

1a/

$$2x^2 - 3xy + y^2 = 12$$

$$u = -3x \quad v = y$$

$$\frac{du}{dx} = -3 \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$4x - 3y - 3x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (2y - 3x) = 4x - 3y$$

$$\frac{dy}{dx} = \frac{3y - 4x}{2y - 3x}$$

b/

$$\text{at } (1, -2) \quad \frac{dy}{dx} = \frac{3(-2) - 4(1)}{2(-2) - 3(1)}$$

$$= \frac{10}{7}$$

gradient of normal:  $\frac{7}{10}$

$$y = \frac{7}{10}x + c \quad (1, -2)$$

$$-2 = \frac{7}{10}(1) + c$$

$$c = -\frac{27}{10}$$

$$y = \frac{7}{10}x - \frac{27}{10}$$

2a/

$$3x^2 + xy + y^2 = 20$$

$$u=x \quad v=y$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$6x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (x + 2y) = -6x - y$$

$$\frac{dy}{dx} = \frac{-6x - y}{x + 2y}$$

$$\frac{-6x - y}{x + 2y} = \frac{4}{3}$$

$$3(-6x - y) = 4(x + 2y)$$

$$-18x - 3y = 4x + 8y$$

$$0 = 22x + 11y$$

$$0 = \underline{\underline{2x + y}}$$

b/

$$y = -2x$$

$$3x^2 + x(-2x) + (-2x)^2 = 20$$

$$3x^2 - 2x^2 + 4x^2 = 20$$

$$5x^2 = 20$$

$$x^2 = 4$$

$$x = \pm 2$$

$$y = -2(2)$$

$$= -4$$

$$y = -2(-2)$$

$$= 4$$

$$\underline{\underline{(2, -4)}} \quad \text{and} \quad \underline{\underline{(-2, 4)}}$$

$$3a) \quad x^2 + 4xy - x + y^2 = 35$$

$$u = 4x \quad v = y$$

$$\frac{du}{dx} = 4 \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$2x + 4y + 4x \frac{dy}{dx} - 1 + 2y \frac{dy}{dx} = \cancel{35} 0$$

$$4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 1 - 2x - 4y$$

$$\frac{dy}{dx} (4x + 2y) = 1 - 2x - 4y$$

$$\frac{dy}{dx} = \frac{1 - 2x - 4y}{4x + 2y}$$

$$b) \quad \text{at } (2, 3) \quad \frac{dy}{dx} = \frac{1 - 2(2) - 4(3)}{4(2) + 2(3)}$$
$$= \frac{-15}{14}$$

$$y = \frac{-15}{14} x + c$$

$$3 = \frac{-15}{14} (2) + c$$

$$c = \frac{36}{7}$$

$$y = \frac{-15}{14} x + \frac{36}{7}$$

4/

$$2 \sin x + 2 \cos y = 3$$

$$2 \cos x - 2 \sin y \frac{dy}{dx} = 0$$

$$2 \cos x = 2 \sin y \frac{dy}{dx}$$

$$\frac{2 \cos x}{2 \sin y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin y}$$

b/

$$\frac{\cos x}{\sin y} = 0$$

$$\cos x = 0$$

$$x = \underline{\underline{\frac{1}{2}\pi}}$$

$$2 \sin\left(\frac{1}{2}\pi\right) + 2 \cos y = 3$$

$$2 + 2 \cos y = 3$$

$$2 \cos y = 1$$

$$\cos y = \frac{1}{2}$$

$$y = \underline{\underline{\frac{\pi}{3}}}$$

$$\underline{\underline{\left(\frac{\pi}{2}, \frac{\pi}{3}\right)}}$$

5a)

$$y = 2^x$$

$$\ln y = \ln 2^x$$

$$\ln y = x \ln 2$$

$$\frac{1}{y} \frac{dy}{dx} = \ln 2$$

$$\frac{dy}{dx} = y \ln 2$$

$$[y = 2^x]$$

$$= \underline{\underline{2^x \ln 2}}$$

b/

$$y = 3^{x^2}$$

$$\frac{dy}{dx} \quad \ln y = \ln 3^{x^2}$$

$$\ln y = x^2 \ln 3$$

$$\frac{1}{y} \frac{dy}{dx} = 2x \ln 3$$

$$\frac{dy}{dx} = y \cdot 2x \ln 3$$

$$= \ln 3^{x^2} \cdot 2x \ln 3$$

when  $x = 2$ 

$$\frac{dy}{dx} = 324 \ln 3$$

$$y = 324 \ln 3 x + C$$

$$81 = 324 \ln 3 (2) + C$$

$$\frac{1}{8} = \ln$$

$$81 - 648 \ln 3 = C$$

$$y = 324 \ln 3 x + 81 - 648 \ln 3$$