Name: _____

Maths Genie Stage 13

Test B

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 Prove algebraically that the sum of the squares of any 2 even positive integers is always a multiple of 4.

$$(2n)^2 + (2m)^2$$

$$4n^2 + 4m^2$$

$$4(n^2 + m^2)$$

(Total for Question 1 is 2 marks)

2 (a) Write $x^2 + 4x + 7$ in the form $(x + a)^2 + b$ where a and b are integers.

$$(x + 2)^2 - 4 + 7$$

$$(x + 2)^2 + 3$$

$(x + 2)^2 + 3$

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = x^2 + 4x + 7$

$(-2, 3)$

(1)

(Total for Question 2 is 3 marks)

3

$$F = 15.6 \text{ N} \quad \text{correct to 3 significant figures}$$
$$A = 4.31 \text{ m}^2 \quad \text{correct to 2 decimal places}$$

By considering bounds, work out the value of p to a suitable degree of accuracy.
Give a reason for your answer.

$$p = \frac{F}{A}$$

p = pressure
F = force
A = area

upper $p = \dfrac{\text{upper } F}{\text{lower } A}$ lower $p = \dfrac{\text{lower } F}{\text{upper } A}$

upper $p = \dfrac{15.65}{4.305}$ lower $p = \dfrac{15.55}{4.315}$

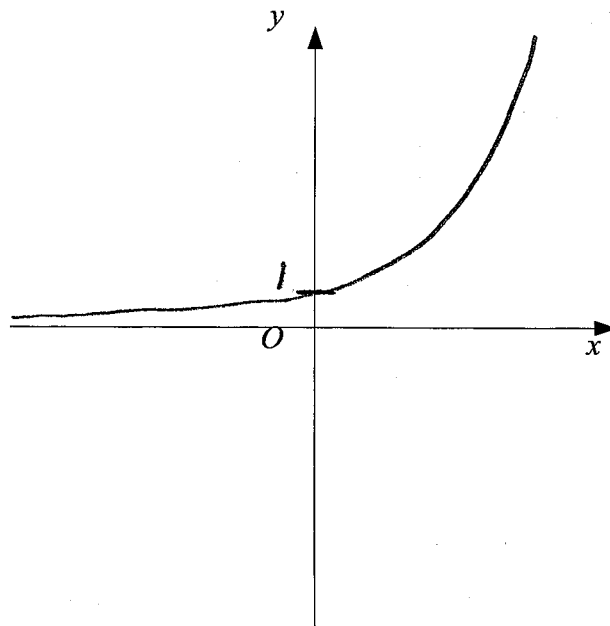$= 3.6353...$ $= 3.6037...$

$p = 3.6 \, (1dp)$ the upper and lower bound both round to $3.6 \, (1dp)$

.........**3.6**.........Nm^{-2}

(Total for Question 3 is 3 marks)

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4   On the grid, sketch the curve with equation $y = 2^x$
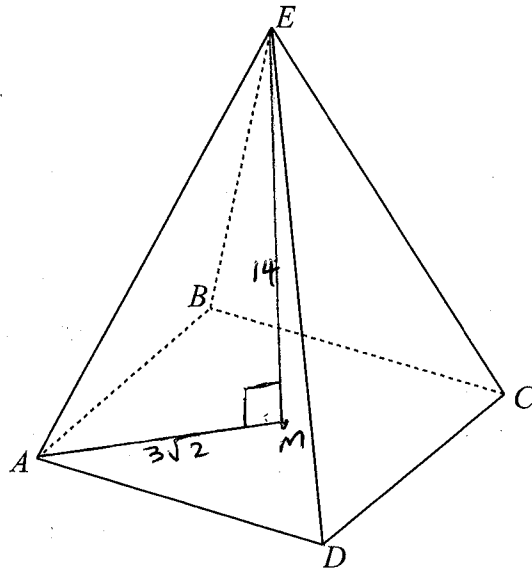Give the coordinates of any points of intersection with the axes.

$(0, 1)$

(Total for Question 4 is 2 marks)

5   The diagram shows a pyramid.
The base of the pyramid *ABCD* is a square.

AB = 6 cm
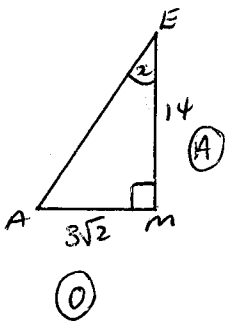The point *E* is 14 cm vertically above the base.

Calculate the size of angle *AEC*.

$$AC^2 = 6^2 + 6^2$$
$$AC = \sqrt{6^2 + 6^2}$$
$$= 6\sqrt{2}$$

$$AM = 3\sqrt{2}$$

$$\tan x = \frac{3\sqrt{2}}{14}$$
$$x = \tan^{-1}\left(\frac{3\sqrt{2}}{14}\right)$$
$$= 16.859°$$

$$AEC = 2 \times 16.859$$
$$= 33.7° \quad 1dp$$

........................33.7........................ °

(Total for Question 5 is 4 marks)

6 60 people were asked if they like tea, coffee and hot chocolate.

Every person liked at least one of the drinks.
21 of the people like all three drinks.

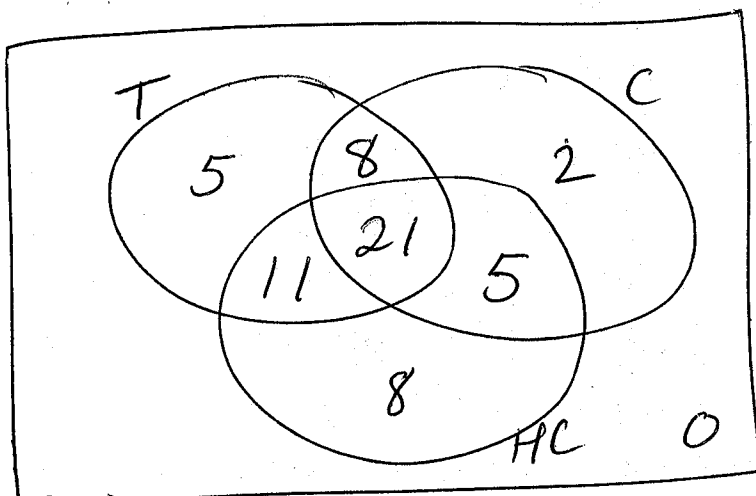45 of the people like hot chocolate
36 of the people like coffee.
29 of the people like tea and coffee.

11 of the people like tea and hot chocolate but not coffee
5 of the people like coffee and hot chocolate but not tea

Two of the 60 people are chosen at random.
Work out the probability that they both like tea.

45 people like tea

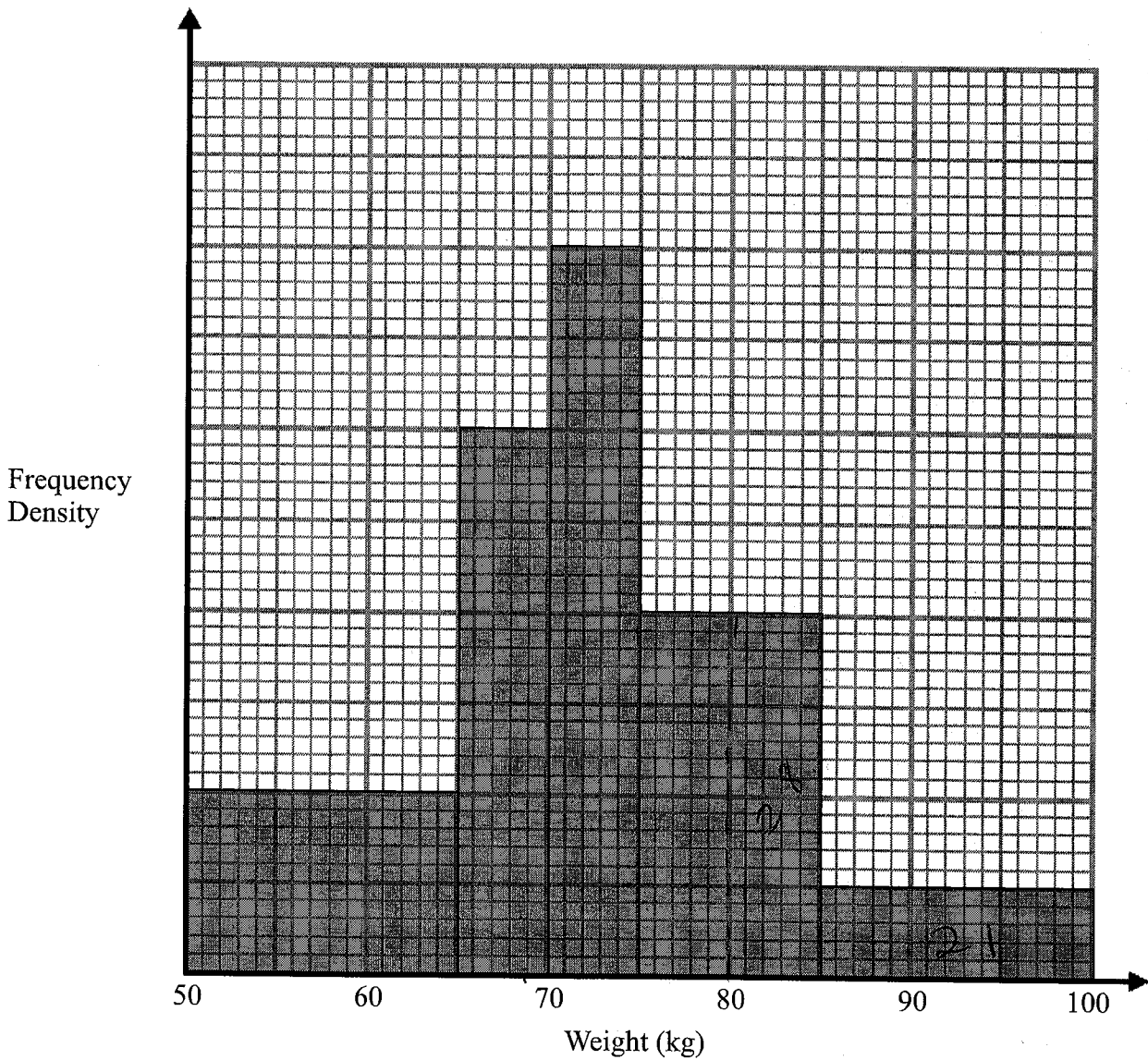$$\frac{45}{60} \times \frac{44}{59}$$

$$\frac{3}{4} \times \frac{44^{11}}{59} = \frac{33}{59}$$

$$\frac{33}{59}$$

(Total for Question 6 is 5 marks)

OR $\frac{1980}{3540}$

7 The histogram shows information about the weight of pigs.

Frequency Density

Weight (kg)

98 pigs weigh between 65 and 75 kg.

(handwritten: 98 pigs = 14 squares)
(handwritten: 7 pigs = 1 square)

(a) Work out an estimate for the number of pigs which weigh more than 80kg.

(handwritten answer: 49)

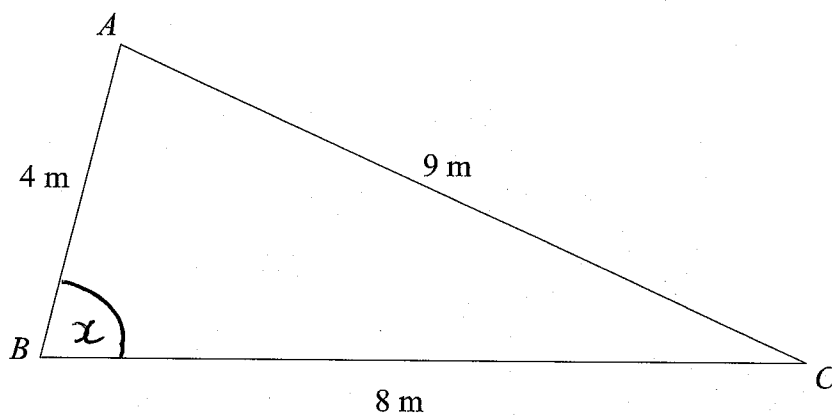......................................

(3)

(b) Explain why your answer to part a is only an estimate.

(handwritten: we do not know how many pigs weigh between 80kg and 85kg — (we assumed half of the 75kg-85kg group))

(1)

(Total for Question 7 is 4 marks)

8

4 m, 9 m, 8 m

Work out the size of angle ABC
Give your answer to the nearest degree.

$$\cos x = \frac{8^2 + 4^2 - 9^2}{2(8)(4)}$$

$$x = \cos^{-1}(\text{Ans})$$

$$= 90.895...^\circ$$

$$= 91^\circ$$

........91........ °

(Total for Question 8 is 3 marks)

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9 Solve $\dfrac{15}{x+1} + \dfrac{10}{2x-1} = 11$

$$\frac{15(2x-1) + 10(x+1)}{(x+1)(2x-1)} = 11$$

$$15(2x-1) + 10(x+1) = 11(x+1)(2x-1)$$

$$30x - 15 + 10x + 10 = 11(2x^2 - x + 2x - 1)$$

$$40x - 5 = 11(2x^2 + x - 1)$$
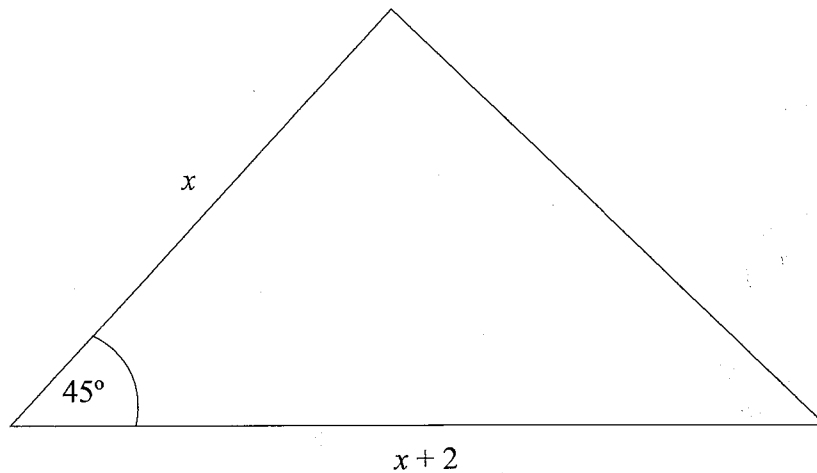
$$40x - 5 = 22x^2 + 11x - 11$$

$$0 = 22x^2 - 29x - 6$$

$$a = 22 \quad b = -29 \quad c = -6$$

$$x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(22)(-6)}}{2(22)}$$

$$x = \frac{3}{2} \text{ or } x = -\frac{2}{11}$$

(Total for Question 9 is 5 marks)

10

The area of the triangle is $56\sqrt{2}$ cm^2.
Work out the value of x.

$$\frac{1}{2}(x)(x+2)\sin 45 = 56\sqrt{2}$$

$$\frac{1}{2}(x^2+2x)\frac{\sqrt{2}}{2} = 56\sqrt{2}$$

$$\frac{1}{4}(x^2+2x) = 56$$

$$x^2 + 2x = 224$$

$$x^2 + 2x - 224 = 0$$

$$(x+16)(x-14) = 0$$

$$x = -16 \qquad x = 14$$

A length cannot be negative $\therefore x = 14$

.............14......... (cm)

(Total for Question 10 is 4 marks)