Edexcel GCE

Core Mathematics C3

Trigonometry

<u>Materials required for examination papers</u>	**<u>Items included with question</u>**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2.$$

(6)

2. (*a*) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(*b*) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0.$$

giving your answers to 2 decimal places.

(5)

3. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

4. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

5. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\,\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 180°$, the equation

$$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$

giving your answers to 1 decimal place.

(6)

6. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

7. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

8. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

9. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

(4)

(*b*) Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

(2)

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

(6)

10. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi,$$

(*a*) express $\arcsin x$ in terms of y.

(2)

(*b*) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

11. (*a*) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that the $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(*b*) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

(2)

(*c*) Solve, for $90° < \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)

12. (*a*) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(*b*) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$.

(4)

13. (*a*) Show that

(i) $\frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \frac{1}{4})\pi,\ n \in \mathbb{Z},$

(2)

(ii) $\frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}$.

(3)

(*b*) Hence, or otherwise, show that the equation

$$\cos\theta \left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(*c*) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

14. **Figure 1**

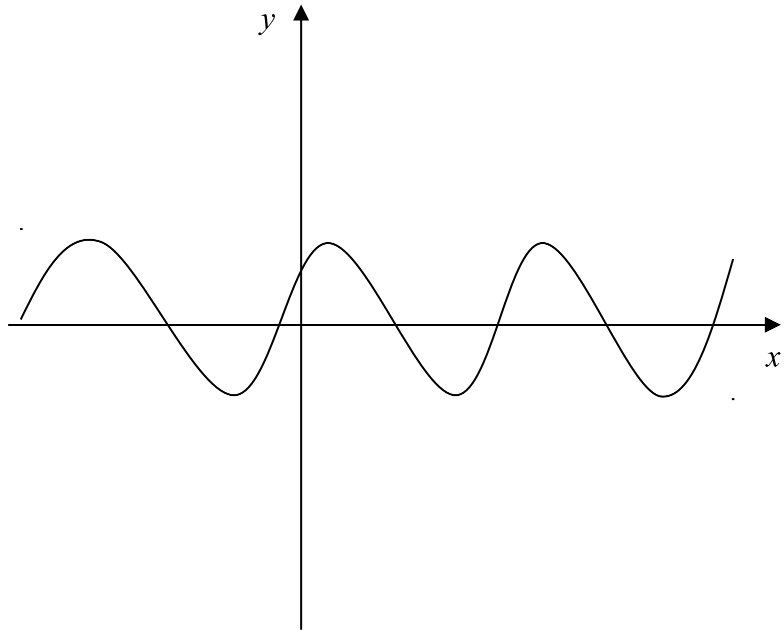

Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation $y = \sqrt{3} \cos x + \sin x$.

(a) Express the equation of the curve in the form $y = R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$.

(4)

15. A curve C has equation

$$y = 3 \sin 2x + 4 \cos 2x, \qquad -\pi \leq x \leq \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.
Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places.

(4)

16. $$f(x) = 12\cos x - 4\sin x.$$

Given that $f(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

17. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

18. $$f(x) = 5 \cos x + 12 \sin x.$$

Given that $f(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(*a*) find the value of R and the value of α to 3 decimal places.

(4)

(*b*) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(*c*) (i) Write down the maximum value of $5 \cos x + 12 \sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

19. (*a*) Use the identity $\cos (A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2 \sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: y = 3 \sin 2x$$
$$C_2: y = 4 \sin^2 x - 2 \cos 2x$$

(*b*) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4 \cos 2x + 3 \sin 2x = 2$$

(3)

(*c*) Express $4\cos 2x + 3 \sin 2x$ in the form $R \cos (2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(*d*) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4 \cos 2x + 3 \sin 2x = 2,$$

giving your answers to 1 decimal place.

(4)

20. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \leq \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

21. (*a*) Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(4)

(*b*) Hence find the greatest value of $(3\sin x + 2\cos x)^4$.

(2)

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3\sin x + 2\cos x = 1,$$

giving your answers to 3 decimal places.

(5)