Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Circles

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

a) $\left(\frac{5+13}{2}, \frac{-1+11}{2}\right)$

$(9, 5)$

b/ $r^2 = 4^2 + 6^2$

$r^2 = 52$

$(x-9)^2 + (y-5)^2 = 52$

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2. The circle C has centre $(3, 1)$ and passes through the point $P(8, 3)$.

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

a) $r^2 = 5^2 + 2^2$

$= 29$

$(x-3)^2 + (y-1)^2 = 29$

b/ gradient of radius $= \frac{3-1}{8-3} = \frac{2}{5}$

gradient of tangent $= -\frac{5}{2}$

$y = -\frac{5}{2}x + c \quad (8, 3)$

$3 = -\frac{5}{2}(8) + c$

$3 = -20 + c$

$c = 23$

$y = -\frac{5}{2}x + 23$

$2y = -5x + 46$

$5x + 2y - 46 = 0$

3. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(*a*) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(*b*) Show that PQ is a diameter of C.

(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(*c*) Find the coordinates of R.

(4)

a) $(x-3)^2 - 9 + (y+2)^2 - 4 = 12$

$(x-3)^2 + (y+2)^2 = 25$

centre $= (3, -2)$ radius $= 5$

b/ $\left(\frac{-1+7}{2}, \frac{1-5}{2}\right) = (3, -2)$ [Midpoint is centre]

c/ point on circumference where $x = 0$

$(-3)^2 + (y+2)^2 = 25$

$(y+2)^2 = 16$

$y + 2 = \pm 4$

positive $\therefore y = 2$

$(0, 2)$

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4. The line joining points $(-1, 4)$ and $(3, 6)$ is a diameter of the circle C.

Find an equation for C.

(6)

centre $\left(\frac{-1+3}{2}, \frac{4+6}{2}\right)$

$(1, 5)$

~~rad~~ $r^2 = 2^2 + 1^2$

$= 5$

$(x-1)^2 + (y-5)^2 = 5$

5.

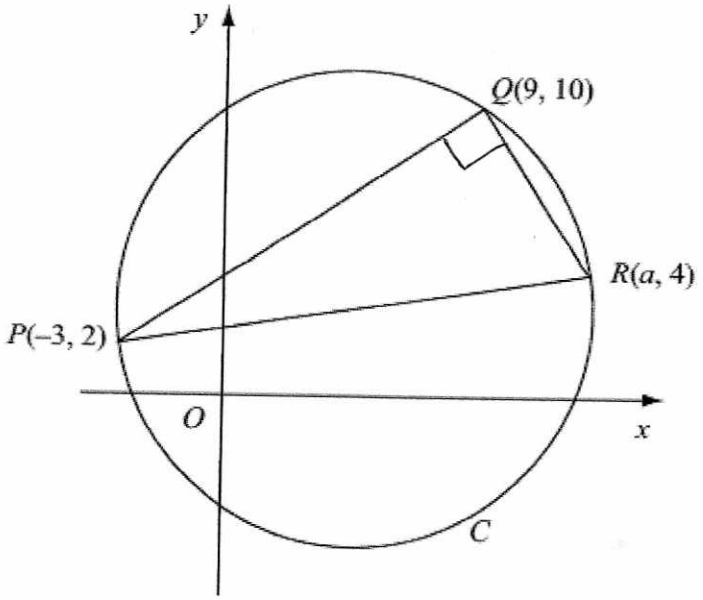


Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2.

Given that PR is a diameter of C,

(*a*) show that $a = 13$,

(3)

(*b*) find an equation for C.

(5)

a) PQ is perpendicular to QR

$$m_{PQ} = \frac{10-2}{9--3} = \frac{8}{12} = \frac{2}{3}$$

$$m_{QR} = \frac{-3}{2} = \frac{10-4}{9-a}$$

$$9-a = \frac{6}{-1.5}$$

$$9-a = -4$$

$$a = 13$$

b) centre $\left(\frac{-3+13}{2}, \frac{2+4}{2}\right)$

$$(5, 3)$$

$$r^2 = 8^2 + 1^2$$

$$= 65$$

$$(x-5)^2 + (y-3)^2 = 65$$

6. A circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = r^2.$$

(2)

Figure 3

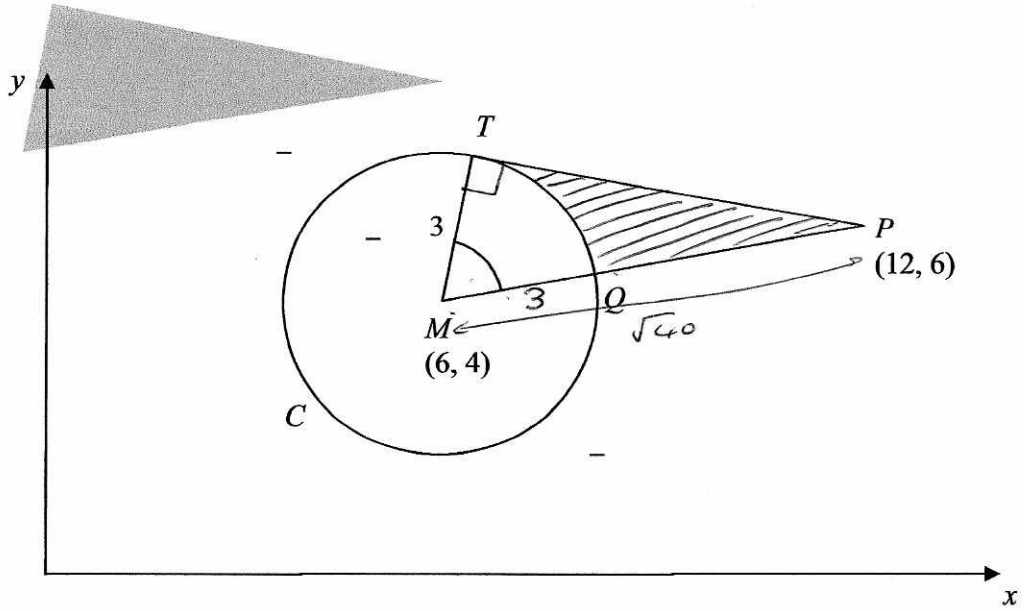


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

a) $(x-6)^2 + (y-4)^2 = 9$

b) $\overrightarrow{MP} = \sqrt{6^2 + 2^2} = \sqrt{40}$

$\cos\theta = \frac{3}{\sqrt{40}}$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{40}}\right)$

$= 1.0766^c$ (4dp)

c) Area of sector $= \frac{\theta}{2} r^2$

$= \frac{\theta}{2}(3)^2 = 4.84461127$

Area of triangle $= \frac{1}{2} ab \sin\theta$

$= \frac{1}{2}(3)(\sqrt{40}) \sin\theta$

$= 8.351646544$

$-$ $=$ 3.507 units^2 (3dp)

7.

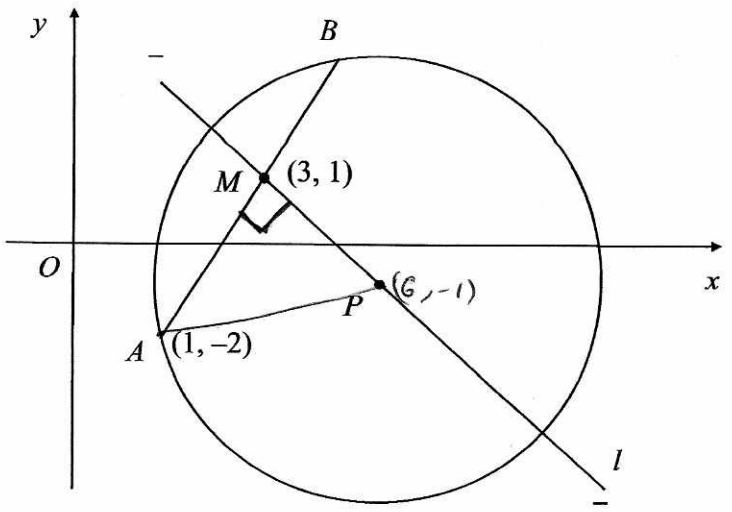


Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

$m_{AM} = \frac{1--2}{3-1} = \frac{3}{2}$

$m_l = -\frac{2}{3}$

$y = -\frac{2}{3}x + c \quad (3, 1)$

$1 = -\frac{2}{3}(3) + c$

$1 = -2 + c$

$c = 3$

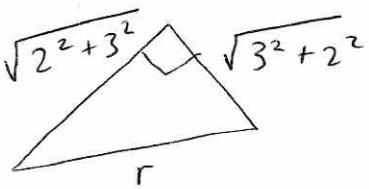
$y = -\frac{2}{3}x + 3$

b/ $y = -\frac{2}{3}(6) + 3$

$= -1$

c/ $(x-6)^2 + (y+1)^2 = r^2$

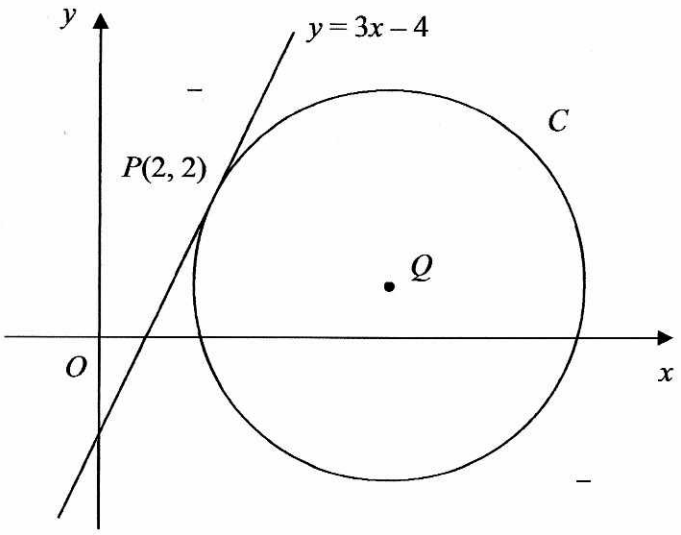
$(x-6)^2 + (y+1)^2 = 26$



$r^2 = 2^2 + 3^2 + 2^2 + 3^2$

$= 26$

8. **Figure 1**



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point P(2, 2), as shown in Figure 1.

The point Q is the centre of C.

(*a*) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

(*b*) show that the x-coordinate of Q is 5,

(1)

(*c*) find an equation for C.

(4)

a) $m = -1/3$ $\quad y = -1/3x + c \quad (2,2)$

$2 = -2/3 + c$

$c = 8/3$

$y = -1/3x + 8/3$

b) $1 = -1/3x + 8/3$

$-5/3 = -1/3x$

$-5 = -x$

$x = 5$

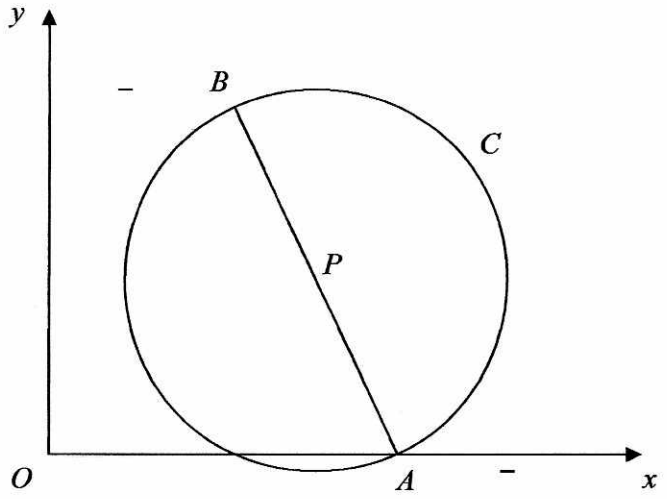
c) $(x-5)^2 + (y-1)^2 = r^2$

$(2,2) \quad (2-5)^2 + (2-1)^2 = r^2$

$r^2 = 10$

$(x-5)^2 + (y-1)^2 = 10$

9. **Figure 1**



In Figure 1, $A(4, 0)$ and $B(3, 5)$ are the end points of a diameter of the circle C.

Find

(a) the exact length of AB,

(2)

(b) the coordinates of the midpoint P of AB,

(2)

(c) an equation for the circle C.

(3)

a) $\sqrt{1^2 + 5^2} = \sqrt{26}$

b) $\left(\frac{4+3}{2}, \frac{0+5}{2}\right)$

$\left(\frac{7}{2}, \frac{5}{2}\right)$

c) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{\sqrt{26}}{2}\right)^2$

$\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{26}{4}$

$\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{13}{2}$

10. The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$.

Find

(a) the coordinates of A,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points at which C crosses the x-axis.

(2)

Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

(d) find an equation of the line which passes through A and T.

(3)

a) $(x-5)^2 - 25 + y^2 + 9 = 0$

$(x-5)^2 + y^2 = 16$

centre $(5, 0)$

b) radius $= \sqrt{16} = 4$

c) $(x-5)^2 = 16$

$x - 5 = \pm 4$

$x = 5 \pm 4$

$(9, 0)$ and $(1, 0)$

d) $m = -\frac{2}{7}$ $\quad y = -\frac{2}{7}x + c \quad (5, 0)$

$0 = -\frac{10}{7} + c$

$c = \frac{10}{7}$

$y = -\frac{2}{7}x + \frac{10}{7}$