C3 Exponential & Log Functions

1a) $P = 80e^{\frac{1}{5}t}$

$\underline{\underline{80}}$

b) $1000 = 80e^{\frac{1}{5}t}$

$\frac{1000}{80} = e^{\frac{1}{5}t}$

$\ln\left(\frac{1000}{80}\right) = \frac{1}{5}t$

$5\ln\left(\frac{1000}{80}\right) = t$

$t = 12.6$ years (to reach 1000)

$\therefore$ <u>13 years to exceed 1000</u>

c) $\frac{dP}{dt} = 16e^{\frac{1}{5}t}$

d) $16e^{\frac{1}{5}t} = 50$

$e^{\frac{1}{5}t} = \frac{50}{16}$

$\frac{1}{5}t = \ln\left(\frac{50}{16}\right)$

$t = 5\ln\left(\frac{50}{16}\right)$

$= 5.69717416$

$P = 80e^{\frac{1}{5}ANS}$

$= \underline{250}$

2 $R = 1000e^{-ct}$

a) 1000

b) $500 = 1000e^{-c(5730)}$

$\frac{1}{2} = e^{-5730c}$

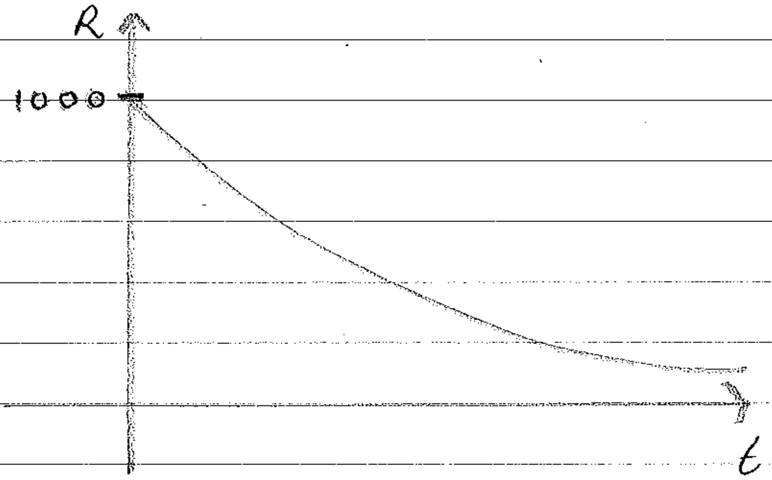
$\ln \frac{1}{2} = -5730c$

$\underline{c = 1.21 \times 10^{-4}}$

c) $R = 1000e^{(-1.21\times10^{-4})(22920)}$

$\underline{= 16000}$

d)



3a) $\ln x + \ln 3 = \ln 6$

$\ln 3x = \ln 6$

$3x = 6$

$x = 2$

3b) $e^{x} + 3e^{-x} = 4$

$e^{x} + \frac{3}{e^{x}} = 4$

$e^{2x} + 3 = 4e^{x}$

$e^{2x} - 4e^{x} + 3 = 0$

$(e^{x} - 3)(e^{x} - 1) = 0$

$e^{x} = 3 \quad e^{x} = 1$

$\underline{x = \ln 3} \quad x = \ln 1$

$\underline{x = 0}$

4) $x = De^{-1/8 t}$

a) $D = 10 \quad t = 5$

$x = 10e^{-1/8(5)}$

$x = 5.35\,\text{mg}$ (2dp)

b) After second dose : 15.35 mg

$x = 15.35\, e^{-1/8(1)}$

$= 13.549$ mg 3dp

c) $x = 3 \quad D = 15.35 \quad t = T$

$3 = 15.35e^{-1/8 T}$

$0.195 = e^{-1/8 T}$

$\ln(0.195) = -1/8\, T$

$T = 13.06$ hours (2dp)

5 $y = 4e^{2x+1}$

a)

$8 = 4e^{2x+1}$

$2 = e^{2x+1}$

$\ln 2 = 2x+1$

$\dfrac{\ln(2) - 1}{2} = x$

b) $\dfrac{dy}{dx} = 8e^{2x+1}$

when $x = \dfrac{\ln(2) - 1}{2}$

$\dfrac{dy}{dx} = 16$

$y = 16x + c$

$8 = 16\left(\dfrac{\ln(2) - 1}{2}\right) + c$

$8 = 8\ln(2) - 8 + c$

$c = 16 - 8\ln(2)$

$y = 16x + 16 - 8\ln(2)$

6 a) when $t = 0$ $T = 425$

b) $300 = 400e^{-0.05t} + 25$

$\dfrac{275}{400} = e^{-0.05t}$

$\ln\left(\dfrac{275}{400}\right) = -0.05t$

$t = -20\ln\left(\dfrac{275}{400}\right) = 7.49$ mins (3sf)

c) $\frac{dT}{dt} = -20e^{-0.05t}$

when $t = 50$

$\frac{dT}{dt} = -20e^{-0.05(50)}$

$\frac{dT}{dt} = -1.64$ °C per min

Decreasing at a rate of 1.64 °C per min

d) The range of e^x is such that it is bigger than zero.

$400e^{-0.05t} > 0$ $\therefore$ the minimum T is 25°c

7 $p = \frac{2800ae^{0.2t}}{1+ae^{0.2t}}$

a) $P = 300$ when $t = 0$

$300 = \frac{2800a}{1+a}$

$300(1+a) = 2800a$

$300 + 300a = 2800a$

$300 = 2500a$

$\underline{a = 0.12}$

b) $$p = \frac{2800 a e^{0.2t}}{1 + ae^{0.2t}}$$

$$1850 = \frac{2800(0.12) e^{0.2t}}{1 + (0.12) e^{0.2t}}$$

$$1850(1 + 0.12e^{0.2t}) = 336 e^{0.2t}$$

$$1850 + 222e^{0.2t} = 336 e^{0.2t}$$

$$1850 = 114 e^{0.2t}$$

$$\frac{1850}{114} = e^{0.2t}$$

$$\ln\left(\frac{1850}{114}\right) = 0.2t$$

$$5 \ln\left(\frac{1850}{114}\right) = t$$

$t = 13.9$ years

$\therefore$ 14 years

c) $$p = \frac{2800(0.12) e^{0.2t}}{1 + 0.12 e^{0.2t}}$$

$$= \frac{336 e^{0.2t}}{1 + 0.12 e^{0.2t}}$$

$\div$ top & bottom by $e^{0.2t}$.

$$p = \frac{336}{e^{-0.2t} + 0.12}$$

$$= \frac{336}{0.12 + e^{-0.2t}}$$

d) ~~e^t~~ $e^{-0.2t} > 0 \quad \therefore \quad p < \frac{336}{0.12}$

$p < 2800$