C4 INTEGRATION

1a)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1.84432	4.81048	8.87207	0

b) $\frac{\pi}{4}\left(1.84432 + 4.81048 + 8.87207\right)$

$= 12.1948$ units^2 (4dp)

2a)

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2

b) $0.4\left(\frac{e^0}{2} + e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28} + \frac{e^2}{2}\right)$

$= 4.922$ units^2 (4sf)

3a)

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1	1.01959	1.08239	1.20269	$\sqrt{2}$

$\frac{\pi}{16}\left(\frac{1}{2} + 1.01959 + 1.08239 + 1.20269 + \frac{\sqrt{2}}{2}\right)$

$= 0.8859$ units^2 (4dp)

b) $\frac{\ln(1+\sqrt{2}) - 0.8859}{\ln(1+\sqrt{2})} \times 100 = 0.51\%$ (2dp)

4a)

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132	1.14805	0

$= \frac{3\pi}{8}\left(\frac{3}{2} + 2.77164 + 2.12132 + 1.14805\right)$

$= \underline{8.884 \text{ units}^2}$ $\underline{\text{3dp}}$

c/ $y = 3\cos\left(\frac{x}{3}\right)$

$\int y\,dx = 9\sin\left(\frac{x}{3}\right)$

$\left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}}$

$9\sin\left(\frac{\pi}{2}\right) - 9\sin(0)$

$= \underline{9 \text{ units}^2}$

5a)

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0	0.44600	0.64359	0.81742	1

b) $\frac{\pi}{16}\left(0.44600 + 0.64359 + 0.81742 + \frac{1}{2}\right)$

$= 0.4726 \text{ units}^2$ (4dp)

c/ $\pi\int_0^{\pi/4} y^2\,dx$

$\pi\int_0^{\frac{\pi}{4}} \tan x\,dx$

$\pi\left[\ln|\sec x| + c\right]_0^{\pi/4}$

$\pi\left[(\ln\sqrt{2}) - \ln(1)\right]$

$= \underline{\pi\ln\sqrt{2}}$

6a) $y = \frac{3}{\sqrt{(1+4x)}}$

$y = 3(1+4x)^{-1/2}$

$\cancel{\pi\int y^2\,dx}$

$$\int_0^2 y \, dx = \left[\frac{3(1+4x)^{1/2}}{1/2} \times \frac{1}{4} \right]_0^2$$

$$= \left[\frac{3}{2}(1+4x)^{1/2} + c \right]_0^2$$

$$= \left[\frac{3}{2}(1+4(2))^{1/2} \right] - \left[\frac{3}{2}(1+4(0))^{\frac{1}{2}} \right]$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \underline{\underline{3}} \text{ units}^2$$

b/ $$\pi \int_0^2 y^2 \, dx = \pi \int_0^2 \left(\frac{3}{\sqrt{1+4x}} \right)^2 dx$$

$$= \pi \int_0^2 \frac{9}{1+4x} \, dx$$

$$= \pi \left[\frac{9}{4} \ln(1+4x) + c \right]_0^2$$

$$= \pi \left[\left(\frac{9}{4} \ln(1+4(2)) \right) - \left(\frac{9}{4} \ln(1+4(0)) \right) \right]$$

$$= \pi \left(\frac{9}{4} \ln 9 - \frac{9}{4} \ln 1 \right)$$

$$= \frac{9}{4} \pi \ln 9 \quad \text{units}^3$$

7a) $$\int_0^{2\pi} 3 \sin \frac{x}{2} \, dx$$

$$\left[-6 \cos \frac{x}{2} + c \right]_0^{2\pi}$$

$$(-6 \cos \pi) - (-6 \cos 0)$$

$$= \underline{\underline{12}} \text{ units}^2$$

b) $\pi \int_0^{2\pi} \left(3 \sin \frac{x}{2}\right)^2 \, dx$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\sin^2 \frac{x}{2} = \frac{1}{2} - \frac{1}{2}\cos x$

$\pi \int_0^{2\pi} 9 \sin^2 \frac{x}{2} \, dx$

$\pi \int_0^{2\pi} 9\left(\frac{1}{2} - \frac{1}{2}\cos x\right) dx$

$\pi \int_0^{2\pi} \frac{9}{2} - \frac{9}{2}\cos x \, dx$

$\pi \left[\left(\frac{9}{2}x - \frac{9}{2}\sin x + c\right)\right]_0^{2\pi}$

$\pi \left[\left(\frac{9}{2}\cdot 2\pi - \frac{9}{2}\sin 2\pi\right) - \left(\frac{9}{2}\cdot 0 - \frac{9}{2}\sin 0\right)\right]$

$\pi \left[9\pi - 0\right]$

$= \underline{\underline{9\pi^2}}$ units^3

8) $\pi \int_1^3 y^2 \, dx$

$\pi \int_1^3 x^2 e^{2x} \, dx$

$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

$u = x^2 \qquad \frac{dv}{dx} = e^{2x}$

$\frac{du}{dx} = 2x \qquad v = \frac{1}{2}e^{2x}$

$\pi \left[\frac{1}{2}x^2 e^{2x} - \int x e^{2x} \, dx\right]_1^3$

$u = x \qquad \frac{dv}{dx} = e^{2x}$

$\frac{du}{dx} = 1 \qquad v = \frac{1}{2}e^{2x}$

$\pi \left[\frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} \, dx\right)\right]_1^3$

$\pi \left[\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x}\right]_1^3$

$$\pi\left(\left(\tfrac{1}{2}(3)^2e^{2(3)}-\tfrac{1}{2}(3)e^{2(3)}+\tfrac{1}{4}e^{2(3)}\right)-\left(\tfrac{1}{2}(1)^2e^{2(1)}-\tfrac{1}{2}(1)e^{2(1)}+\tfrac{1}{4}e^{2(1)}\right)\right)$$

$$\pi\left(\left(\tfrac{9}{2}e^6-\tfrac{3}{2}e^6+\tfrac{1}{4}e^6\right)-\left(\tfrac{1}{2}e^2-\tfrac{1}{2}e^2+\tfrac{1}{4}e^2\right)\right)$$

$$\pi\left(\left(\tfrac{13}{4}e^6\right)-\left(\tfrac{1}{4}e^2\right)\right)$$

$$\underline{\underline{\tfrac{1}{4}\pi\left(13e^6-e^2\right)}}$$

9/ $$\pi\int_a^b y^2 \, dx$$

$$\pi\int_a^b (2x+1)^{-2} \, dx$$

$$\pi\left[-\tfrac{1}{2}(2x+1)^{-1}+c\right]_a^b$$

$$\pi\left(-\tfrac{1}{2}(2b+1)^{-1} - -\tfrac{1}{2}(2a+1)^{-1}\right)$$

$$\pi\left(\frac{-1}{2(2b+1)}+\frac{1}{2(2a+1)}\right)$$

$$\frac{\pi}{2(2a+1)}-\frac{\pi}{2(2b+1)}$$

$$\frac{\pi(2b+1)}{2(2a+1)(2b+1)}-\frac{\pi(2a+1)}{2(2a+1)(2b+1)}$$

$$\frac{2b\pi+\pi-2a\pi-\pi}{2(2a+1)(2b+1)}$$

$$\frac{\cancel{2}b\pi-\cancel{2}a\pi}{\cancel{2}(2a+1)(2b+1)}=\frac{b\pi-a\pi}{(2a+1)(2b+1)}$$

10)

$$\pi \int_{-1/4}^{1/2} y^2 \, dx$$

$$\pi \int_{-1/4}^{1/2} \left(\frac{1}{9(1+2x)^2} \right) dx$$

$$\pi \int_{-1/4}^{1/2} \frac{1}{9}(1+2x)^{-2} \, dx$$

$$\pi \left[-\frac{1}{18}(1+2x)^{-1} \right]_{-1/4}^{1/2}$$

$$\pi \left[\left(-\frac{1}{18}(1+2(\tfrac{1}{2}))^{-1} \right) - \left(-\frac{1}{18}(1+2(-\tfrac{1}{4}))^{-1} \right) \right.$$

$$\pi \left[-\frac{1}{36} + \frac{1}{9} \right)$$

$$\underline{\underline{\frac{1}{12}\pi}}$$

b) Height of original $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

Scale factor $\frac{3}{3/4} = \underline{\underline{4}}$

$$\frac{1}{12}\pi \times 4^3 = \underline{\underline{\frac{16}{3}\pi}} \text{ cm}^3$$

11a) $\int \tan^2 x \, dx$

$\tan^2 x + 1 = \sec^2 x$

$\int \sec^2 x - 1 \, dx$

$\underline{\underline{\tan x - x + c}}$

b) $\int \frac{1}{x^3} \ln x \, dx$

$u = \ln x$ $\qquad \frac{dv}{dx} = x^{-3}$

$\frac{dv}{dx} = \frac{1}{x}$ $\qquad v = -\frac{1}{2}x^{-2}$

$$-\frac{1}{2}x^{-2}\ln x - \int -\frac{1}{2}x^{-3}\ dx$$

$$-\frac{1}{2}x^{-2}\ln x - \frac{1}{4}x^{-2} + C$$

11c/ $u = 1 + e^{x}$

$$\int \frac{e^{3x}}{1+e^{x}}\ dx \qquad \frac{du}{dx} = e^{x}$$

$$\int \frac{e^{3x}}{u}\frac{dx}{du}\ du \qquad \frac{dx}{du} = \frac{1}{e^{x}}$$

$$\int \frac{e^{3x}}{u}\cdot\frac{1}{e^{x}}\ du$$

$$\int \frac{e^{2x}}{u}\ du \qquad u-1 = e^{x}$$

$$(u-1)^2 = e^{2x}$$

$$\int \frac{(u-1)^2}{u}\ du$$

$$\int \frac{u^2-2u+1}{u}\ du$$

$$\int u - 2 + u^{-1}\ du$$

$$\frac{1}{2}u^2 - 2u + \ln u + C$$

$$\frac{1}{2}(1+e^{x})^2 - 2(1+e^{x}) + \ln(1+e^{x}) + C$$

$$\frac{1}{2}(1+2e^{x}+e^{2x}) - 2 - 2e^{x} + \ln(1+e^{x}) + C$$

$$\frac{1}{2} + e^{x} + \frac{1}{2}e^{2x} - 2 - 2e^{x} + \ln(1+e^{x}) + C$$

$$\frac{1}{2}e^{2x} - e^{x} + \ln(1+e^{x}) - \frac{3}{2} + C$$

12a/ $$\frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

$$4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$$

Let $x=-1$

$$6 = B(-1)(2)$$

$$6 = -2B$$

$$\underline{\underline{B=-3}}$$

Let $x=-3$

$$10 = C(-5)(-2)$$

$$10 = 10C$$

$$\underline{C=1}$$

Let $x = -1/2$

$$5 = A(1/2)(5/2)$$

$$5 = \frac{5}{4}A$$

$$\underline{\underline{A=4}}$$

$$\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3}$$

bi/ $\int f(x)\,dx$

$$= \frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + c$$

$$= 2\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + c$$

ii $\int_0^2 f(x)\,dx$

$$\left(2\ln(5) - 3\ln(3) + \ln(5)\right) - \left(2\ln(1) - 3\ln(1) + \ln(3)\right)$$

$$\left(\ln(5)^2 + \ln(5) - \ln(3)^3\right) - \ln(3)$$

$$\ln\left(\frac{5^3}{3^4}\right) = \underline{\underline{\ln\left(\frac{125}{81}\right)}}$$

13a/ $\int (5-x)^{1/2} \, dx$

$-\frac{2}{3}(5-x)^{3/2}$

bi/ $u = x - 1$ $\quad \frac{dv}{dx} = (5-x)^{1/2}$

$\frac{du}{dx} = 1$ $\quad v = -\frac{2}{3}(5-x)^{3/2}$

$$\int (x-1)\sqrt{5-x}\, dx = -\frac{2}{3}(x-1)(5-x)^{3/2} - \int -\frac{2}{3}(5-x)^{3/2} \, dx$$

$$= -\tfrac{2}{3}(x-1)(5-x)^{3/2} - \frac{4}{15}(5-x)^{5/2}$$

ii $\left[-\frac{2}{3}(x-1)(5-x)^{3/2} - \frac{4}{15}(5-x)^{5/2}\right]_1^5$

$$\left((0) - \left(-\frac{4}{15}(4)^{5/2}\right)\right)$$

$$\frac{4}{15}(4)^{5/2} = \underline{\underline{\frac{128}{15}}}$$

14a/ $\int x e^x \, dx$

$u = x$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 1$ $\quad v = e^x$

$$= xe^x - \int e^x \, dx$$

$$= \underline{xe^x - e^x + c}$$

b/ $\int x^2 e^x \, dx$

$u = x^2$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 2x$ $\quad v = e^x$

$$x^2e^x - \int 2xe^x \, dx + c$$

$$x^2e^x - 2\int xe^x \, dx$$

$$x^2e^x - 2(xe^x - e^x) + c$$

$$x^2e^x - 2xe^x + 2e^x + c$$

15 i) $\int \ln\left(\frac{x}{2}\right) dx$

$u = \ln\left(\frac{x}{2}\right) \quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \cancel{\frac{2}{x}}\frac{1}{x} \quad v = x$

$$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int 1 \; dx$$

$$= x\ln(x/2) - x + c$$

ii) $\int_{\pi/4}^{\pi/2} \sin^2 x \; dx$

$\int_{\pi/4}^{\pi/2} 1/2 - 1/2\cos 2x \; dx$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = (1 - \sin^2 A) - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = 1/2 - 1/2\cos 2A$

$$\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_{\pi/4}^{\pi/2}$$

$$\left(\frac{\pi}{4} - \frac{1}{4}\sin\pi\right) - \left(\frac{1}{8}\pi - \frac{1}{4}\sin\frac{\pi}{2}\right)$$

$$\frac{\pi}{4} - \frac{1}{8}\pi + \frac{1}{4}$$

$$\frac{1}{8}\pi + \frac{1}{4}$$

16a) $\frac{dv}{dt} = 1600 - \overset{c}{k}\sqrt{h}$ $\quad v = 4000h$

$\frac{dv}{dh} = 4000$

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$$

$$\frac{dh}{dt} = \frac{1}{4000}\left(1600 - \cancel{k}^{c}\sqrt{h}\right)$$

$$= \frac{1600}{4000} - \frac{\cancel{k}^{c}}{4000}\sqrt{h}$$

$$= 0.4 - \frac{\cancel{k}^{c}}{4000}\sqrt{h} \qquad \left(k = \frac{\cancel{k}^{c}}{4000}\right)$$

b/ when $h = 25$

$$\frac{dv}{dt} = 1600 - \boxed{c\sqrt{h}}$$

$$\frac{dv}{dt} = 1600 - \boxed{400}$$

$$= \cancel{1200}\ cm^3/s$$

~~120~~

$$\cancel{k}^{c}\sqrt{h} = 400$$

$$\cancel{k}^{c}\sqrt{25} = 400$$

$$5\cancel{k}^{c} = 400$$

$$\cancel{k}^{c} = 80$$

$$k = \frac{80}{4000} = \underline{\underline{0.02}}$$

c/

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h}$$

$$\int \frac{1}{0.4 - 0.02\sqrt{h}}\, dh = \int 1 \, dt$$

$$\int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh = t$$

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh = t \qquad (\times 50 \text{ top and bottom})$$

$$h = (20 - x)^2$$

$$\int_{20}^{10} \frac{50}{20 - \sqrt{h}} \frac{dh}{dx}\, dx$$

$$\frac{dh}{dx} = -2(20-x)^{1}$$

$$\int_{10}^{20} \frac{50}{20-\sqrt{h}} \left(2(20-x)\right) \, dx$$

$$\int_{10}^{20} \frac{50}{20-(20-x)} (40-2x) \, dx$$

$$\int_{10}^{20} \frac{50}{x} (40-2x) \, dx$$

$$\int_{10}^{20} \frac{2000-100x}{x} \, dx$$

$$\int_{10}^{20} 2000x^{-1} - 100 \, dx$$

$$\left[2000\ln x - 100x + c\right]_{10}^{20}$$

$$\left(2000\ln(20) - 100(20)\right) - \left(2000\ln 10 - 100(10)\right)$$

$$\left(2000\ln 20 - 2000\right) - \left(2000\ln 10 - 1000\right)$$

$$2000\ln 20 - 2000\ln 10 - 1000$$

$$2000(\ln 20 - \ln 10) - 1000$$

$$\underline{\underline{2000\ln 2 - 1000}}$$

e) $2000 \ln 2 - 1000 = 386$ mins (nearest min)

$= \underline{\underline{6 \text{ mins } 26 \text{ seconds}}}$

17 $u = 2^{x}$

$\frac{du}{dx} = 2^{x} \ln 2$ $\longrightarrow$ $2^{1} = 2$

$$\int_{0}^{1} \frac{2^{x}}{(2^{x}+1)^{2}} \frac{dx}{du} \, du$$

$2^{0} = 1$

$$\int_1^2 \frac{2^x}{(2^x+1)^2} \frac{dx}{du} \, du$$

$$\int_1^2 \frac{u}{(u+1)^2} \frac{1}{2^x \ln 2} \, du$$

$$\int_1^2 \frac{\cancel{u}}{(u+1)^2} \frac{1}{\cancel{u} \ln 2} \, du$$

$$\frac{1}{\ln 2} \int_1^2 \frac{1}{(u+1)^2} \, du$$

$$\frac{1}{\ln 2} \int_1^2 (u+1)^{-2} \, du$$

$$\frac{1}{\ln 2} \Big[-(u+1)^{-1} \Big]_1^2$$

$$\frac{1}{\ln 2} \Big[-(2+1)^{-1} + (1+1)^{-1} \Big]$$

$$\frac{1}{\ln 2} \left(-\frac{1}{3} + \frac{1}{2} \right)$$

$$\underline{\underline{\frac{1}{6 \ln 2}}}$$

18 a/ $\int x \cos 2x \, dx$

$u = x \qquad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 1 \qquad v = \frac{1}{2} \sin 2x$

$$\int x \cos 2x \, dx = \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$$

$$= \underline{\underline{\frac{1}{2} x \sin x + \frac{1}{4} \cos 2x + c}}$$

b) $\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$\frac{1}{2}\cos 2x + \frac{1}{2} = \cos^2 x$

$u = x$ $\qquad$ $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$

$\frac{du}{dx} = 1$ $\qquad$ $v = \frac{1}{4}\sin 2x + \frac{1}{2}x$

$$\int x\cos^2 x \, dx = \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int \frac{1}{4}\sin 2x + \frac{1}{2}x \, dx$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \left(-\frac{1}{8}\cos 2x + \frac{1}{4}x^2\right) + C$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 + \frac{1}{8}\cos 2x - \frac{1}{4}x^2 + C$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{4}x^2 + \frac{1}{8}\cos 2x + C$$

19) $\frac{2(4x^2+1)}{(2x+1)(2x-1)} = \frac{8x^2+2}{4x^2-1}$

$$= \frac{2(4x^2-1) + 4}{4x^2-1}$$

$\boxed{\text{OR USE LONG DIVISION}}$ $\qquad$ $= \frac{2\cancel{(4x^2-1)}}{\cancel{4x^2-1}} + \frac{4}{4x^2-1}$

$A = 2$

$$\frac{4}{(2x+1)(2x-1)} = \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

~~Let $x = \frac{1}{2}$~~

$4 = B(2x-1) + C(2x+1)$

Let $x = \frac{1}{2}$

$4 = 2C$

$C = 2$

Let $x = -\frac{1}{2}$

$4 = -2B$

$B = -2$

b/ $\int_1^2 2 - \frac{2}{2x+1} + \frac{2}{2x-1} \, dx$

$\left[2x - \ln(2x+1) + \ln(2x-1) + c\right]_1^2$

$\left(4 - \ln(5) + \ln(3)\right) - \left(2 - \ln(3) + \ln(1)\right)$

$2 - \ln(5) + 2\ln(3)$

$2 + \ln(3)^2 - \ln(5)$

$2 + \ln\left(\frac{9}{5}\right)$

$k = 9/5$

20

x	0	1	2	3	4	5
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4

b/ $1\left(\frac{e^1}{2} + e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}} + \frac{e^4}{2}\right)$

$= 110.6$ units^2

c/ $\int_0^5 e^{\sqrt{3x+1}} \, dx$

$t = \sqrt{3x+1}$

$t = (3x+1)^{1/2}$

$\sqrt{3(5)+1} \rightarrow$

$\sqrt{3(0)+1}$

$\int_1^4 e^t \frac{dx}{dt} \, dt$

$\frac{dt}{dx} = \frac{3}{2}(3x+1)^{-1/2}$

$\frac{dx}{dt} = \frac{2(3x+1)^{1/2}}{3}$

$\int_1^4 e^t \frac{2}{3}(3x+1)^{1/2} \, dt$

$\int_1^4 e^t \times \frac{2}{3} t \, dt$

$\int_1^4 \frac{2}{3} t e^t \, dt$

$a = 1 \quad b = 4 \quad k = 2/3$

d/ $u = \frac{2}{3}t \qquad \frac{dv}{dx} = e^t$

$\frac{du}{dx} = \frac{2}{3} \qquad v = e^t$

$$\left[\frac{2}{3}t\,e^t - \int \frac{2}{3}e^t\,dt + c\right]_1^4$$

$$\left[\frac{2}{3}t e^t - \frac{2}{3}e^t + c\right]_1^4$$

$$\left(\frac{2}{3}(4)e^4 - \frac{2}{3}e^4\right) - \left(\frac{2}{3}e^1 - \frac{2}{3}e^1\right)$$

$$\cancel{= 109} = \underline{109.2 \text{ units}^2}$$

21
$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx \qquad u^2 = 2x - 1$$

$u = \sqrt{2x-1}$

$= (2x-1)^{1/2}$

$$\int_1^3 \frac{3x}{u}\frac{dx}{du}\,du \qquad \frac{du}{dx} = (2x-1)^{-1/2}$$

$$\int_1^3 \frac{3x}{u}\frac{1}{(2x-1)^{1/2}}\,du \qquad \frac{dx}{du} = (2x-1)^{1/2}$$

$x = \frac{u^2}{2} + \frac{1}{2}$

$$\int_1^3 \frac{3x(2x-1)^{1/2}}{u}\,du$$

$$\int_1^3 \frac{3\left(\frac{1}{2}u^2 + \frac{1}{2}\right)u}{u}\,du$$

$$\int_1^3 \frac{\frac{3}{2}u^3 + \frac{3}{2}u}{u}\,du$$

$$\int_1^3 \frac{3}{2}u^2 + \frac{3}{2}\,du$$

$$\left[\frac{1}{2}u^3 + \frac{3}{2}u + c\right]_1^3$$

$$\cancel{\left(\frac{9}{2} - \frac{1}{2}\right) - \left(\frac{3}{2} - \frac{3}{2}\right)}$$

$$\left(\frac{27}{2} + \frac{9}{2}\right) - \left(\frac{1}{2} + \frac{3}{2}\right) = \underline{16}$$

22a/

x	1	1.5	2	2.5	3
y	0	$0.5\ln 1.5$	$\ln 2$	$1.5\ln 2.5$	$2\ln 3$

b i/ $1\left(\frac{0}{2} + \ln 2 + \frac{2\ln 3}{2}\right)$

$= 1.792$ (4sf) units^2

ii/ $0.5\left(\frac{0}{2} + 0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5 + \frac{2\ln 3}{2}\right)$
$= 1.684$ units^2 (4sf)

c/ more trapeziums will be a closer approximation - less space over the graph

d/ $\int_1^3 (x-1)\ln x \, dx$

$u = \ln x \quad \frac{dv}{dx} = x - 1$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{1}{2}x^2 - x$

$\left[\left(\frac{1}{2}x^2 - x\right)\ln x - \int \frac{1}{x}\left(\frac{1}{2}x^2 - x\right) dx\right.$

$\left(\frac{1}{2}x^2 - x\right)\ln x - \int \frac{1}{2}x - 1 \, dx$

$\left[\left(\frac{1}{2}x^2 - x\right)\ln x - \left(\frac{1}{4}x^2 - x\right)\right]_1^3$

$\left[\left(\frac{1}{2}x^2 - x\right)\ln x - \frac{1}{4}x^2 + x\right]_1^3$

$\left(\left(\frac{9}{2} - 3\right)\ln(3) - \frac{1}{4}(3)^2 + 3\right) - \left(\left(\frac{1}{2} - 1\right)\ln 1 - \frac{1}{4}(1)^2 + (1)\right)$

$\left(\frac{3}{2}\ln(3) - \frac{9}{4} + 3\right) - \left(-\frac{1}{4} + 1\right)$

$\frac{3}{2}\ln 3 + \frac{3}{4} - \frac{3}{4}$

$\underline{\underline{\frac{3}{2}\ln 3}}$