

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1a)  $f(x) = x^5 + 3x^2 + x - 10$

$$f'(x) = 5x^4 + 6x + 1$$

b)  $x_1 = 1.25 - \frac{(1.25)^5 + 3(1.25)^2 + 1.25 - 10}{5(1.25)^4 + 6(1.25) + 1}$

$$= 1.298811545$$

$$x_2 = 1.296399201$$

$$x_3 = 1.296392881$$

$$x_4 = 1.296392881$$

$$x = \underline{\underline{1.296}} \quad 3dp$$

2/

$$f(x) = \ln(2x+1) + x^2 - 5$$

$$f(1.8) = \ln(2(1.8)+1) + (1.8)^2 - 5 = -0.2339\dots$$

$$f(1.9) = \ln(2(1.9)+1) + (1.9)^2 - 5 = 0.1786\dots$$

change of sign  $\therefore$  root in interval  $[1.8, 1.9]$

b/

$$f'(x) = \frac{2}{2x+1} + 2x$$

c/

$$\begin{aligned} x_1 &= 1.8 - \frac{\ln(2(1.8)+1) + (1.8)^2 - 5}{\frac{2}{2(1.8)+1} + 2(1.8)} \\ &= \underline{\underline{1.858}} \quad 3dp \end{aligned}$$

3a/

$$f(x) = x^3 + 3x^2 - 2x^{\frac{1}{2}}$$

$$f(0.6) = (0.6)^3 + 3(0.6)^2 - 2(0.6)^{\frac{1}{2}} = -0.253\dots$$

$$f(0.7) = (0.7)^3 + 3(0.7)^2 - 2(0.7)^{\frac{1}{2}} = 1.396\dots$$

change of sign  $\therefore$  root in interval  $[0.6, 0.7]$

b/

$$f'(x) = 3x^2 + 6x - x^{-\frac{1}{2}}$$

c/

$$x_1 = 0.65 - \frac{(0.65)^3 + 3(0.65)^2 - 2(0.65)^{\frac{1}{2}}}{3(0.65)^2 + 6(0.65) - (0.65)^{-\frac{1}{2}}}$$

$$= \underline{\underline{0.650}}$$

$$= \underline{\underline{0.668}}$$

4a/

$$f(x) = \sin 2x + \ln x$$

$$f'(x) = 2 \cos 2x + \frac{1}{x}$$

stationary point where  $f'(x) = 0$

$$f'(1) = 2 \cos(2) + \frac{1}{1} = 0.1677\dots$$

$$f'(1.1) = 2 \cos(2.2) + \frac{1}{1.1} = -0.2679\dots$$

change of sign  $\therefore$  stationary point in interval  $[1, 1.1]$

b/

$$f'(x) = 2 \cos 2x + x^{-1}$$

$$f''(x) = -4 \sin 2x - x^{-2}$$

$$x_1 = (1.05) - \frac{2 \cos(2(1.05)) + (1.05)^{-1}}{-4 \sin(2(1.05)) - (1.05)^{-2}}$$

$$= 1.036854815$$

$$x_2 = 1.036966397$$

$$\cancel{x_3 = 1.036966405}$$