Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Differentiation and Integration

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Calculators may NOT be used in this examination.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k,

(4)

(c) the value of the y-coordinate of A.

(2)

2. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB.

(4)

3. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(*a*) Sketch C, showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(*b*) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(*c*) Find the x-coordinates of these two points.

(6)

4. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(*a*) find $\mathrm{f}(x)$ and simplify your answer.

(6)

(*b*) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

5. The curve C with equation $y = \mathrm{f}(x)$ passes through the point $(5, 65)$.

Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$,

(*a*) use integration to find $\mathrm{f}(x)$.

(4)

(*b*) Hence show that $\mathrm{f}(x) = x(2x + 3)(x - 4)$.

(2)

(*c*) Sketch C, showing the coordinates of the points where C crosses the x-axis.

(3)

6. The curve C has equation $y = x^2(x - 6) + \dfrac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

7. The curve C has equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $\mathrm{f}(x)$.

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Show that the point $P(4, 8)$ lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

9. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$,

(*a*) find $\mathrm{f}(x)$.

(5)

(*b*) Verify that $\mathrm{f}(-2) = 5$.

(1)

(*c*) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

10. The curve C has equation $y = 4x^2 + \dfrac{5 - x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at P is 3.

(5)

(*b*) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k.

(2)

11. The gradient of the curve C is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(*a*) Find an equation of the normal to C at P.

(4)

(*b*) Find an equation for the curve C in the form $y = \mathrm{f}(x)$.

(5)

(*c*) Using $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.

(2)

12. **Figure 2**

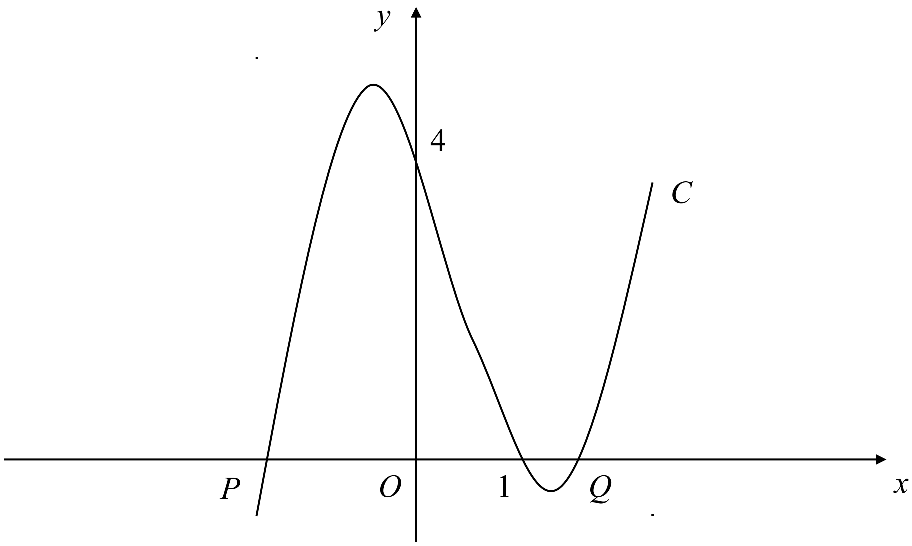

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P and the x-coordinate of Q.
(2)

(b) Show that $\dfrac{dy}{dx} = 3x^2 - 2x - 4$.
(3)

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$.
(2)

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R.
(5)

13. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates (3, 0).

(*a*) Show that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(*c*) Find the coordinates of Q.

(5)

14. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0.$$

The point P has coordinates (2, 7).

(*a*) Show that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(*c*) show that the x-coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$.

(5)

END