Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Sequences and Series

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Calculators may NOT be used in this examination.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \quad n \geq 1,$$

where k is a constant.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(*c*) find the value of k.

(4)

2. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(*a*) the value of d,

(3)

(*b*) the value of a,

(2)

(*c*) the total number of houses built in Oldtown over the 40-year period.

(3)

3. The first term of an arithmetic sequence is 30 and the common difference is -1.5.

(*a*) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(*b*) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(*c*) Find the largest positive value of S_n.

(3)

4. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(*a*) Use this information to write down two equations for a and d.
(2)

(*b*) Show that $a = -17.5$ and find the value of d.
(2)

The sum of the first n terms of the series is 2750.

(*c*) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$
(4)

(*d*) Hence find the value of n.
(3)

5. A sequence x_1 , x_2 , x_3 , … is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(*a*) Find an expression for x_2 in terms of a.
(1)

(*b*) Show that $x_3 = a^2 - 3a - 3$.
(2)

Given that $x_3 = 7$,

(*c*) find the possible values of a.
(3)

6. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(*a*) Show that on the 4th Saturday of training she runs 11 km.

(1)

(*b*) Find an expression, in terms of n, for the length of her training run on the nth Saturday.

(2)

(*c*) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.

(3)

On the nth Saturday Sue runs 43 km.

(*d*) Find the value of n.

(2)

(*e*) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

7. A sequence is given by

$$x_1 = 1,$$

$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(*a*) Find x_2 in terms of p.

(1)

(*b*) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(*c*) find the value of p,

(3)

(*d*) write down the value of x_{2008}.

(2)

8. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(*a*) Find the amount she saves in Week 200.

(3)

(*b*) Calculate her total savings over the complete 200 week period.

(3)

9. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(*a*) Find the value a_2 and the value of a_3.

(2)

(*b*) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

10. A sequence $a_1,\ a_2,\ a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \quad n \geq 1,$$

where *k* is a positive integer.

(*a*) Write down an expression for a_2 in terms of *k*.

(1)

(*b*) Show that $a_3 = 9k + 20$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of *k*.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

(4)

11. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 

Row 2 

Row 3 

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(*a*) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(*b*) Find the total number of sticks Ann uses in making these 10 rows.

(3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k + 1)$th row,

(*c*) show that k satisfies $(3k - 100)(k + 35) < 0$.

(4)

(*d*) Find the value of k.

(2)

12. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of a and the value of d.

(7)

13. The sequence of positive numbers $u_1, u_2, u_3, \ldots$, is given by

$$u_{n+1} = (u_n - 3)^2, \qquad u_1 = 1.$$

(*a*) Find u_2, u_3 and u_4.

(3)

(*b*) Write down the value of u_{20}.

14. The rth term of an arithmetic series is $(2r - 5)$.

(*a*) Write down the first three terms of this series.

(2)

(*b*) State the value of the common difference.

(1)

(*c*) Show that $\sum_{r=1}^{n} (2r - 5) = n(n - 4)$.

(3)

15. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(*a*) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200.

(1)

(*b*) Find the amount of Alice's annual allowance on her 18th birthday.

(2)

(*c*) Find the total of the allowances that Alice had received up to and including her 18th birthday.

(3)

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(*d*) Find how old Alice was when she received her last allowance.

(7)

16. An arithmetic series has first term a and common difference d.

(*a*) Prove that the sum of the first n terms of the series is

$$\frac{1}{2}n[2a + (n - 1)d].$$

(4)

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

(*b*) Find the amount Sean repays in the 21st month.

(2)

Over the n months, he repays a total of £5000.

(*c*) Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0.$$

(3)

(*d*) Solve the equation in part (*c*).

(3)

(*e*) State, with a reason, which of the solutions to the equation in part (*c*) is **not** a sensible solution to the repayment problem.

(1)

END