Edexcel GCE

Core Mathematics M1

Moments

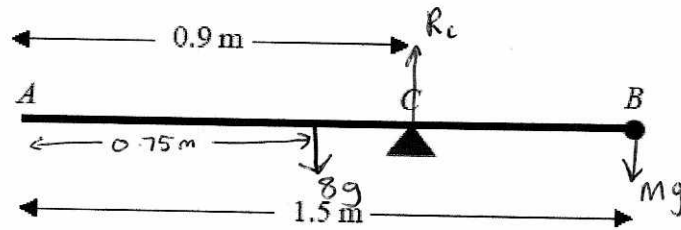| **Materials required for examination**	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1.

Figure 3

A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where $AC = 0.9$ m, and the system is in equilibrium with AB horizontal, as shown in Figure 2.

(a) Show that $m = 2$.

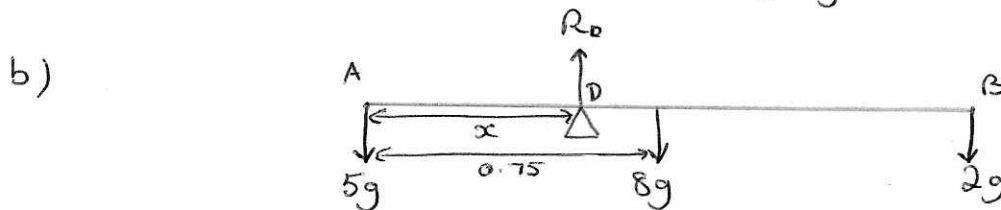**(4)**

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(b) Find the distance AD.

(5)

a) Taking moments about C:

clockwise moments = anticlockwise moments

$$0.6mg = 0.15(8g)$$
$$0.6mg = 1.2g$$
$$m = \frac{1.2g}{0.6g} = \underline{\underline{2}}$$
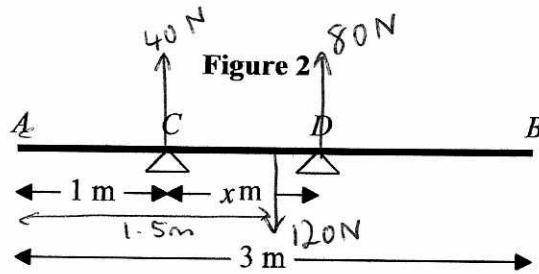
b)

$R_D = 15g$ (Forces up = forces down)

Taking moments about A:

$$0.75(8g) + 1.5(2g) = x(15g)$$
$$9g = x \cdot 15g$$
$$x = \frac{9}{15} = \frac{3}{5} = \underline{\underline{0.6\ m}}$$

2

2.

Figure 2

40 N 80 N

A   C   D   B

← 1 m → ← x m →

1.5m

120 N

3 m

A uniform plank *AB* has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports *C* and *D*, where *AC* = 1 m and *CD* = *x* m, as shown in Figure 2. The reaction of the support on the plank at *D* has magnitude 80 N. Modelling the plank as a rod,

(*a*) show that *x* = 0.75.

(3)

A rock is now placed at *B* and the plank is on the <u>point of tilting about *D*.</u> Modelling the rock as a particle, find $R_C = 0$

(*b*) the weight of the rock,

(4)

(*c*) the magnitude of the reaction of the support on the plank at *D*.

(2)

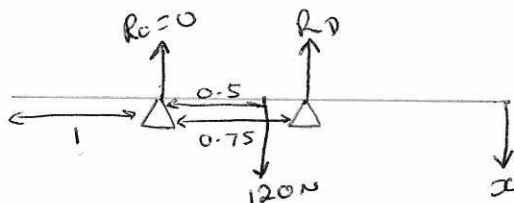(*d*) State how you have used the model of the rock as a particle.

(1)

a) $R_C = 40N$ (Forces up = Forces down)

Taking moments about C:

$$0.5(120) = x(80)$$
$$60 = x(80)$$
$$x = \frac{6}{8} = 0.75$$

b)

$R_C = 0$ R_D

0.5

1   0.75

120 N   x

Taking moments about D

$$1.25x = 0.25(120)$$
$$1.25x = 30$$
$$x = \underline{24 N}$$

c) Forces up = forces down

$$R_D = 120 + 24$$
$$= \underline{144 N}$$ 3

d) The weight acts at one point.

3.

Figure 1
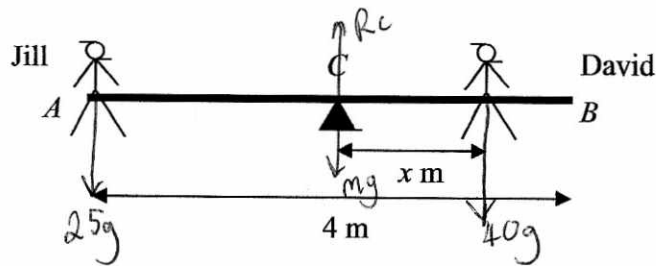
A seesaw in a playground consists of a beam AB of length 4 m which is supported by a smooth pivot at its centre C. Jill has mass 25 kg and sits on the end A. David has mass 40 kg and sits at a distance x metres from C, as shown in Figure 1. The beam is initially modelled as a uniform rod. Using this model,

(a) find the value of x for which the seesaw can rest in equilibrium in a horizontal position.

(3)

(b) State what is implied by the modelling assumption that the beam is uniform.
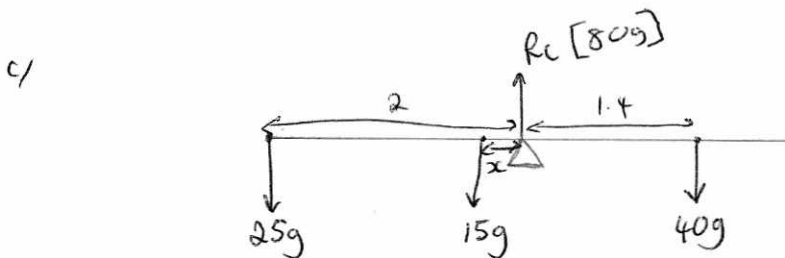
(1)

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from C for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

(c) find the distance of the centre of mass of the beam from C.

(4)

a) Taking moments about C

$$x(40g) = 2(25g)$$
$$40x = 50$$
$$x = 1.25 \text{ m}$$

b/ The weight of the beam acts at the centre.

c/

$R_C = 80g$

Forces up = Forces down

Taking moments about C

$$1.4(40g) = x(15g) + 2(25g)$$
$$56g = 15g \, x + 50g$$
$$6g = 15g \, x$$
$$6 = 15x$$
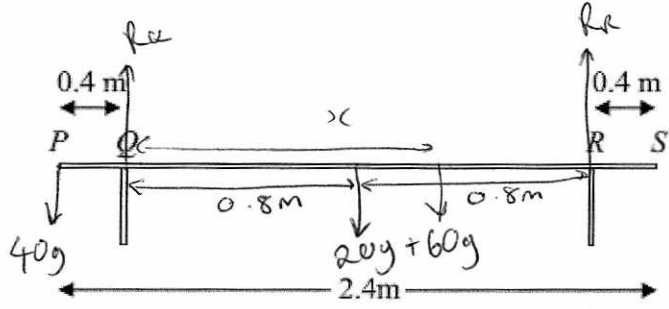$$x = \frac{6}{15} = \frac{2}{5} = 0.4 \text{ m}$$

4.

Figure 1

A bench consists of a plank which is resting in a horizontal position on two thin vertical legs. The plank is modelled as a uniform rod PS of length 2.4 m and mass 20 kg. The legs at Q and R are 0.4 m from each end of the plank, as shown in Figure 1.

Two pupils, Arthur and Beatrice, sit on the plank. Arthur has mass 60 kg and sits at the middle of the plank and Beatrice has mass 40 kg and sits at the end P. The plank remains horizontal and in equilibrium. By modelling the pupils as particles, find

(a) the magnitude of the normal reaction between the plank and the leg at Q and the magnitude of the normal reaction between the plank and the leg at R.

(7)

Beatrice stays sitting at P but Arthur now moves and sits on the plank at the point X. Given that the plank remains horizontal and in equilibrium, and that the magnitude of the normal reaction between the plank and the leg at Q is now twice the magnitude of the normal reaction between the plank and the leg at R,

(b) find the distance QX.

(6)

a) Taking moments about Q

$$0.8(80g) = 1.6 R_R + 0.4(40g)$$
$$64g = 1.6 R_R + 16g$$
$$48g = 1.6 R_R$$
$$R_R = 30g$$
$$R_Q = 90g \quad (\text{forces up} = \text{forces down})$$

b/

Taking moments about Q

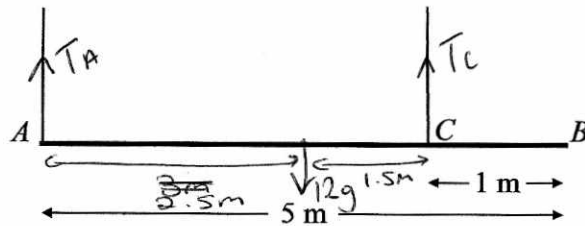$$0.8(20g) + x(60g) = 1.6(40g) + 0.4(40g)$$
$$16g + 60g \, x = 64g + 16g$$
$$60x = 64$$
$$x = \frac{64}{60} = \frac{16}{15} \text{ m}$$

5.

Figure 2

A beam *AB* has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to *A*, the other to the point *C* on the beam, where *BC* = 1 m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(*a*) Find

    (i)   the tension in the rope at *C*,

    (ii) the tension in the rope at *A*.

(5)

A small load of mass 16 kg is attached to the beam at a point which is *y* metres from *A*. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(*b*) find, in terms of *y*, an expression for the tension in the rope at *C*.

(3)

The rope at *C* will break if its tension exceeds 98 N. The rope at *A* cannot break.

(*c*) Find the range of possible positions on the beam where the load can be attached without the rope at *C* breaking.

(3)

a i/ Taking Moments about A

$$2.5(12g) = 4T_c$$
$$30g = 4T_c$$
$$\underline{7.5g = T_c}$$

ii/ $T_A = 12g - 7.5g$
$$= \underline{4.5g}$$

b/ Taking moments about A:

$$2.5(12g) + y(16g) = 4T_c$$
$$30g + 16g(y) = 4T_c$$
$$\underline{7.5g + 4g\,y = T_c}$$

c/ $7.5g + 4g\,y \leq 98$
$$4g\,y \leq 24.5$$
$$\underline{y \leq \tfrac{5}{8}\,m}$$

$$\underline{0 \leq y \leq \tfrac{5}{8}}$$

6

6.
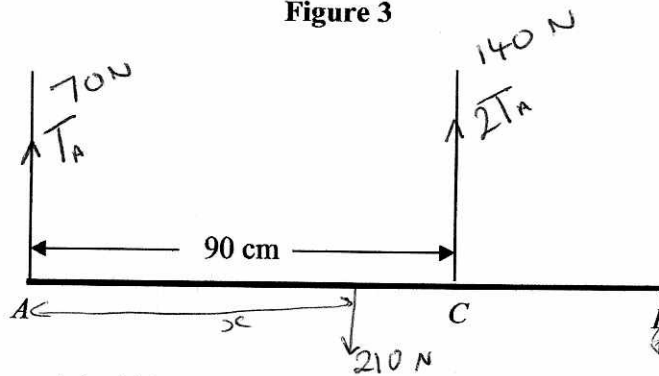
Figure 3

A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other able is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(a) the tension in the cable at A,

(2)

(b) show that $AB = 120$ cm.

(4)

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.
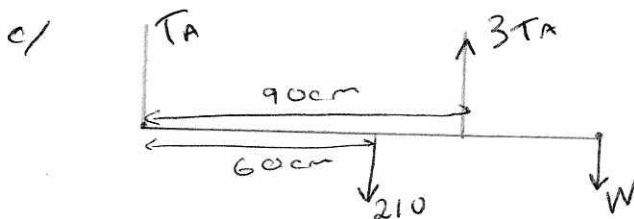
(c) Find the value of W.

(7)

a/ $\quad T_A + 2T_A = 210$

$\qquad 3T_A = 210,$

$\qquad T_A = 70$ N

b/ $\quad$ moments about A.

$\qquad 210x = 90(140)$

$\qquad x = 60$ cm

$\qquad \therefore AB = 120$ cm $\quad$ (uniform rod $\Rightarrow$ weight acts at centre)

c/

$4T_A = 210 + W$

$T_n = \dfrac{210 + W}{4}$

Taking moments about C

$\qquad 30W + \cancel{90}\left(\dfrac{210 + W}{4}\right) = 30(210)$

$\qquad 120W + 90(210 + W) = 25200$

$\qquad \cancel{+210W = 24990}$

$\qquad 120W + 18900 + 90W = 25200$

$\qquad 210W = 6300$

$\qquad W = 30$ N

7.

Figure 2
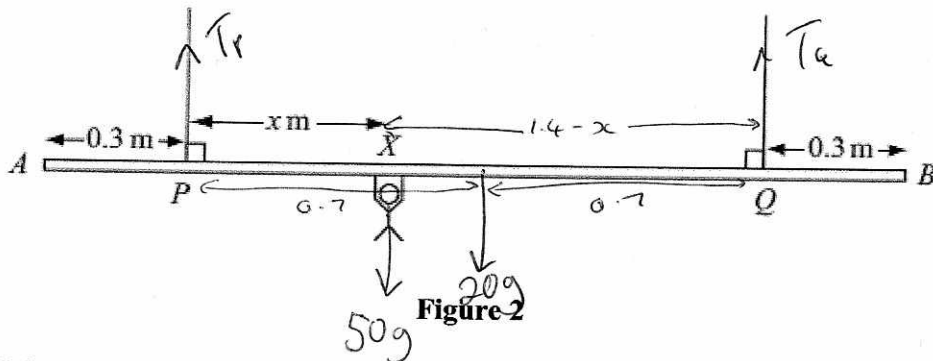
A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(b) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x.

(3)

Taking moments about Q

$$1.4(T_P) = 0.7(20g) + (1.4 - x)(50g)$$
$$1.4\, T_P = 137.2 + 686 - 490x$$
$$1.4\, T_P = 823.2 - 490x$$
$$\underline{T_P = 588 - 350x} \quad N$$

b/ $T_a = 70g - (588 - 350x)$
$$= \underline{98 + 350x}$$

c/ ~~588~~ ~~≤ T_P ≤ 58 8~~
~~98~~ $0 < x < 1.4$
                       $98 < T_P < 588$
~~98 ≤ T_a ≤ 588~~ $98 < T_a < 588$

d/ $98 + 350x = 3(588 - 350x)$
$$98 + 350x = 1764 - 1050x$$
$$1400x = 1666$$
$$\underline{x = 1.19\, m}$$

8.

8)Figure 2
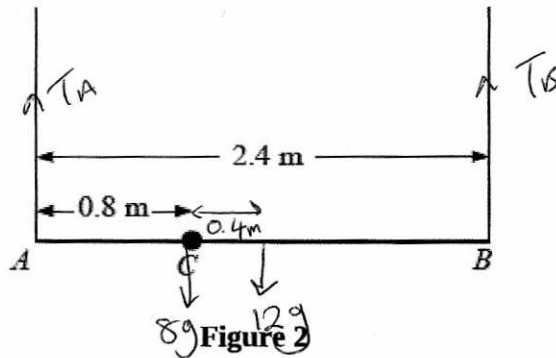
A plank AB has mass 12 kg and length 2.4 m. A load of mass 8 kg is attached to the plank at the point C, where $AC = 0.8$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in Figure 2. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.
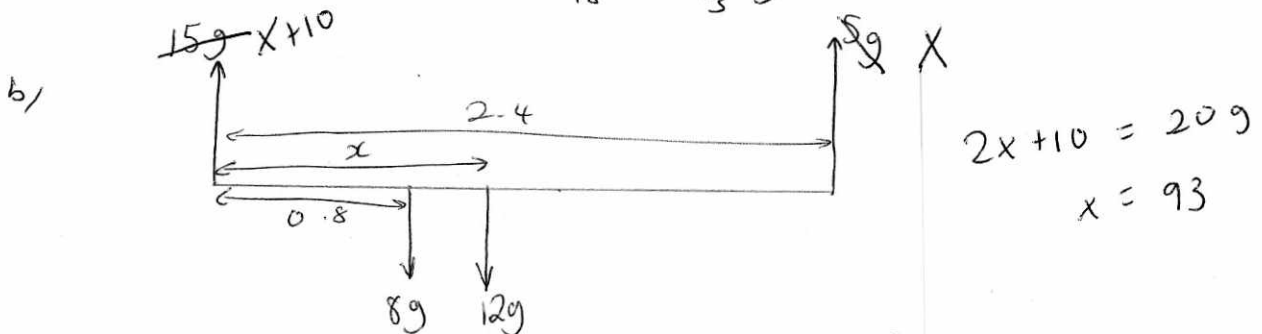
(a) Find the tension in the rope attached at B.

(4)

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(b) Find the distance of the centre of mass of the plank from A.

(6)

Taking moments about A:

$$0.8(8g) + 1.2(12g) = 2.4\, T_B$$
$$6.4g + 14.4g = 2.4\, T_B$$
$$20.8g = 2.4\, T_B$$
$$T_B = \frac{26}{3}g$$

b/

$$2x + 10 = 20g$$
$$x = 93$$

Taking moments about A:

$$0.8(8g) + x(12g) = 2.4\left(\frac{5}{5}g\right)$$
$$6.4g + 12xg = \quad 223.2$$
$$x = \frac{7}{5} \not x$$
$$\underline{\underline{1.4\,m}}$$

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