Edexcel GCE

Core Mathematics C4

Differential Equations

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(*b*) Hence obtain the solution of

$$2\cot x\,\frac{\mathrm{d}y}{\mathrm{d}x} = (4-y^2)$$

for which $y = 0$ at $x = \dfrac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$.

(8)

[June 2008]

2. A population growth is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(*a*) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(*b*) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P\cos\lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(*c*) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(*d*) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)

[June 2007]

3. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(b) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)y.$$

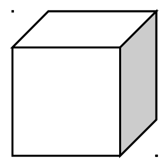
(5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = \mathrm{f}(x)$.

(4)

[January 2007]

4.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(b) $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)

[June 2006]

5. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \dfrac{4}{3}\pi r^3$.

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(*b*) Using the chain rule, or otherwise, find an expression in terms of *r* and *t* for $\frac{dr}{dt}$.

(2)

(*c*) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$, to obtain *V* in terms of *t*.

(4)

(*d*) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

(3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}.

(2)

[January 2006]