1) $(1+x)^{-2}$

$$1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3$$

$$1 - 2x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{6}x^3$$

$$1 - 2x + 3x^2 - 4x^3$$

2a) $4^{-\frac{1}{2}}\left(1+\frac{3}{2}x\right)^{-\frac{1}{2}}$

$$\frac{1}{2}\left(1+\frac{3}{2}x\right)^{-\frac{1}{2}}$$

$$\frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(\frac{3}{2}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{3}{2}x\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6}\left(\frac{3}{2}x\right)^3\right)$$

$$\frac{1}{2}\left(1 - \frac{3}{4}x + \frac{27}{32}x^2 - \frac{135}{128}x^3\right)$$

$$\frac{1}{2} - \frac{3}{8}x + \frac{27}{64}x^2 - \frac{135}{256}x^3$$

b) $|x| < \frac{2}{3}$

3a) $8^{\frac{1}{3}}\left(1+\frac{3}{2}x\right)^{\frac{1}{3}}$

$$2\left(1+\frac{3}{2}x\right)^{\frac{1}{3}}$$

$$2\left[1 + \left(\frac{1}{3}\right)\left(\frac{3}{2}x\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}\left(\frac{3}{2}x\right)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6}\left(\frac{3}{2}x\right)^3\right)$$

$$2\left(1 + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{5}{24}x^3\right)$$

$$2 + x - \frac{1}{2}x^2 + \frac{5}{12}x^3$$

b) $|x| < \frac{2}{3}$

4a) $(1+2x)^{\frac{1}{3}}$

$$1 + \left(\tfrac{1}{3}\right)(2x) + \frac{\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)}{2}(2x)^2 + \frac{\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)\left(-\tfrac{5}{3}\right)}{6}(2x)^3$$

$$1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3$$

$|x| < \frac{1}{2}$

b) $(1+2x)^{\frac{1}{3}} = 1.1^{\frac{1}{3}}$

$1 + 2x = 1.1$

$2x = 0.1$

$x = 0.05$

$$1 + \frac{2}{3}(0.05) - \frac{4}{9}(0.05)^2 + \frac{40}{81}(0.05)^3$$

$= 1.03228$ (5dp)

5a) $(2+3x)^{-2}$

$2^{-2}\left(1+\frac{3}{2}x\right)^{-2}$

$\frac{1}{4}\left(1+\frac{3}{2}x\right)^{-2}$

$$\frac{1}{4}\left(1 + (-2)\left(\tfrac{3}{2}x\right) + \frac{(-2)(-3)}{2}\left(\tfrac{3}{2}x\right)^2\right)$$

$$\frac{1}{4}\left(1 - 3x + \frac{27}{4}x^2\right)$$

$$\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 \qquad \cancel{+x+}$$

b) $|x| < \frac{2}{3}$

6a) $3^{\frac{1}{2}}\left(1+\frac{2}{3}x\right)^{\frac{1}{2}}$

$3^{\frac{1}{2}}\left(1+\left(\frac{1}{2}\right)\left(\frac{2}{3}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{2}{3}x\right)^2}{2}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{2}{3}x\right)^3}{6}\right)$

$3^{\frac{1}{2}}\left(1+\frac{1}{3}x-\frac{1}{18}x^2+\frac{1}{54}x^3\right)$

$\sqrt{3}+\frac{\sqrt{3}}{3}x-\frac{\sqrt{3}}{18}x^2+\frac{\sqrt{3}}{54}x^3$

b) $|x|<\frac{3}{2}$

c) $x=0.1 \qquad (3.2)^{\frac{1}{2}}=\frac{4\sqrt{5}}{5}$

$\frac{4\sqrt{5}}{5}=\sqrt{3}+\frac{\sqrt{3}}{3}(0.1)-\frac{\sqrt{3}}{18}(0.1)^2+\frac{\sqrt{3}}{54}(0.1)^3$

$\frac{4\sqrt{5}}{5}=1.788855659$

$\sqrt{5}=2.236 \quad (3\,dp)$

7a) $(1+3x)^{-3}$

$1 + (-3)(3x) + \frac{(-3)(-4)(3x)^2}{2} + \frac{(-3)(-4)(-5)(3x)^3}{6}$

$1 - 9x + 54x^2 - 270x^3$

b) $|x| < \frac{1}{3}$

c) $(2x+1)(1 - 9x + 54x^2 - 270x^3)$

$2x - 18x^2 + 108x^3 + 1 - 9x + 54x^2 - 270x^3$

$1 - 7x + 36x^2 - 162x^3$

8a) $(9-2x)^{\frac{1}{2}}$

$9^{\frac{1}{2}}(1 - \frac{2}{9}x)^{\frac{1}{2}}$

$3(1 - \frac{2}{9}x)^{\frac{1}{2}}$

$3\left(1 + (\frac{1}{2})(-\frac{2}{9}x) + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{2}{9}x)^2}{2} + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{2}{9}x)^3}{6}\right)$

$3(1 - \frac{1}{9}x - \frac{1}{162}x^2 - \frac{1}{1458}x^3)$

$3 - \frac{1}{3}x - \frac{1}{54}x^2 - \frac{1}{486}x^3$

b) $|x| < \frac{9}{2}$

c) $(9-2x)^{\frac{1}{2}} = 8 \cdot 9^{\frac{1}{2}}$

$9 - 2x = 8 \cdot 9$

$0.1 = 2x$

$x = 0.05$

$3 - \frac{1}{3}(0.05) - \frac{1}{54}(0.05)^2 - \frac{1}{486}(0.05)^3$

$\underline{2.9833}$