1a) $f(x) = x^3 + 2x^2 + x + 1$

$f(-1.5) = (-1.5)^3 + 2(-1.5)^2 + (-1.5) + 1$

$= \frac{5}{8}$

$f(-2) = (-2)^3 + 2(-2)^2 + (-2) + 1$

$= -1$

change of sign $\therefore$ root between -1.5 and -2.

b/ $0 = x^3 + 2x^2 + x + 1$

$\cancel{-x - 1 = x(x^2 + 2x)}$

$-x - 1 = x^3 + 2x^2$

$x^3 = -x - 1 - 2x^2$

$x = \frac{-x-1}{x^2} - 2$

$= -\left(\frac{x+1}{x^2} + 2\right)$

c/ $x_1 = -\left(\frac{-2+1}{(-2)^2} + 2\right)$

$= -1.75$

$x_2 = -1.755102041 \quad (-1.755)$

$x_3 = -1.754867496 \quad (-1.755)$

d/ $f(-1.7545) = 1.22 \times 10^{-3}$

$f(-1.7555) = -2.00 \times 10^{-3}$

change of sign $\therefore$ $\alpha = -1.755$ to 3dp

2/ $f(x) = e^{x+2} + x - 10$

$0 = e^{x+2} + x - 10$

$10 - x = e^{x+2}$

$\ln(10 - x) = x + 2$

$\ln(10 - x) - 2 = x$

$x = \ln(10 - x) - 2$

b/ $x_1 = \ln(10 - (0.5)) - 2$

$= 0.251$

$x_2 = \ln(10 - \text{Ans}) - 2$

$= 0.277$

$x_3 = 0.274$

c/ $f(0.2745) = -2.44 \times 10^{-3}$

$f(0.2755) = 8.28 \times 10^{-3}$

change of sign $\therefore$ $\alpha = 0.275$ to 3dp.

3a) $$y = (12 - x)\ln x$$

$$u = 12 - x \qquad v = \ln x$$

$$\frac{du}{dx} = -1 \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x}(12 - x) - \ln x$$

$$= 12x^{-1} - 1 - \ln x$$

b) ~~max~~ stationary point where $\frac{dy}{dx} = 0$

$$\frac{12}{4.5} - 1 - \ln 4.5 = 0.16\ldots$$

$$\frac{12}{5} - 1 - \ln 5 = -0.209\ldots$$

change of sign $\therefore$ P lies between 4.5 and 5.

c) $$\frac{12}{x} - 1 - \ln x = 0$$

$$\frac{12}{x} = 1 + \ln x$$

$$\frac{12}{1 + \ln x} = x$$

d) $$x_1 = \frac{12}{1 + \ln(4.75)}$$

$$= 4.691$$

$$x_2 = 4.714$$

$$x_3 = 4.705$$

4/ $y = e^x$ $y = 5x - 1$

$$e^x = 5x - 1$$
$$x = \ln(5x - 1)$$

b/ $x_1 = \ln(5(2.5) - 1)$

$= 2.442$

$x_2 = \ln(5(\text{Ans}) - 1)$

$= 2.417$

$x_3 = \ln(5(\text{Ans}) - 1)$

$= 2.406$