Name: _____

Maths Genie Stage 12

Test D

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

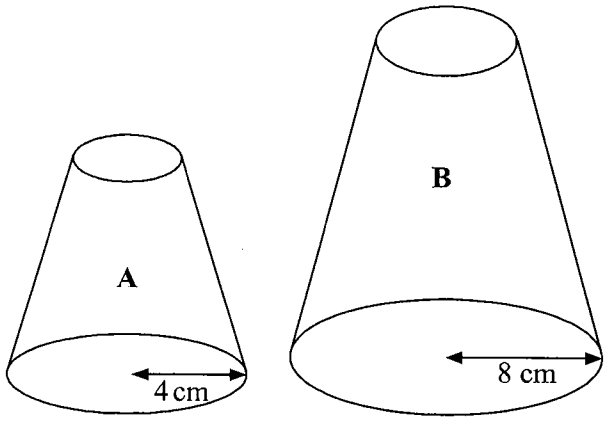
1 There are 14 boys and 16 girls in a class.
One boy and one girl will be selected to represent the class on the student council.
Work out the total number of ways of choosing a boy and a girl.

14×16

224

(Total for Question 1 is 2 marks)

2



Two solid shapes, A and B, are mathematically similar.

The base of shape A is a circle with radius 4 cm.
The base of shape B is a circle with radius 8 cm.
The volume of shape A is 140 cm^3.

Work out the volume of shape B.

scale factor for length 2

s.f volume $= 2^3 = 8$

$140 \times 8 = 1120\ cm^3$

1120 cm^3

(Total for Question 2 is 3 marks)

3 Alex invests some money for 4 years in a savings account.
She gets 2.6% per annum compound interest.

Alex has £4709.54 at the end of 4 years, work how much she invested.

$$x \times 1.026^4 = 4709.54$$

$$x = \frac{4709.54}{1.026^4}$$

$$= 4250$$

£ 4250

(Total for Question 3 is 3 marks)

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4 x is directly proportional to the cube of y

When $x = 64$, $y = 0.5$

Find the value of y when $x = 1728$

$$x \propto y^3$$

$$x = ky^3$$

$$64 = k(0.5)^3$$

$$64 = \frac{1}{8}k$$

$$k = 512$$

$$x = 512y^3$$

$$1728 = 512y^3$$

$$\frac{27}{8} = y^3$$

$$y = 1.5$$

$y =$ 1.5

(Total for Question 4 is 3 marks)

5 Factorise fully $5x^2 - 80$

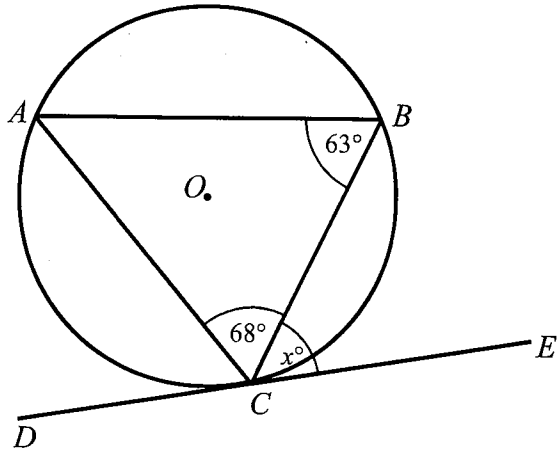
$5(x^2 - 16)$

$5(x + 4)(x - 4)$

$5(x + 4)(x - 4)$

(Total for Question 5 is 2 marks)

6



A, *B* and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle $ABC = 63°$
Angle $ACB = 68°$
Angle $BCE = x°$

Find the value of x.
Give reasons for each stage of your working.

$BAC = 180 - 63 - 68$

$= 49°$ Angles in a triangle add to 180°

$BCE = 49°$ Alternate segment theorem

49 °

(Total for Question 6 is 3 marks)

7 Here are the first 5 terms of a quadratic sequence.

1	4	8	13	19

Find an expression, in terms of n, for the nth term of this sequence.

an^2+bn+c

$a+b+c \rightarrow$ 1 4 8 13

$3a+b \rightarrow$ 3 4 5

$2a \rightarrow$ 1 1

$2a = 1$ $\quad a = 0.5$

$3(0.5)+b = 3$ $\quad 1.5+b = 3$ $\quad b = 1.5$

$a+b+c = 1$ $\quad 0.5+1.5+c = 1$ $\quad 2+c = 1$ $\quad c = -1$

$0.5n^2+1.5n-1$

(Total for Question 7 is 4 marks)

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8 Given that $f(x) = 3x+1$ and $g(x) = x^2-6$

(a) Work out an expression for $gf(x)$

$gf(x) = (3x+1)^2 - 6$

$= (3x+1)(3x+1) - 6$

$= 9x^2+3x+3x+1-6$

$= 9x^2+6x-5$

$9x^2+6x-5$

(2)

(b) Solve $gf(x) = 0$

Give your answers correct to 3 significant figures.

$9x^2+6x-5 = 0$

$a = 9 \quad b = 6 \quad c = -5$

$x = \dfrac{-(6) \pm \sqrt{(6)^2-4(9)(-5)}}{2(9)}$

$x = 0.483 \quad x = -1.15$

$x = 0.483$ or $x = -1.15$

(3)

(Total for Question 8 is 5 marks)

9 (a) Show that the equation $x^3 + 5x = 2$ has a solution between $x = 0$ and $x = 1$.

$(0)^3 + 5(0) = 0$

$(1)^3 + 5(1) = 6$

One below 2 and one above 2 ∴ solution is between 0 and 1.

(2)

(b) Show that the equation $x^3 + 5x = 2$ can be rearranged to give: $x = \frac{2}{5} - \frac{x^3}{5}$

$5x = 2 - x^3$

$x = \frac{2}{5} - \frac{x^3}{5}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{5} - \frac{x_n^3}{5}$ twice to find an estimate for the solution to $x^3 + 5x = 2$

$x_1 = \frac{2}{5} - \frac{(0)^3}{5} = 0.4$

$x_2 = \frac{2}{5} - \frac{Ans^3}{5} = 0.3872$

0.3872

(2)

(Total for Question 9 is 5 marks)