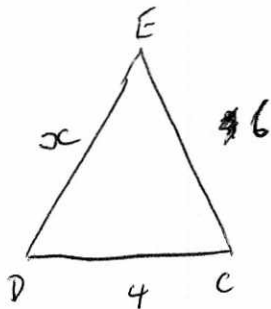
$$\frac{6}{4} = 1.5$$

$$\frac{9}{1.5} = \frac{18}{3} = 6$$

.....6.....cm
 (1)

$AD = 12.5$ cm

(b) Find the length of AE



$$x + 1.5x = 12.5$$

$$2.5x = 12.5$$

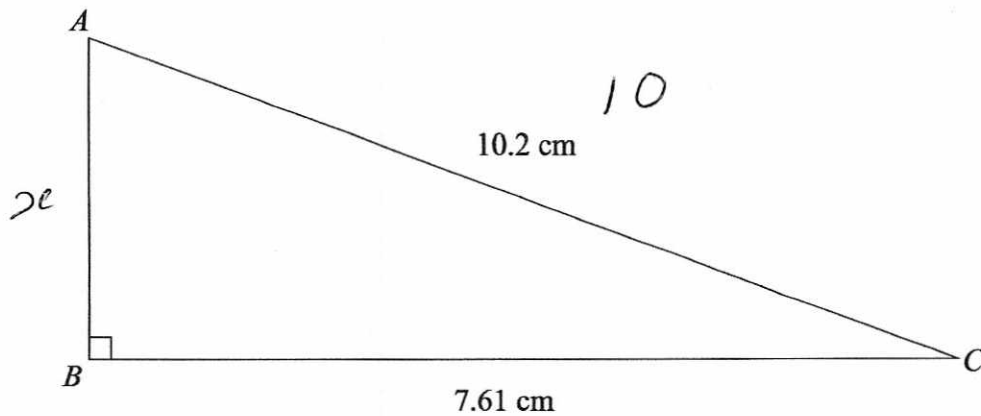
$$\underline{\underline{x = 5}}$$

$$\underline{\underline{1.5x = 7.5}}$$

.....7.5.....cm
 (2)

(Total for question 5 is 3 marks)

6



(a) Estimate the length of AB

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 8^2 + x^2 &= 10^2 \\
 64 + x^2 &= 100 \\
 x^2 &= 36 \\
 x &= 6
 \end{aligned}$$

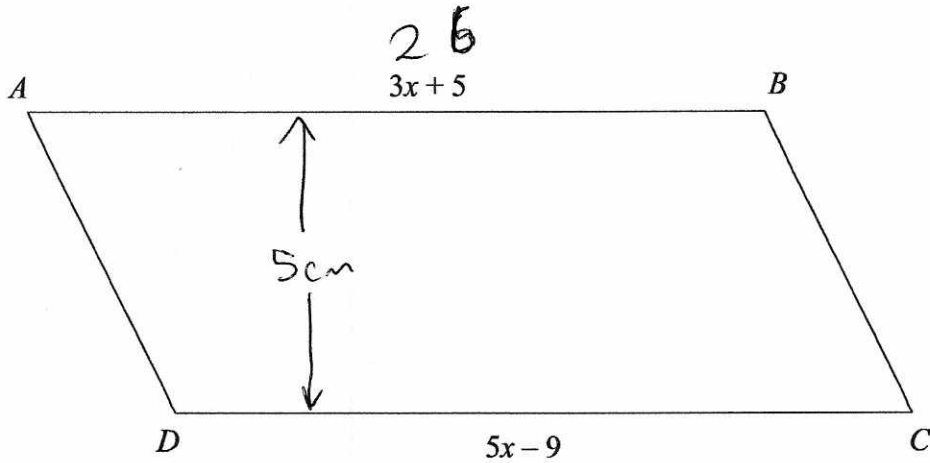
.....6.....cm
(2)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer.

.....underestimate. Rounded 10.2 down and
.....7.61 up. Bigger number - Smaller ^{number} = Bigger Answer.....
(1)

(Total for question 6 is 3 marks)

7



$ABCD$ is a parallelogram
 All measurements are in centimetres.
 The height of the parallelogram is 5cm.

Find the area of $ABCD$

$$\begin{aligned} 3x + 5 &= 5x - 9 \\ -3x & \quad -3x \\ 5 &= 2x - 9 \\ +9 & \quad +9 \end{aligned}$$

$$14 = 2x$$

$$x = 7$$

$$3(7) + 5 = 26$$

$$26 \times 5$$

$$130 \text{ cm}^2$$

(Total for question 7 is 4 marks)

8 Solve the simultaneous equations

$$5x + 2y = 24 \quad \times 3$$

$$3x - y = 21 \quad \times 5$$

$$15x + 6y = 72$$

$$15x - 5y = 105$$

$$11y = -33$$

$$y = -3$$

$$3x - (-3) = 21$$

$$3x + 3 = 21$$

$$3x = 18$$

$$x = 6$$

$$x = 6$$

$$y = -3$$

(Total for question 8 is 3 marks)

9 A shop decreases prices by 10% and then by a further 20%.

Rachel says: "Prices have now decreased by 30%".

Is Rachel correct?

You must show your working.

£100

10% DECREASE
 $10\% = \text{£}10$
 $\text{£}100 - \text{£}10 = \text{£}90$

20% DECREASE
 $10\% = \text{£}9$
 $20\% = \text{£}18$
 $\text{£}90 - \text{£}18 = \underline{\underline{\text{£}72}}$

30% DECREASE
 £100

10% = £10
 30% = £30
 $\text{£}100 - \text{£}30 = \underline{\underline{\text{£}70}}$

Prices have not decreased by 30%.
 Rachel is wrong

(Total for question 9 is 2 marks)

10 In a box there are black pens, red pens and green pens.

The ratio of black pens to pens that are not black is 9:11

The ratio of green pens to pens that are not green is 3:5

Find the ratio of black pens to red pens to green pens.

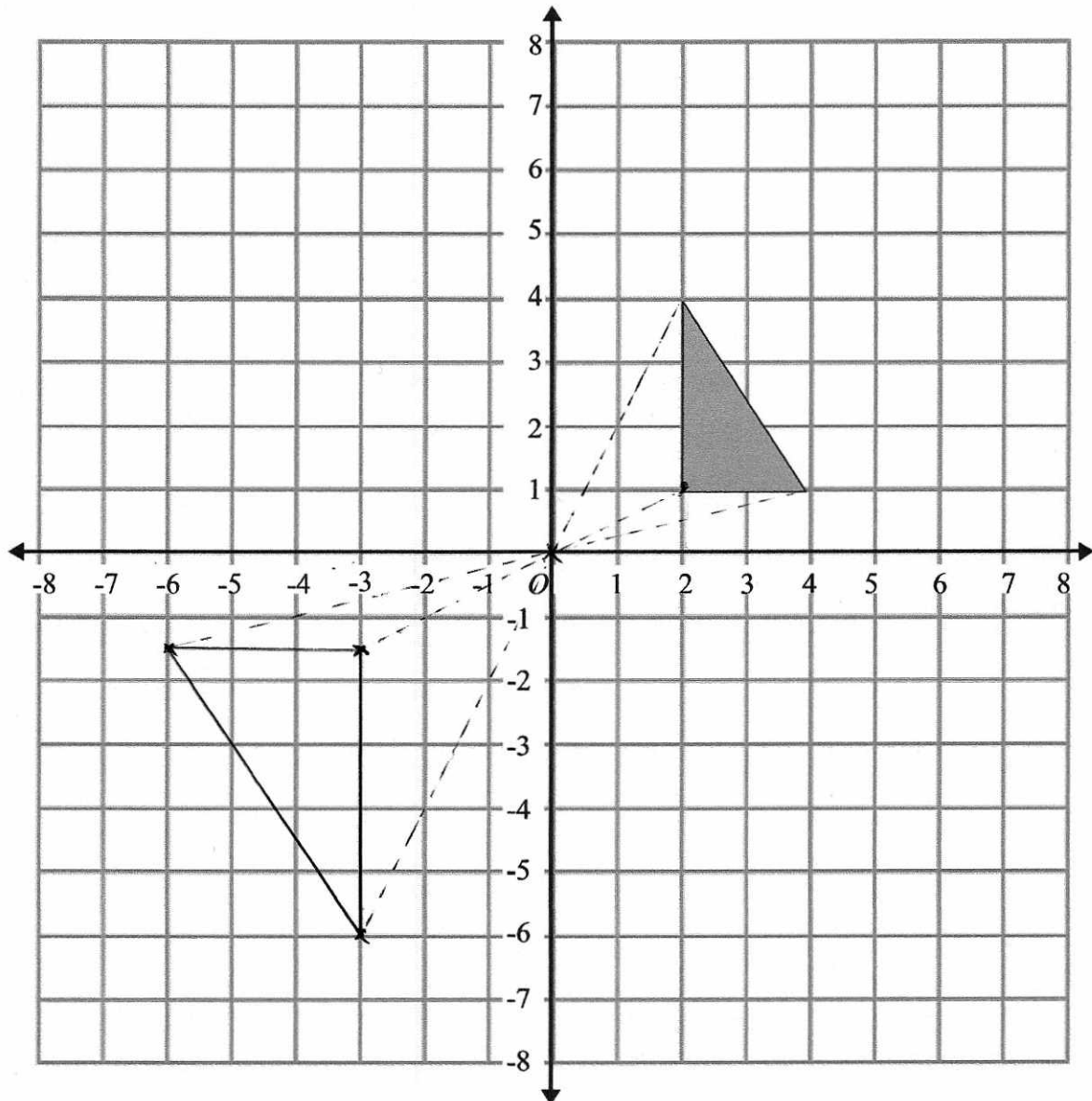
Black	Red	Green
9		11
	5	3
18	22	
25		15
18	7	15

20 parts x 2
 8 parts x 5
 40 parts
 40 parts

18 : 7 : 15

(Total for question 10 is 3 marks)

11



On the grid, enlarge the triangle by scale factor -1.5 , centre O .

(Total for question 11 is 2 marks)

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \times -1.5 = \begin{pmatrix} -3 \\ -1.5 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} \times -1.5 = \begin{pmatrix} -6 \\ -1.5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \times -1.5 = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

12 (a) Write down the value of $64^{\frac{1}{2}}$

8

(1)

(b) Write down the value of 16^0

1

(1)

(c) Work out the value of $\left(\frac{125}{8}\right)^{-\frac{2}{3}}$

$$\left(\frac{5}{2}\right)^{-2}$$

$$\left(\frac{25}{4}\right)^{-1}$$

$$\frac{4}{25}$$

(2)

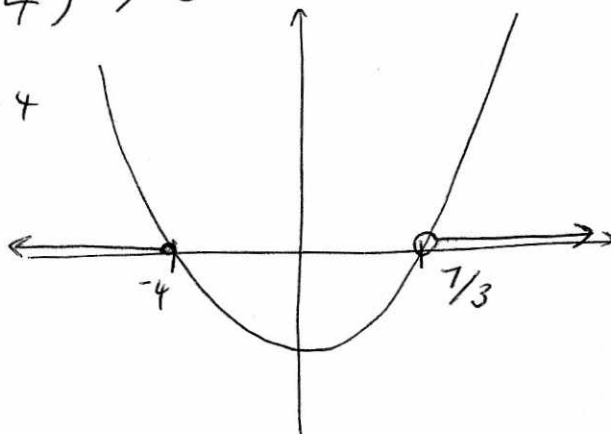
(Total for question 12 is 4 marks)

13 Solve $3x^2 + 5x - 28 > 0$

$$(3x - 7)(x + 4) > 0$$

$$x = \frac{7}{3}$$

$$x = -4$$



$$x < -4 \quad \text{or} \quad x > \frac{7}{3}$$

(Total for question 13 is 3 marks)

- 14 Find the value of $0.\dot{5}\dot{3} + 0.\dot{2}$
Give your answer as a fraction in its simplest form.

$$\begin{aligned} 0.\dot{5}\dot{3} &= x \\ 53.\dot{5}\dot{3} &= 100x \\ 53 &= 99x \\ x &= \frac{53}{99} \end{aligned}$$

$$\begin{aligned} 0.\dot{2} &= y \\ 2.\dot{2} &= 10y \\ 2 &= 9y \\ y &= \frac{2}{9} \end{aligned}$$

$$\frac{53}{99} + \frac{2}{9} \times 11$$

$$\frac{53}{99} + \frac{22}{99}$$

$$\frac{75}{99} = \frac{25}{33}$$

$$\frac{25}{33}$$

(Total for question 14 is 3 marks)

- 15 Make x the subject of the formula $a = \frac{x+4}{2x-1}$

$$a(2x-1) = x+4$$

$$2ax - a = x + 4$$

$$2ax = x + 4 + a$$

$$2ax - x = 4 + a$$

$$x(2a-1) = 4+a$$

$$x = \frac{4+a}{2a-1}$$

$$x = \frac{4+a}{2a-1}$$

(Total for question 15 is 3 marks)

- 16 (a) Prove algebraically that the difference between the squares of two consecutive numbers is always odd.

Two consecutive numbers n and $n+1$

Difference of squares

$$\begin{aligned} & (n+1)^2 - n^2 \\ & (n+1)(n+1) - n^2 \\ & n^2 + 2n + 1 - n^2 \end{aligned}$$

$$\underline{\underline{2n + 1}}$$

$2n$ is even

even + 1 is odd.

(2)

- (b) Use your answer to (a) to work out $72^2 - 71^2$

$$n = 71$$

$$2n + 1$$

$$2(71) + 1$$

$$\begin{array}{r} \dots\dots\dots 143 \\ \dots\dots\dots (1) \end{array}$$

(Total for question 16 is 3 marks)

- 17 Find the coordinates of the turning point of the curve with the equation $y = x^2 - x + 8$

You must show all your working.

$$\begin{aligned} y &= x^2 - x + 8 \\ &= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 8 \\ &= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 8 \\ &= \left(x - \frac{1}{2}\right)^2 + 7.75 \end{aligned}$$

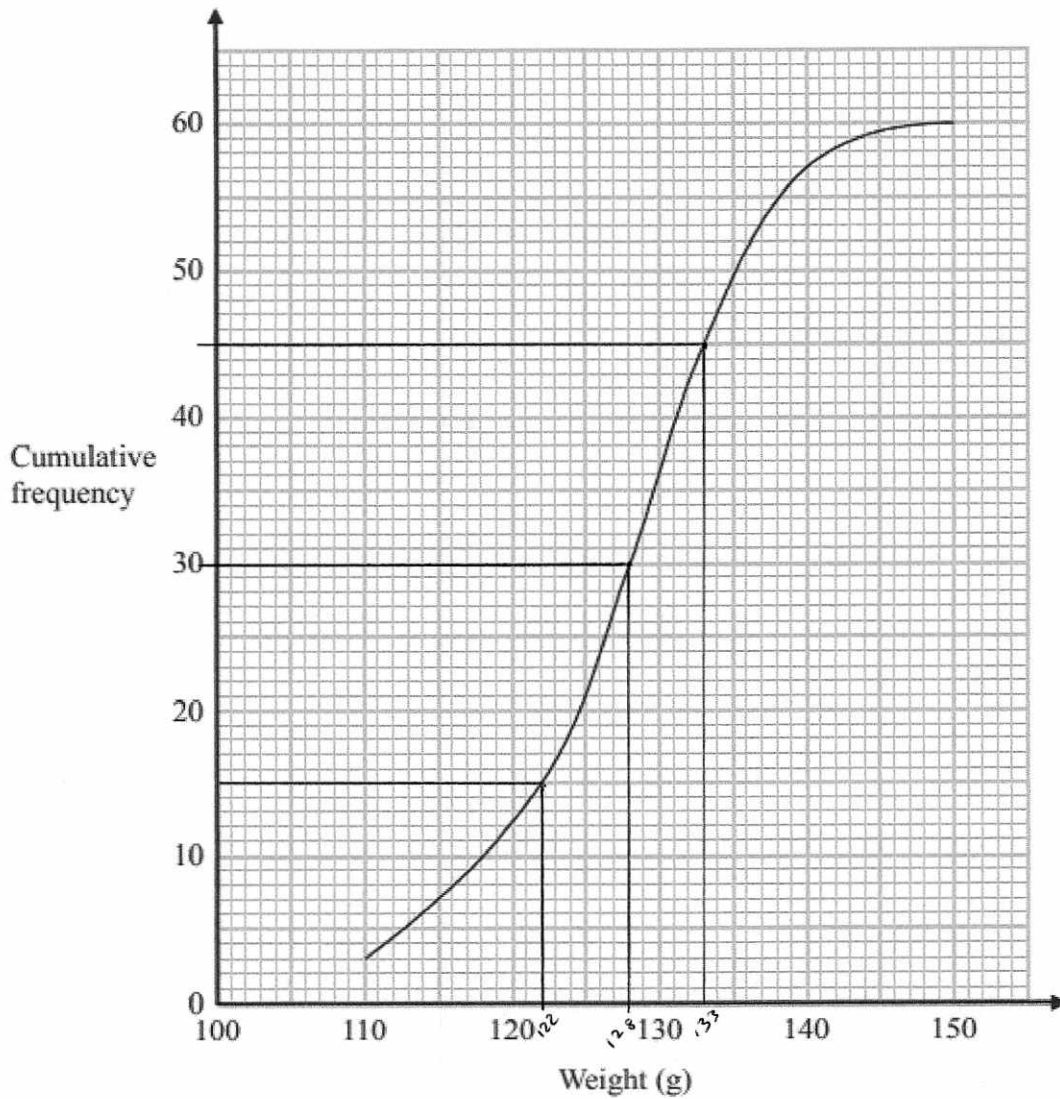
At turning point is when $\left(x - \frac{1}{2}\right) = 0$

$$x = \frac{1}{2} \quad y = 7.75$$

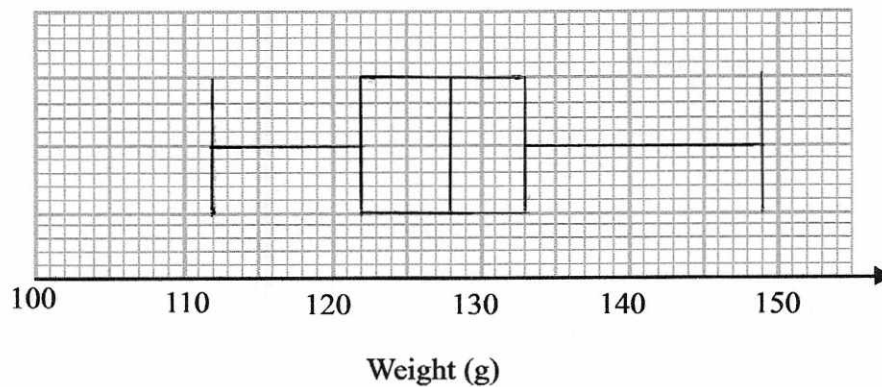
$$(\dots\dots\dots 0.5), (\dots\dots\dots 7.75)$$

(Total for question 17 is 3 marks)

18 The cumulative frequency graph shows the weight, in grams, of 60 pears.



The 60 pears had a minimum weight of 112 grams and a maximum weight of 149 grams. Draw a box plot to show the distribution of the weights of the pears.



(Total for question 18 is 3 marks)

19 a is directly proportional to the square of b

When $a = 12$, $b = 6$

Find a value of b when $a = 15$

Give your answer as a simplified surd.

$$\begin{aligned} a &= kb^2 \\ 12 &= k(6)^2 \\ 12 &= 36k \\ k &= \frac{12}{36} \\ &= \frac{1}{3} \end{aligned}$$

$$a = \frac{1}{3}b^2$$

$$\begin{aligned} 15 &= \frac{1}{3}b^2 \\ 45 &= b^2 \\ b &= \sqrt{45} \\ &= \sqrt{9} \sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

$$b = \dots 3\sqrt{5} \dots$$

(Total for question 19 is 3 marks)

20 Write $\frac{\sqrt{8} + \sqrt{18}}{\sqrt{2} - 1}$ in the form $a + b\sqrt{2}$ where a and b are integers

$$\begin{aligned} \sqrt{8} &= \sqrt{4} \times \sqrt{2} = 2\sqrt{2} \\ \sqrt{18} &= \sqrt{9} \times \sqrt{2} = 3\sqrt{2} \end{aligned}$$

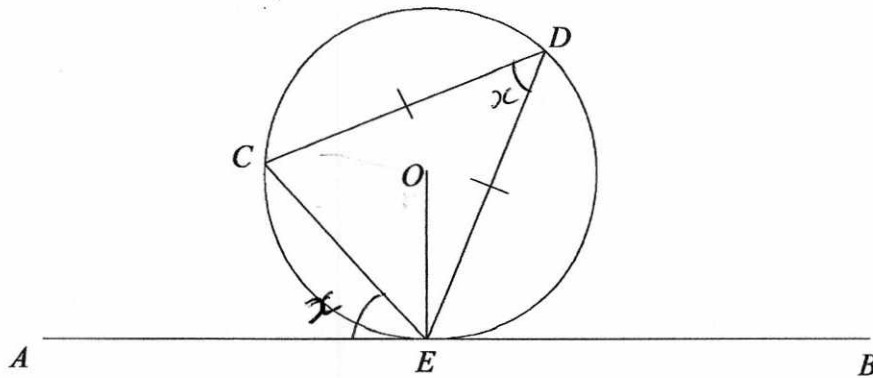
$$\frac{(2\sqrt{2} + 3\sqrt{2})(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$

$$\frac{4 + 2\sqrt{2} + 6 + 3\sqrt{2}}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$\frac{10 + 5\sqrt{2}}{1}$$

$$\underline{\underline{10 + 5\sqrt{2}}}$$

(Total for question 20 is 3 marks)



C, D and E are points on a circle, centre O .
 AEB is a tangent to the circle at E .

$CD = DE$
 Angle $AEC = x^\circ$

Find the size of angle OED in terms of x .

$CDE = x$ Alternate segment theorem

DCE and $DEC = \frac{180 - x}{2}$ Angles at the base of an isosceles triangle are equal

$OEA = 90^\circ$ Tangent meets radius

$OEC = 90 - x$

$$\begin{aligned} OED &= \frac{180 - x}{2} - (90 - x) \\ &= 90 - \frac{1}{2}x - 90 + x \\ &= \frac{1}{2}x \end{aligned}$$

$$\frac{1}{2}x$$

(Total for question 21 is 4 marks)

- 22 The line l_1 passes through the points (x_1, y_1) and (x_2, y_2)
The line l_2 has the equation $4x - 2y = 3$

Show that lines l_1 and l_2 are perpendicular.

$$\begin{aligned}l_1: m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 3}{12 - 2} \\ &= \frac{-5}{10} \\ &= \underline{\underline{-\frac{1}{2}}}\end{aligned}$$

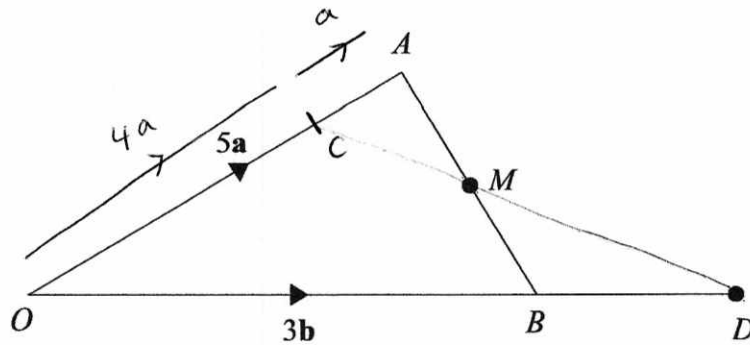
$$\begin{aligned}l_2: 4x - 2y &= 3 \\ 4x &= 2y + 3 \\ 4x - 3 &= 2y \\ 2x - \frac{3}{2} &= y \\ y &= 2x - \frac{3}{2}\end{aligned}$$

$$m = 2$$

If perpendicular gradients multiply to give -1 .

$$-\frac{1}{2} \times 2 = -1 \quad \therefore \text{perpendicular.}$$

(Total for question 22 is 5 marks)



$$\vec{OA} = 5a$$

$$\vec{OB} = 3b$$

C is the point such that $OC:CA = 4:1$

M is the midpoint of AB

OBD is a straight line

$$\vec{OD} = k\vec{OB} \text{ where } k \text{ is a scalar quantity.}$$

Given that CMD is a straight line, find the value of k .

$$\vec{AB} = -5a + 3b$$

$$\vec{AM} = -\frac{5}{2}a + \frac{3}{2}b$$

$$\begin{aligned} \vec{CM} &= a - \frac{5}{2}a + \frac{3}{2}b \\ &= -\frac{3}{2}a + \frac{3}{2}b \end{aligned}$$

$$\begin{aligned} \vec{CD} &= -4a + k(3b) \\ &= -4a + 3kb \end{aligned}$$

CMD is a straight line

$$\therefore \vec{CD} = x\vec{CM}$$

$$-4a + 3kb = x\left(-\frac{3}{2}a + \frac{3}{2}b\right)$$

$$-4a + 3kb = -\frac{3}{2}xa + \frac{3}{2}xb$$

a only

$$-4 = -\frac{3}{2}x$$

$$-8 = -3x$$

$$x = \frac{8}{3}$$

b only

$$3k = \frac{3}{2}\left(\frac{8}{3}\right)$$

$$3k = 4$$

$$k = \frac{4}{3}$$

24 There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 4:1.

$$4x : x$$

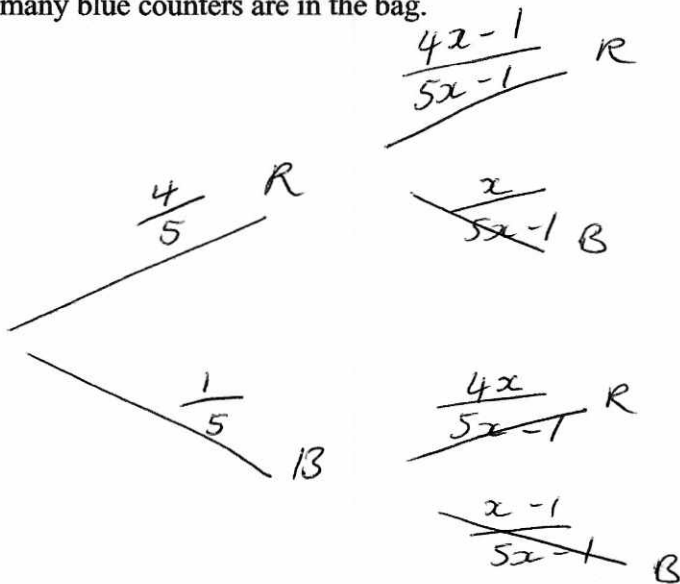
Two counters are removed at random.

5x counters in total

The probability that both the counters taken are red is $\frac{22}{35}$

Work how many blue counters are in the bag.

$$\frac{22}{35}$$



$$\frac{4}{5} \times \frac{4x-1}{5x-1} = \frac{22}{35}$$

$$\frac{16x-4}{25x-5} = \frac{22}{35}$$

$$35(16x-4) = 22(25x-5)$$

$$140(4x-1) = 110(5x-1)$$

$$14(4x-1) = 11(5x-1)$$

$$56x-14 = 55x-11$$

$$x-14 = -11$$

$$x = 3$$

3

(Total for question 24 is 5 marks)