Edexcel GCE

Core Mathematics C4

Parametric Equations

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1.

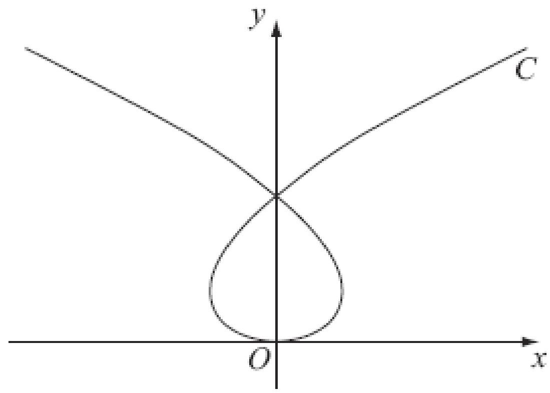


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

[January 2009]

2.

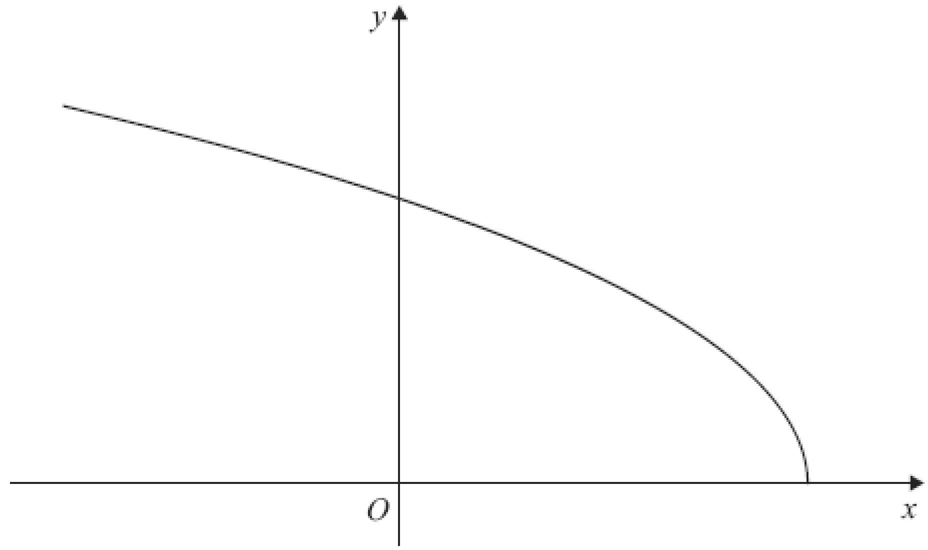


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(*a*) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(*b*) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leq x \leq k,$$

stating the value of the constant k.

(4)

(*c*) Write down the range of $\mathrm{f}(x)$.

(2)

[June 2009]

3. (*a*) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.

(2)

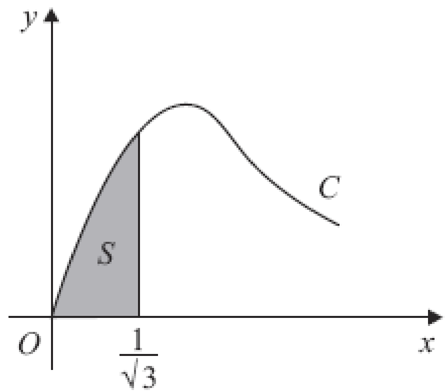


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Show that the volume of the solid of revolution formed is given by the integral

$$k \int \sin^2\theta \, d\theta ,$$

where k is a constant.

(5)

(*c*) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

[June 2009]

4.

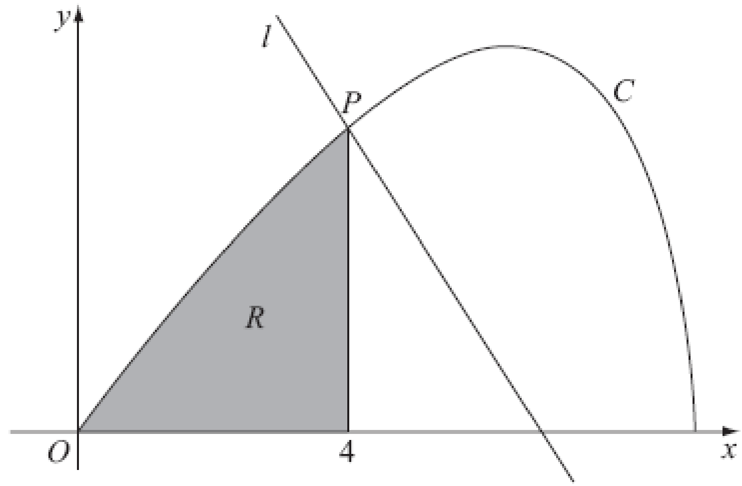


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \le t \le \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int 64\sin^2 t\cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

[June 2008]

5.

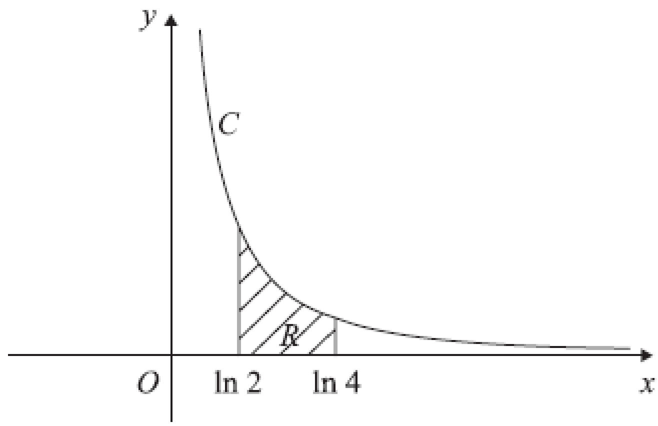


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int \frac{1}{(t+1)(t+2)}\, dt \quad .$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

[January 2008]

6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$.

(4)

[June 2007]

7. A curve has parametric equations

$$x = 7\cos t - \cos 7t, \quad y = 7\sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

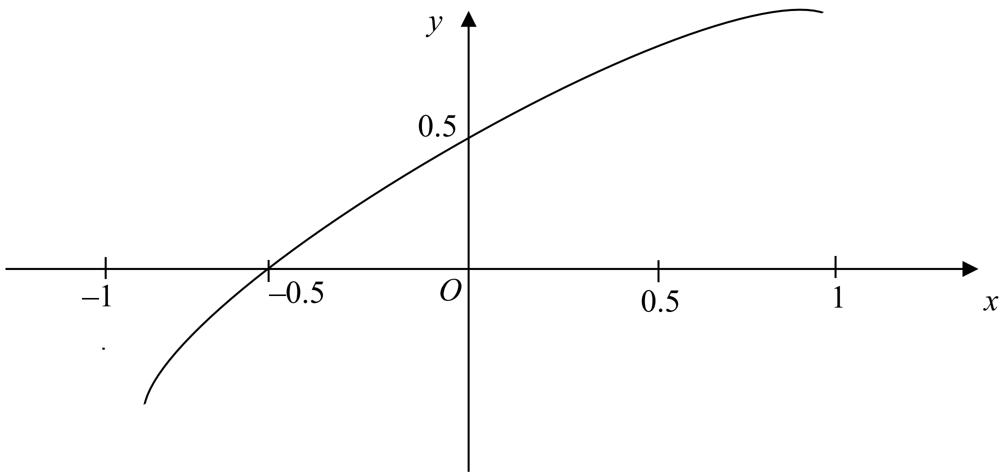
(*b*) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.

Give your answer in its simplest exact form.

(6)

[January 2007]

8. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(*a*) Find an equation of the tangent to the curve at the point where $t = \dfrac{\pi}{6}$.

(6)

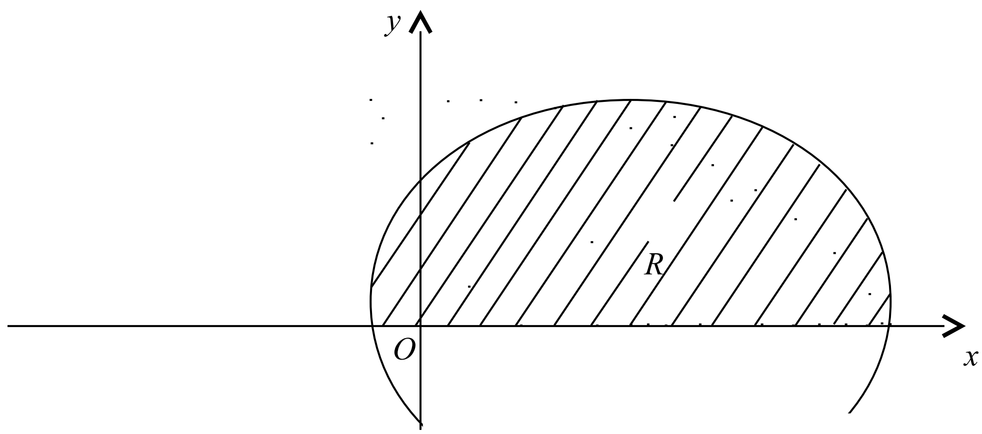
(*b*) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

[June 2006]

9. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \le t \le 2\pi.$$

(*a*) Show that the curve crosses the *x*-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region *R* is enclosed by the curve and the *x*-axis, as shown shaded in Figure 2.

(*b*) Show that the area *R* is given by the integral

$$\int (1 - 2\cos t)^2 \, dt \, .$$

(3)

(*c*) Use this integral to find the exact value of the shaded area.

(7)

[January 2006]