

Write your name here

Surname

Other Names

AS/A Level Mathematics

Newton-Raphson

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled..
- Answer the questions in the spaces provided
– there may be more space than you need.
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1

$$f(x) = x^5 + 3x^2 + x - 10$$

- (a) Find $f'(x)$ (2)

The root of $f(x) = 0$ lies in the interval $[1, 1.5]$

- (b) Use the Newton-Raphson method, starting with $x_0 = 1.25$, to find the root of $f(x) = 0$ to 3 decimal places. (5)

(Total for question 1 is 7 marks)

2

$$f(x) = \ln(2x + 1) + x^2 - 5 \quad x > -0.5$$

- (a) Show that $f(x) = 0$ has a root in the interval $[1.8, 1.9]$ (2)

- (b) Find $f'(x)$ (3)

- (c) Using $x_0 = 1.8$ as a first approximation, apply the Newton-Raphson procedure to find a second approximation, giving your answer to 3 decimal places. (3)

(Total for question 2 is 8 marks)

3

$$f(x) = x^3 + 3x^2 - 2\sqrt{x} \quad x > 0$$

- (a) Show that $f(x) = 0$ has a root in the interval $[0.6, 0.7]$ (2)

- (b) Find $f'(x)$ (3)

- (c) Starting with $x_0 = 0.65$, apply the Newton-Raphson procedure once to find an approximate solution to the equation $f(x) = 0$ giving your answer to 3 decimal places. (3)

(Total for question 3 is 8 marks)

4

$$f(x) = \sin(2x) + \ln(x) \quad x > 0$$

- (a) Show that $y = f(x)$ has a stationary point in the interval $[1, 1.1]$ (5)

- (b) Starting with $x_0 = 1.05$, apply the Newton-Raphson procedure twice to find an approximation for the x coordinate of the stationary point in the interval $[1, 1.1]$. (4)

(Total for question 4 is 9 marks)
