Edexcel GCE

Core Mathematics C3

Functions

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Simplify $\dfrac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(*b*) Hence, or otherwise, express $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

2.

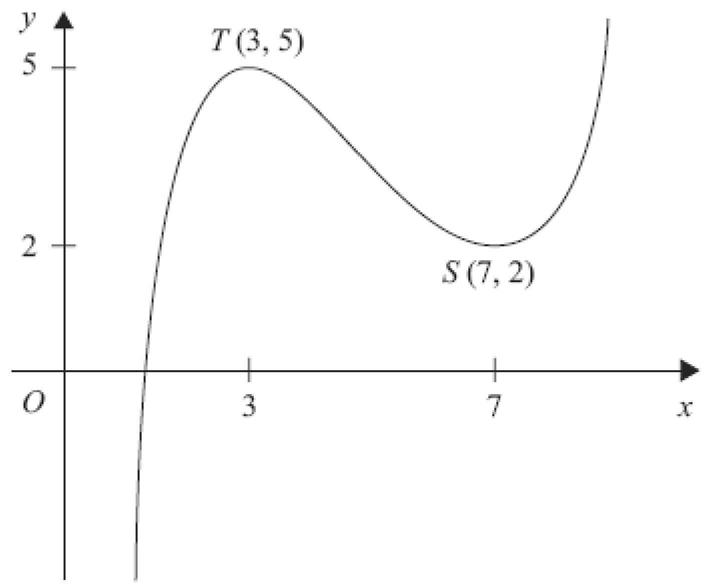

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2\mathrm{f}(x) - 4$,

(3)

(*b*) $y = |\mathrm{f}(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

3. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g.

(1)

(*b*) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(*c*) Write down the range of fg.

(1)

4. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $\mathrm{f}(x) = \frac{1}{x+1}, \quad x > 3$.

(4)

(*b*) Find the range of f.

(2)

(*c*) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $\mathrm{fg}(x) = \frac{1}{8}$.

(3)

5.

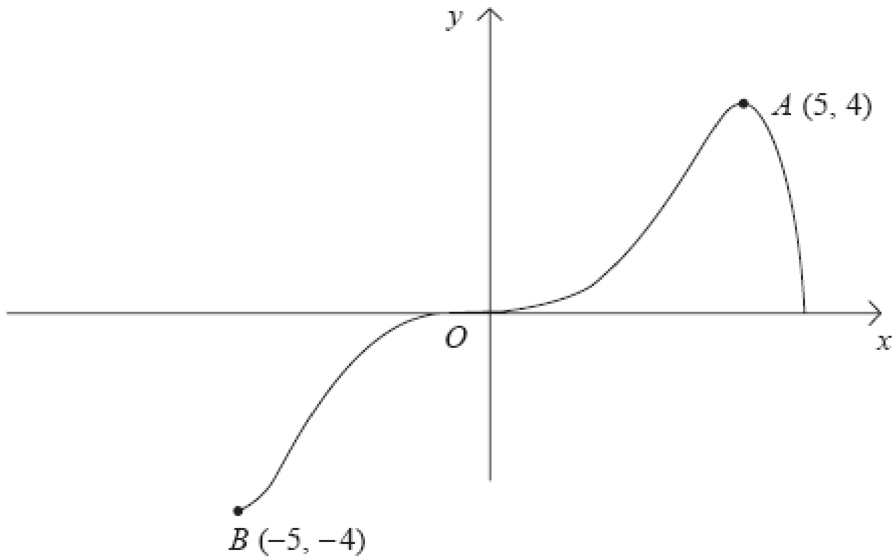

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}(|x|)$,

(3)

(*c*) $y = 2\mathrm{f}(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

6. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$$

$$\mathrm{g} : x \mapsto \frac{3}{x} - 4, \quad x > 0,\ x \in \mathbb{R}.$$

(*a*) Find the inverse function f^{-1}.

(2)

(*b*) Show that the composite function gf is

$$\mathrm{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(*c*) Solve $\mathrm{gf}(x) = 0$.

(2)

7. The functions f and g are defined by

$$\mathrm{f} : \mapsto \ln(2x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$\mathrm{g} : \mapsto \frac{2}{x - 3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(*a*) Find the exact value of fg(4).

(2)

(*b*) Find the inverse function $\mathrm{f}^{-1}(x)$, stating its domain.

(4)

(*c*) Sketch the graph of $y = |\mathrm{g}(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the *y*-axis.

(3)

(*d*) Find the exact values of *x* for which $\left| \frac{2}{x - 3} \right| = 3$.

(3)

8. $$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(*a*) Show that $f(x) = \dfrac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(4)

(*b*) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(*c*) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

9. $$f(x) = x^4 - 4x - 8.$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(*b*) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(*c*) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants a, b and c.

(3)

(*d*) Sketch the graph of $y = f(x)$.

(3)

(*e*) Hence sketch the graph of $y = |f(x)|$.

(1)

10. **Figure 1**

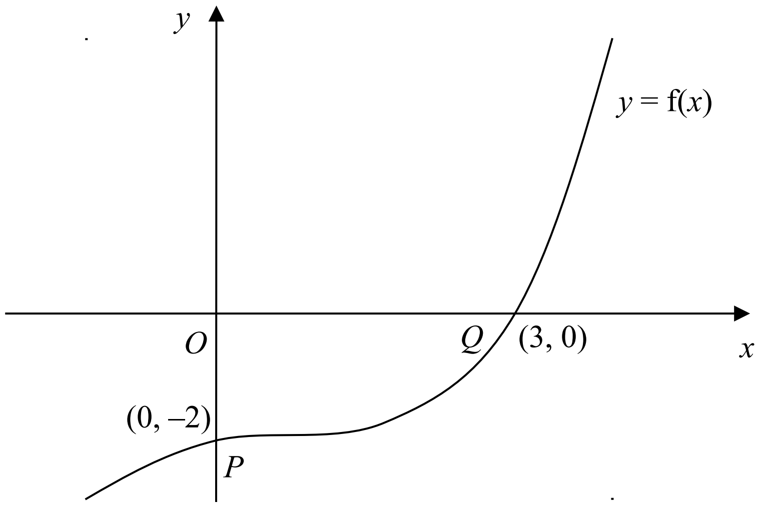

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$,

(3)

(*c*) $y = \frac{1}{2}\mathrm{f}(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

11. For the constant k, where $k > 1$, the functions f and g are defined by

$$\text{f}: x \mapsto \ln(x+k), \quad x > -k,$$
$$\text{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(*a*) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(*b*) Write down the range of f.

(1)

(*c*) Find $\text{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \text{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(*d*) Find the value of k.

(4)

12. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)

13. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

14. **Figure 1**

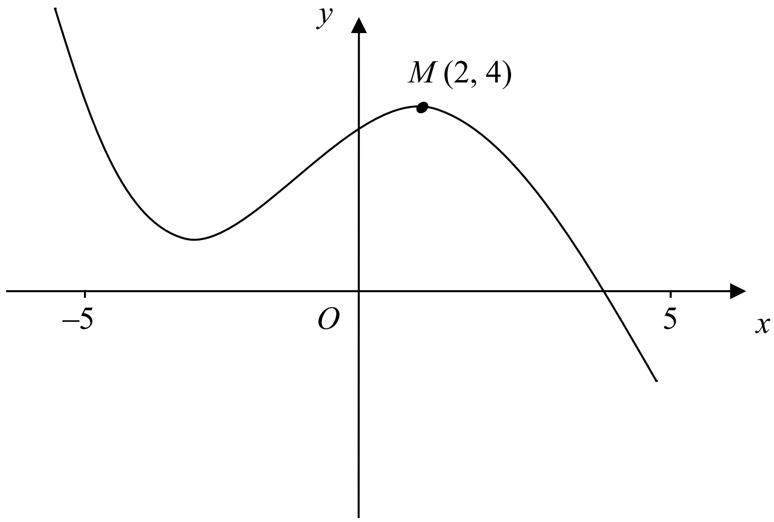

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \leq x \leq 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = \mathrm{f}(x) + 3$,

(2)

(b) $y = |\mathrm{f}(x)|$,

(2)

(c) $y = \mathrm{f}(|x|)$.

(3)

Show on each graph the coordinates of any maximum turning points.

15. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 2x + \ln 2, \qquad x \in \mathbb{R},$$

$$\mathrm{g}: x \mapsto \mathrm{e}^{2x}, \qquad x \in \mathbb{R}.$$

(*a*) Prove that the composite function gf is

$$\mathrm{gf}: x \mapsto 4\mathrm{e}^{4x}, \qquad x \in \mathbb{R}.$$

(4)

(*b*) Sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(*c*) Write down the range of gf.

(1)

16. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{2}{x-1}$, $x > 1$.

(4)

(*b*) Find $\mathrm{f}^{-1}(x)$.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*b*) Solve $\mathrm{fg}(x) = \dfrac{1}{4}$.

(3)

17. **Figure 1**

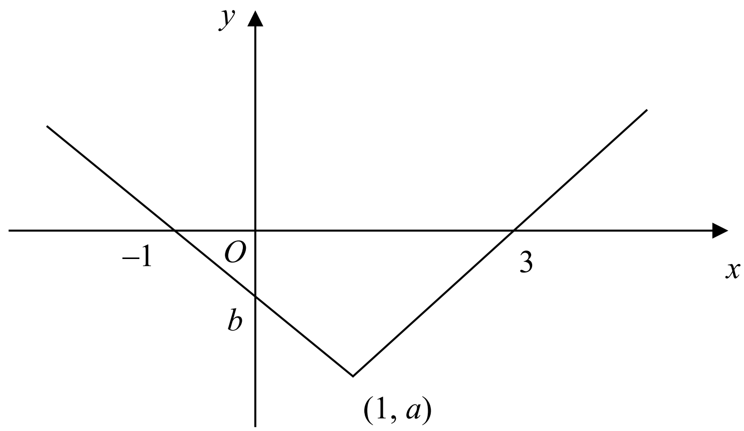

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = \mathrm{f}(x + 1)$,

(2)

(*b*) $y = \mathrm{f}(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $\mathrm{f}(x) = |x - 1| - 2$, find

(*c*) the value of a and the value of b,

(2)

(*d*) the value of x for which $\mathrm{f}(x) = 5x$.

(4)

18.

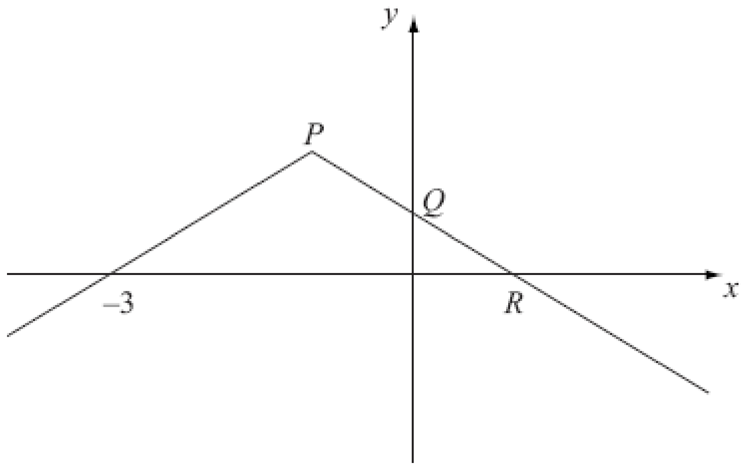

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points $(–3, 0)$ and R.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$,

(2)

(b) $y = \mathrm{f}(-x)$.

(2)

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P, Q and R,

(3)

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$.

(5)

19.

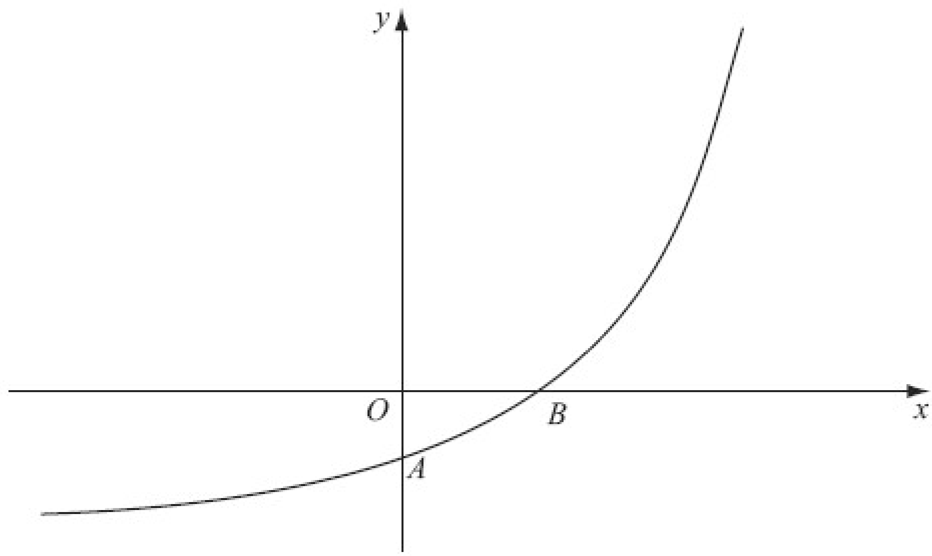

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0,\ 1 - k)$ and $B(\frac{1}{2}\ln k,\ 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(*c*) state the range of f,

(1)

(*d*) find $\mathrm{f}^{-1}(x)$,

(3)

(*e*) write down the domain of f^{-1}.

(1)