Name: ______________________________

GCSE (1 – 9)

Iteration

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– there may be more space than you need.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1. The equation $x^3 + 7x - 2 = 55$ has a solution between 3 and 4.

Use trial and improvement to find this solution.
Give your answer to 1 decimal place.

x	$x^3 + 7x - 2$	comment
3.5	$(3.5)^3 + 7(3.5) - 2$ $= 65.375$	too big
3.3	$(3.3)^3 + 7(3.3) - 2$ $= 57.037$	too big
3.2	$(3.2)^3 + 7(3.2) - 2$ $= 53.168$	too small
3.25	$(3.25)^3 + 7(3.25) - 2$ $= 55.078125$	too big

......3.2...... (4)

2. Use trial and improvement to solve $x^3 - x^2 = 85$

Give your answer to 1 decimal place.

x	$x^3 - x^2$	comment
5	$(5)^3 - (5)^2 = 100$	too big
4.8	$(4.8)^3 - (4.8)^2 = 87.552$	too big
4.7	$(4.7)^3 - (4.7)^2 = 81.733$	too small
4.75	$(4.75)^3 - (4.75)^2 = 84.609375$	too small

4.8 .... (4)

3. Use trial and improvement to solve $x^3 + 5x = 70$

Give your answer to 1 decimal place.

x	$x^3 + 5x$	comment
4	$(4)^3 + 5(4)$ $= 84$	too high
3.8	$(3.8)^3 + 5(3.8)$ $= 73.872$	too high
3.7	$(3.7)^3 + 5(3.7)$ $= 69.153$	too low
3.75	$(3.75)^3 + 5(3.75)$ $= 71.484375$	too high

3.7 .... (4)

4. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \sqrt{(x_n) + 10} \quad \text{and} \quad x_1 = 3$$

a) Work out the values of x_2 and x_3

$x_2 = \sqrt{(3) + 10} = \sqrt{13} = 3.61$ (2dp)

$x_3 = \sqrt{(\sqrt{13}) + 10} = 3.69$ (2dp)

$\sqrt{13}$ 3.69 (2dp) (2)

b) Work out the solution to 3 decimal places

$x_4 = 3.6998068$

$x_5 = 3.701325006$

$x_6 = 3.70153009$

$x_7 = 3.701557792$

$x_8 = 3.701561534$

3.702 (1)

5. An approximate solution to an equation is found using this iterative process:

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = -1$$

a) Work out the values of x_2 and x_3

$$x_2 = \frac{(-1)^3 - 3}{8} = -\frac{1}{2}$$

$$x_3 = \frac{-25}{64}$$

$-\frac{1}{2}$ .... $-\frac{25}{64}$ .... (2)

b) Work out the solution to 6 decimal places

$x_4 = -0.3824505806$

$x_5 = -0.3819925565$

$x_6 = -0.3819674637$

$x_7 = -0.3819660907$

$x_8 = -0.3819660156$

-0.381966 .... (1)

6. A sequence is defined by the term-to-term rule:

$$U_{n+1} = U_n^2 - 8U_n + 17$$

a) Given that $U_1 = 4$, find U_2 and U_3

$U_2 = (4)^2 - 8(4) + 17 = 1$

$U_3 = 10$

1 .... 10 .... (2)

b) Given instead that $U_1 = 2$, find U_2, U_3 and U_{100}

$U_2 = 5$

$U_3 = 2$

$U_4 = 5$

$U_2 = 5$ .... $U_3 = 2$ .... $U_{100} = 5$ .... (3)

7.(a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 4x - 1 = 0$$

$$(0)^3 + 4(0) - 1 = -1$$

$$(1)^3 + 4(1) - 1 = 4$$

change of sign ∴ there is a solution between 0 and 1 .... (2)

(b) Show that the equation $x^3 + 4x = 1$ can be rearranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$x^3 + 4x = 1$$

$$4x = 1 - x^3$$

$$x = \frac{1}{4} - \frac{x^3}{4}$$

.... (1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate to the solution of $x^3 + 4x = 1$

$$x_1 = \frac{1}{4} - \frac{(0)^3}{4} = \frac{1}{4}$$

$$x_2 = 0.24609375$$

$$x_3 = 0.246274009\mathrm{3}$$

.... (3)