Edexcel GCE

Core Mathematics C4

Integration

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1.

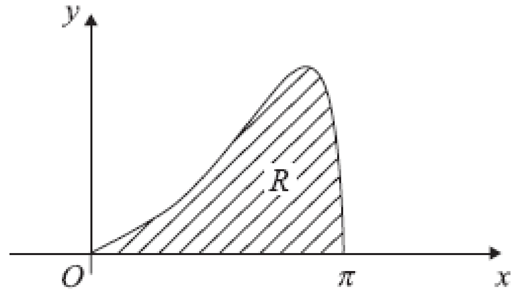


Figure 1

The curve shown in Figure 1 has equation $e^x\sqrt{(\sin x)}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Copy and complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

[January 2008]

2.

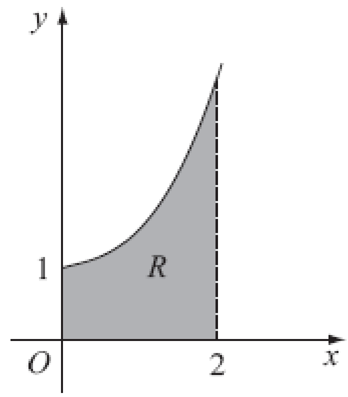


Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

[June 2008]

3. (*a*) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(*b*) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \; \mathrm{d}x$. Show all the steps of your working and give your answer to 4 decimal places.

(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \; \mathrm{d}x$ is $\ln(1 + \sqrt{2})$.

(*c*) Calculate the % error in using the estimate you obtained in part (*b*).

(2)

[January 2006]

4.

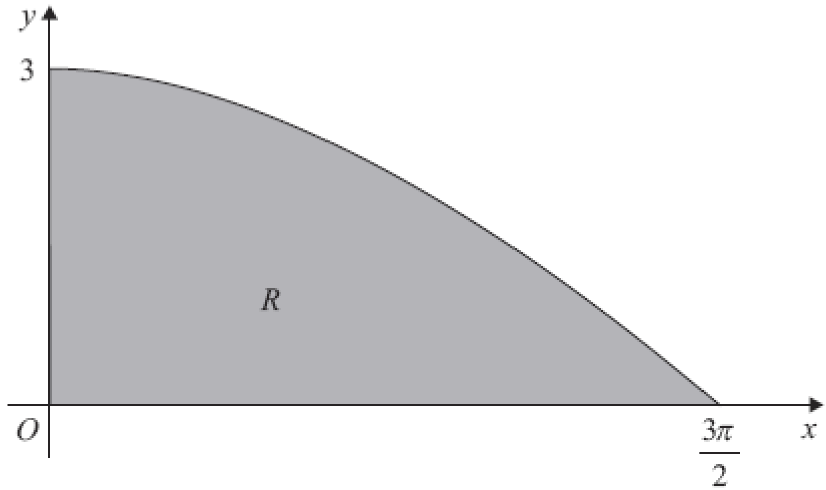


Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \le x \le \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Copy and complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(c) Use integration to find the exact area of R.

(3)

[June 2009]

5.

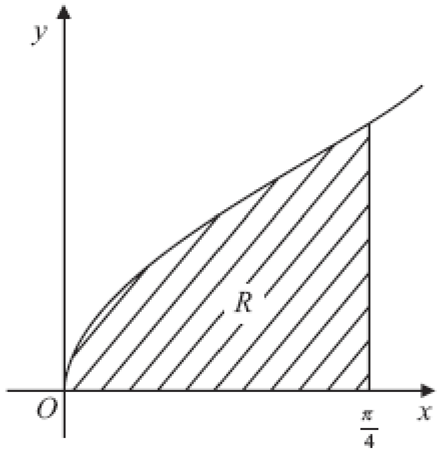


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(*a*) Given that $y = \sqrt{(\tan x)}$, copy and complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(*c*) Use integration to find an exact value for the volume of the solid generated.

(4)

[June 2007]

6.

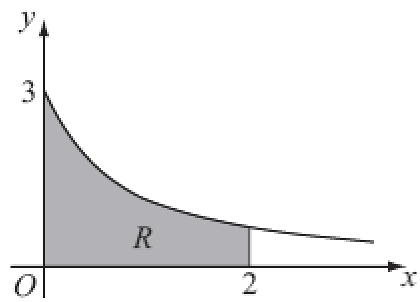


Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

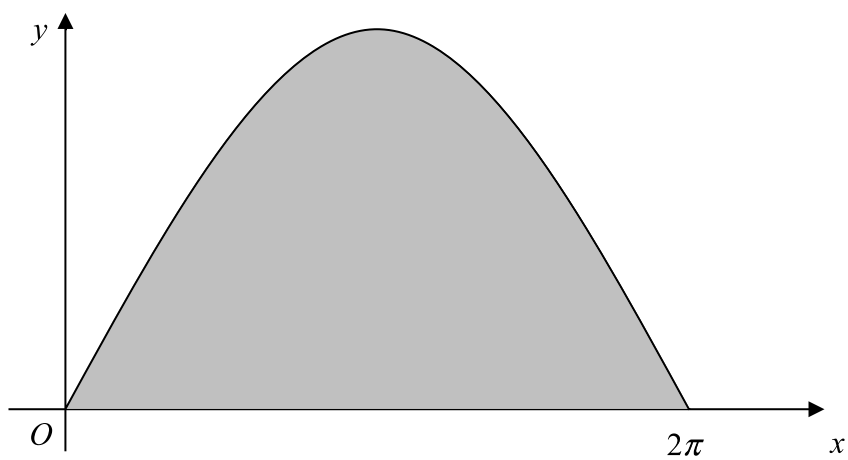
The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

[January 2009]

7. **Figure 3**



The curve with equation $y = 3 \sin \dfrac{x}{2}$, $0 \le x \le 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)

[June 2006]

8. **Figure 1**

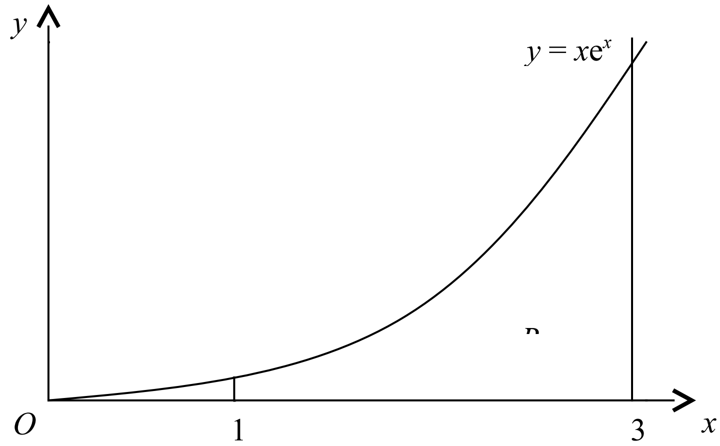


Figure 1 shows the finite region R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

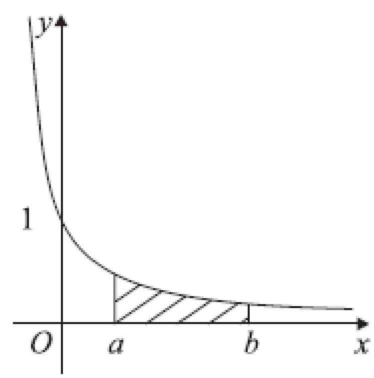
The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

[January 2006]

9.



The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

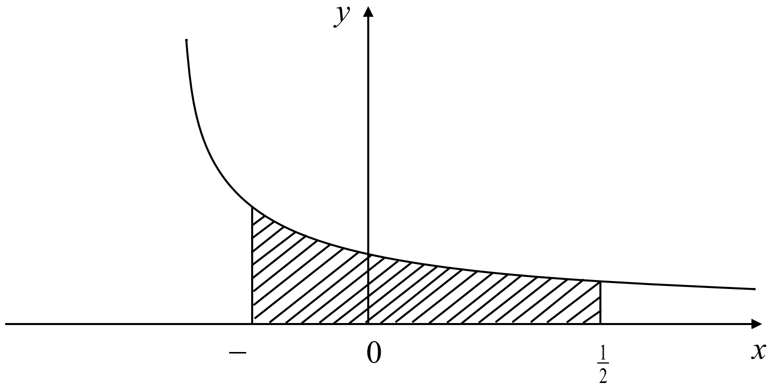
Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

[January 2008]

10.

Figure 1



The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\dfrac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\dfrac{1}{4}$, $x = \dfrac{1}{2}$, the x-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

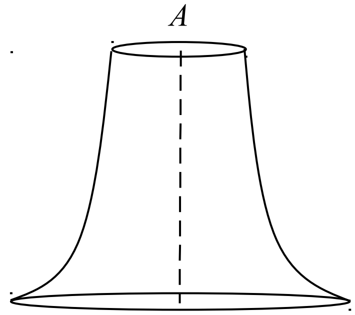


Figure 2 shows a paperweight with axis of symmetry AB where AB = 3 cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)

[January 2007]

11. (*a*) Find $\int \tan^2 x \ \mathrm{d}x$.

(2)

(*b*) Use integration by parts to find $\int \frac{1}{x^3} \ln x \ \mathrm{d}x$.

(4)

(*c*) Use the substitution $u = 1 + \mathrm{e}^x$ to show that

$$\int \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^x} \ \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1 + \mathrm{e}^x) + k,$$

where k is a constant.

(7)

[January 2009]

12.

$$\mathrm{f}(x) = \frac{4 - 2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) (i) Hence find $\int \mathrm{f}(x) \ \mathrm{d}x$.

(3)

(ii) Find $\int_0^2 \mathrm{f}(x) \ \mathrm{d}x$ in the form $\ln k$, where k is a constant.

(3)

[June 2009]

13. (*a*) Find $\int \sqrt{(5-x)}\ \mathrm{d}x$.

(2)

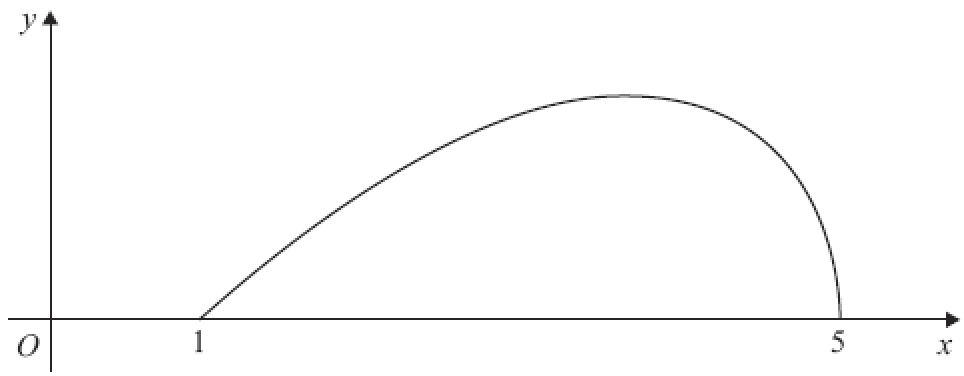


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{(5 - x)}, \quad 1 \leq x \leq 5$$

(*b*) (i) Using integration by parts, or otherwise, find $\int (x-1)\sqrt{(5-x)}\ \mathrm{d}x$.

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\ \mathrm{d}x$..

(2)

[June 2009]

14. (*a*) Use integration by parts to find $\int x\mathrm{e}^x\ \mathrm{d}x$.

(3)

(*b*) Hence find $\int x^2\mathrm{e}^x\ \mathrm{d}x$.

(3)

[June 2008]

15. (i) Find $\int \ln\left(\frac{x}{2}\right) \, dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$.

(5)

[January 2008]

16. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 cm^3s^{-1} and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(*a*) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h},$$

where k is a positive constant.

(3)

When $h = 25$, water is leaking out of the hole at 400 cm^3s^{-1}.

(*b*) Show that $k = 0.02$.

(1)

(*c*) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h}$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}} \, dh .$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(*d*) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}} \, dh$.

(6)

(*e*) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

[January 2008]

17. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2}\, dx.$$

(6)

[June 2007]

18. (*a*) Find $\int x \cos 2x \ dx$.

(4)

(*b*) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \ dx$.

(3)

[June 2007]

19.

$$\frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)} \equiv A + \frac{B}{(2x + 1)} + \frac{C}{(2x - 1)}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) Hence show that the exact value of $\int_1^2 \frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)}\, dx$ is $2 + \ln k$, giving the value of the constant k.

(6)

[June 2007]

20.

$$I = \int_0^5 e^{\sqrt{(3x+1)}}\, dx\,.$$

(*a*) Given that $y = e^{\sqrt{(3x+1)}}$, copy and complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.

(3)

(*c*) Use the substitution $t = \sqrt{(3x + 1)}$ to show that I may be expressed as $\int_a^b kte^t\, dt$, giving the values of a, b and k.

(5)

(*d*) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)

[June 2007]

21. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\, dx\,.$$

(8)

[January 2006]

22. **Figure 3**

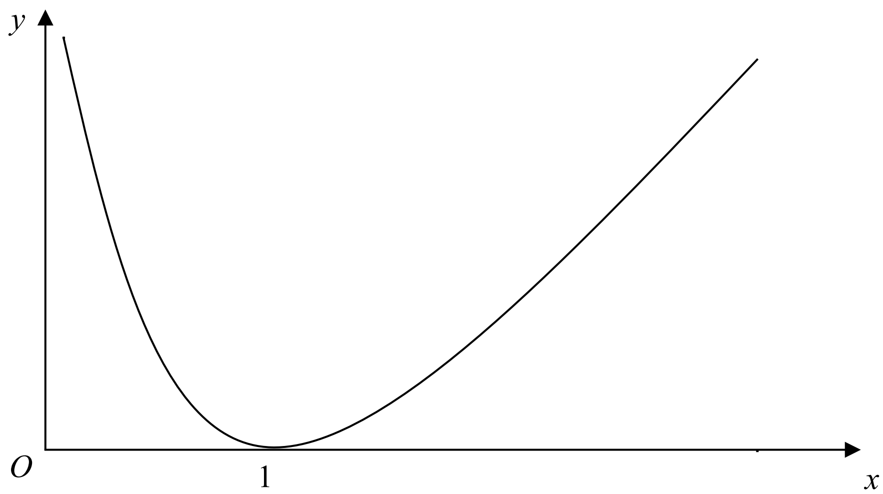


Figure 3 shows a sketch of the curve with equation $y = (x - 1) \ln x$, $x > 0$.

(*a*) Copy and complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		ln 2		2 ln 3

(1)

Given that $I = \int_1^3 (x - 1) \ln x \, dx$,

(*b*) use the trapezium rule

(i) with values at y at x = 1, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values at y at x = 1, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

(*c*) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(*d*) Show, by integration, that the exact value of $\int_1^3 (x - 1) \ln x \, dx$ is $\frac{3}{2} \ln 3$.

(6)

[June 2006]