Write your name here

Surname	Other Names

Mathematics

Paper 1 (Non-Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may not be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium = $\frac{1}{2}(a + b)\, h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle = $2\pi r = \pi d$

Area of a circle = πr^2

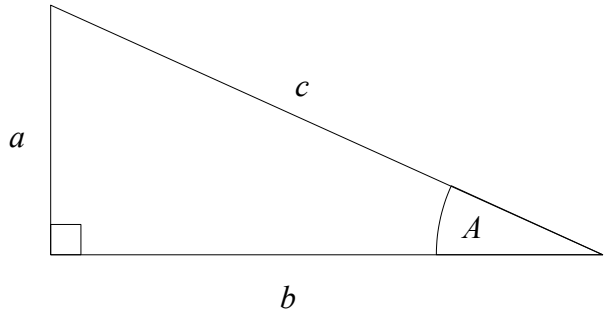
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

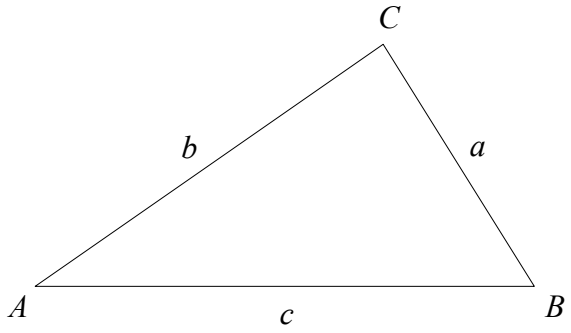


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued = $P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

END OF EXAM AID

1 Work out $6.84 \div 0.12$

$684 \div 12$

12
24
36
48
60
72
84
96
108
120

$$12 \overline{)\,6^{6}8^{8}4} \quad 057$$

57

(Total for Question 1 is 3 marks)

300

2 A plane travels at a speed of 298 miles per hour.

(a) Work out an estimate for the number of seconds the plane takes to travel 1 mile.

300 miles in 60 minutes
÷60 ÷60
5 miles in 1 minute
(60 seconds)
÷5 ÷5
1 mile in 12 seconds

12 seconds

(3)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer

underestimate — the speed is actually lower ∴ it will take longer

(1)

(Total for Question 2 is 4 marks)

3 Simon writes down three numbers a, b and c

$$a : b = 2 : 3$$
$$b : c = 5 : 4$$

(a) Find $a : b : c$

5x

$a : b$	$b : c$
$2 : 3$	$5 : 4$
$10 : 15$	$15 : 12$

x 3

$10 : 15 : 12$

(2)

Alvin writes down three numbers d, e and f

$d = 3e$ when $e = 1$ $d = 3$
$e = 4f$ when $f = 1$ $e = 4$

(b) Find $d : f$

4x

$d : e$	$e : f$
$3 : 1$	$4 : 1$
$12 : 4$	

$d : e : f$

$12 : 4 : 1$

$12 : 1$

(2)

(Total for Question 3 is 4 marks)

4 (a) Work out $2\frac{3}{4} + 3\frac{1}{3}$

Give your answer as a mixed number.

$3\times \frac{11}{4} + \frac{10}{3} \times 4$

3×4

$\frac{33}{12} + \frac{40}{12} = \frac{73}{12} = 6\frac{1}{12}$

$6\frac{1}{12}$

(2)

(b) Show that $1\frac{3}{5} \div 6 = \frac{4}{15}$

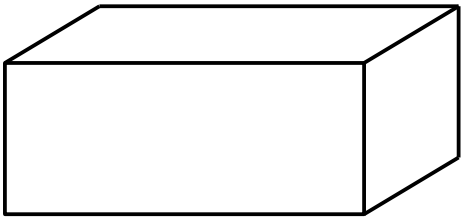
$\frac{8}{5} \div \frac{6}{1}$

$\frac{8}{5} \times \frac{1}{6} = \frac{8}{30} = \frac{4}{15}$

(2)

(Total for Question 4 is 4 marks)

5



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

A large box exerts a force of 900 newtons on the ground.

The base of the box in contact with the ground is a 3 m by 2 m rectangle.

Work out the pressure on the ground due to the box. $3 \times 2 = 6\ m^2$

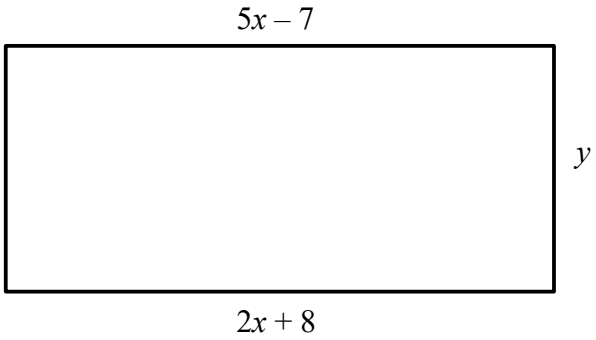
$$\text{pressure} = \frac{900}{6}$$

$$= 150\ N/m^2$$

150 newtons/m^2

(Total for Question 5 is 2 marks)

6 Here is a rectangle.



All measurements are in centimetres.

The area of the rectangle is 36 cm^2.

Find the value of y

$5x - 7 = 2x + 8$

$3x - 7 = 8$

$3x = 15$

$x = 5$

Length $= 2(5) + 8$

$= 18\text{cm}$

$18 \times y = 36$

$y = 2$

2

(Total for Question 6 is 4 marks)

7 Larry has 9 boxes.

The mean weight of the 9 boxes is 1.5 kg.
The mean weight of 4 of the boxes is 2 kg.

Work out the mean weight of the other 5 boxes.

$9 \times 1.5 = 13.5$ kg

$4 \times 2 = 8$ kg

$13.5 - 8 = 5.5$ kg

$5.5 \div 5 = 1.1$ kg

1.1 kg

(Total for Question 7 is 3 marks)

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8 Solve the simultaneous equations

$6x - 5y = 11$
$2x - 3y = 9$ $\times 3$

$6x - 5y = 11$
$6x - 9y = 27$

$4y = -16$
$y = -4$

$2x - 3(-4) = 9$
$2x + 12 = 9$
$2x = -3$
$x = -1.5$

$x =$ −1.5

$y =$ −4

(Total for Question 8 is 4 marks)

9 (a) Work out $16^{\frac{1}{2}} - 27^{\frac{1}{3}}$

$4 - 3$

1

(2)

(b) Simplify $\sqrt{27} + \sqrt{48}$

$\sqrt{9}\sqrt{3} + \sqrt{16}\sqrt{3}$

$3\sqrt{3} + 4\sqrt{3} = 7\sqrt{3}$

$7\sqrt{3}$

(2)

(Total for Question 9 is 4 marks)

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10 Write these numbers in order of size.
Start with the smallest number.

0.372×10^3 3720×10^{-2} 372×10^2 0.00372×10^6

372 37.2 37200 3720

3720×10^{-2}, 0.372×10^3, 0.00372×10^6, 372×10^2

(Total for Question 10 is 2 marks)

11 The table shows some information about the profit made each day at a cricket club on 100 days.

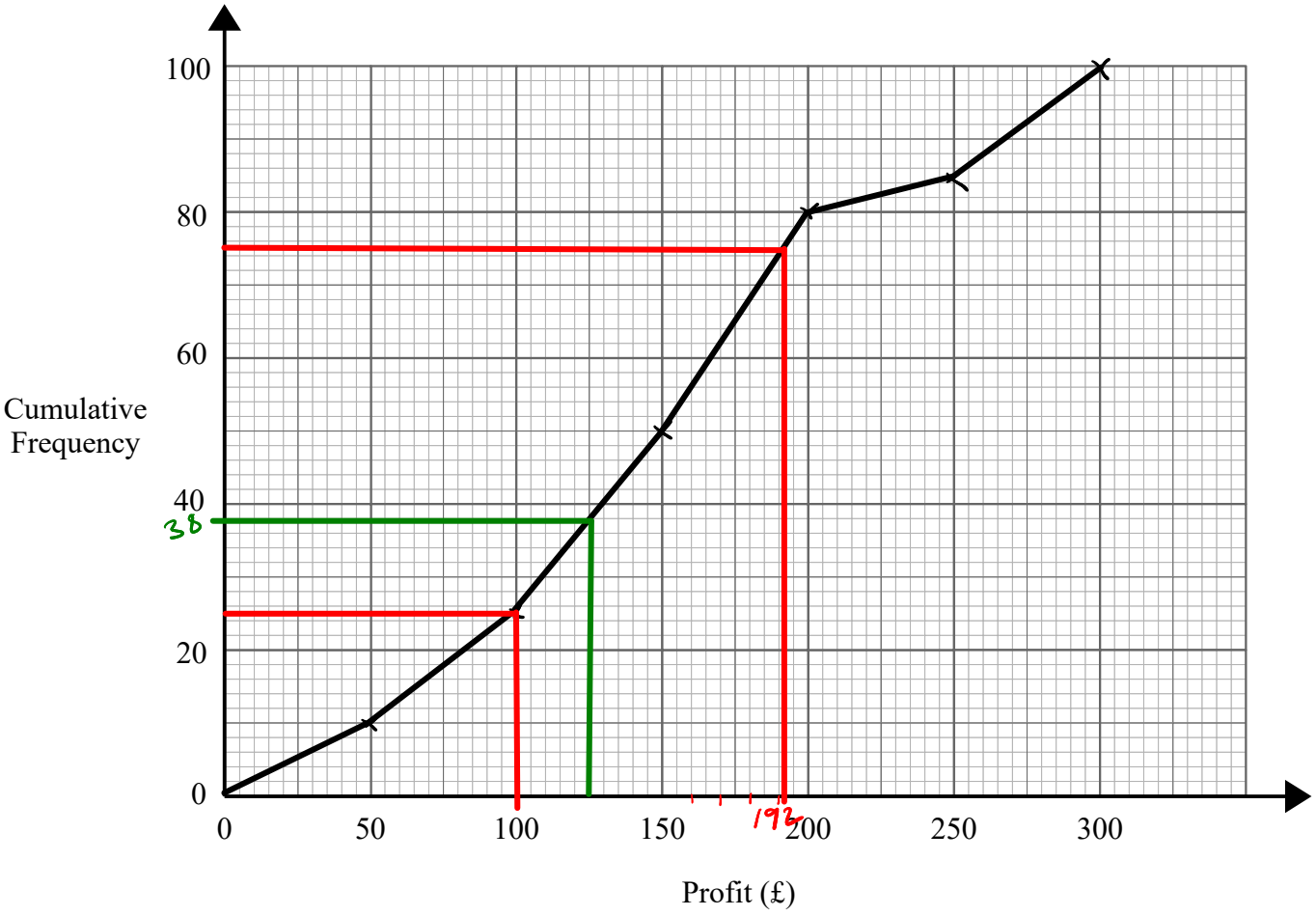
Profit (£x)	**Frequency**
$0 \leqslant x < 50$	10
$50 \leqslant x < 100$	15
$100 \leqslant x < 150$	25
$150 \leqslant x < 200$	30
$200 \leqslant x < 250$	5
$250 \leqslant x < 300$	15

(a) Complete the cumulative frequency table.

Profit (£x)	**Cumulative Frequency**
$0 \leqslant x < 50$	10
$0 \leqslant x < 100$	25
$0 \leqslant x < 150$	50
$0 \leqslant x < 200$	80
$0 \leqslant x < 250$	85
$0 \leqslant x < 300$	100

(1)

(b) On the grid, draw a cumulative frequency graph for this information.



(2)

(c) Use your graph to find an estimate for the number of days the profit was less than £125

38 days

(1)

(d) Use your graph to find an estimate for the interquartile range.

192 − 100 = 92

£ 92

(2)

(Total for Question 11 is 6 marks)

12 The equation of the line L_1 is $y = 2x + 5$
The equation of the line L_2 is $6y + kx + 10 = 0$

L_1 is perpendicular to L_2

Find the value of k
You must show all your working.

perpendicular gradient $= -\frac{1}{2}$

$6y = -kx - 10$

$y = -\frac{k}{6}x - \frac{10}{6}$

$-\frac{k}{6} = -\frac{1}{2}$ $\quad k = 3$

$k =$ 3

(Total for Question 12 is 3 marks)

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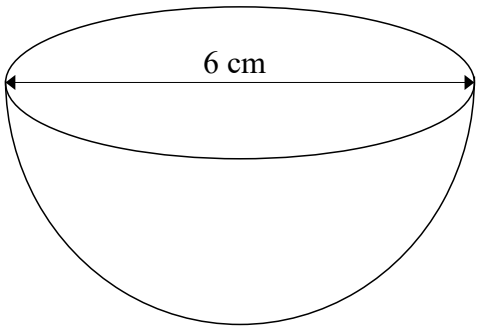
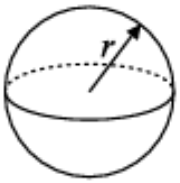
13 The diagram shows a hemisphere with diameter 6 cm.



Volume of sphere $= \frac{4}{3}\pi r^3$



Work out the volume of the hemisphere.
Give your answer in terms of π.

$r = 3$

$\frac{2}{3}\pi(3)^3$

$\frac{2}{3}\pi(27)$

18π

18π cm^3

(Total for Question 13 is 2 marks)

14 y is directly proportional to $\sqrt{x}$

$y = 9$ when $x = 4$

Find the value of y when $x = 16$

$y = k\sqrt{x}$

$9 = k\sqrt{4}$

$9 = 2k$

$k = 4.5$

$y = 4.5\sqrt{x}$

$y = 4.5\sqrt{16}$

$= 4.5 \times 4$

$= 18$

18

(Total for Question 14 is 3 marks)

15 Prove that $(3n + 1)^2 - (3n - 1)^2$ is always a multiple of 12, for all positive integer values of n.

$(3n+1)(3n+1) - (3n-1)(3n-1)$

$(9n^2 + 3n + 3n + 1) - (9n^2 - 3n - 3n + 1)$

$(9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

$9n^2 + 6n + 1 - 9n^2 + 6n - 1$

$12n$

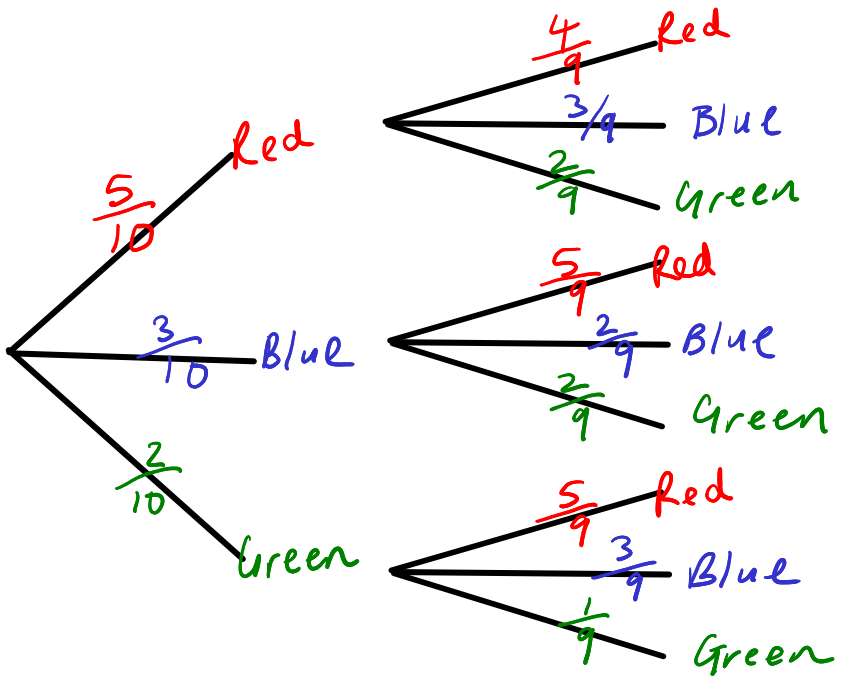
(Total for Question 15 is 2 marks)

16 There are 10 counters in a bag.

5 of the counters are red.
3 of the counters are blue.
2 of the counters are green.

Billie takes two counters at random from the bag.

Work out the probability that both of the counters Billie takes are the same colour.
You must show your working.



$P(RR) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$

$P(BB) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

$P(GG) = \frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$

$\frac{20}{90} + \frac{6}{90} + \frac{2}{90}$

$\frac{28}{90}$

(Total for Question 16 is 4 marks)

17 $f(x) = \dfrac{3x-4}{2}$

(a) Find $f^{-1}(x)$

input → [× 3] → [− 4] → [÷ 2] → output

$f^{-1}(x) =$ $\dfrac{2x+4}{3}$

(2)

For all values of x

$g(x) = x^2 + 3x$ and $h(x) = 3 - 2x$

(b) Work out the value of gh(4)

$h(4) = 3 - 2(4)$

$= -5$

$g(-5) = (-5)^2 + 3(-5)$

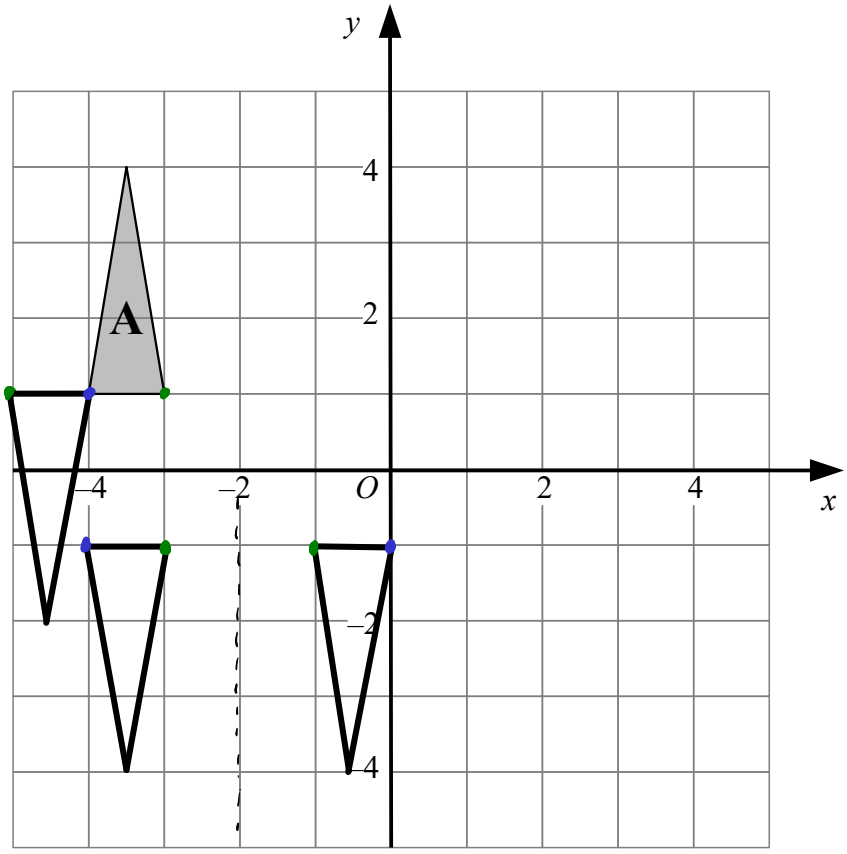
$= 10$

$gh(4) =$ 10

(2)

(Total for Question 17 is 4 marks)

18



Triangle **A** is transformed by the combined transformation of a reflection in the x-axis, followed by a reflection in the line $x = -2$, followed by a translation with vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

One point on triangle **A** is invariant under the combined transformation.
Find the coordinates of this point.

You must show your working.

(−4 , 1)

(Total for Question 18 is 2 marks)

19 Find the coordinates of the turning point on the curve with equation $y = x^2 + 8x + 11$

You must show all your working

$y = (x+4)^2 - 16 + 11$

$= (x+4)^2 - 5$

(−4 , −5)

(Total for Question 19 is 3 marks)

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20 **A**, **B** and **C** are three spheres.

The surface area of sphere **A** is 25 cm^2
The surface area of sphere **B** is 36 cm^2

The ratio of the radius of sphere **B** to the radius of sphere **C** is 2 : 3

Work out the ratio of the volume of sphere **A** to the volume of sphere **C**.

S.a A : B
25 : 36

length 5 : 6 (scale factor for area is scale factor for length squared)

Radius

A : B
5 : 6

B : C
2 : 3 x 3
6 : 9

A : C
length 5 : 9
volume 125 : 729

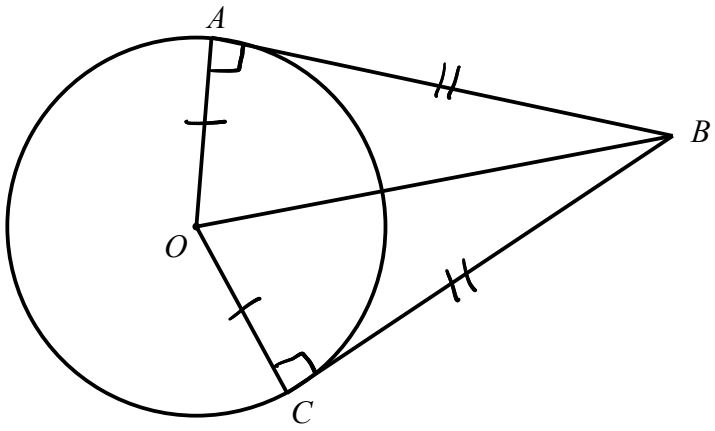
S.F for volume is S.F cubed

9^1
$\times 9$
729

125 : 729

(Total for Question 20 is 3 marks)

21 *A* and *C* are points on a circle, centre *O*.
AB and *BC* are tangents to the circle.



Prove that triangle *ABO* is congruent to triangle *BCO*.

OAB and OCB are both 90° tangent meets radius at 90°

OA and OC are both radius

AB and BC are equal tangents from the same point are equal.

SAS ∴ congruent

(Total for Question 21 is 4 marks)

22 n is an integer such that $6n - 10 \geq 8$ and $2n^2 < 13n - 6$

Find all the possible values of n.

$6n \geq 18$

$n \geq 3$

$2n^2 - 13n + 6 < 0$

$(2n - 1)(n - 6) < 0$

$n = \frac{1}{2}$ $n = 6$

$\frac{1}{2} < n < 6$

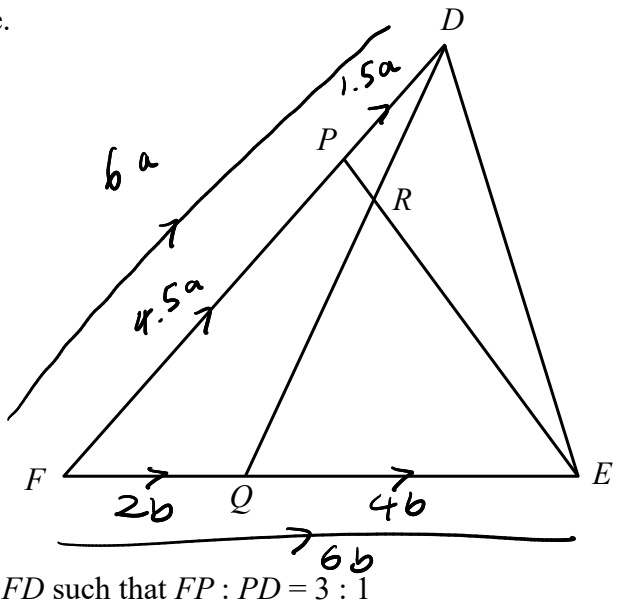
$3 \leq n < 6$

3, 4 and 5

3, 4, 5

(Total for Question 22 is 5 marks)

23 *DEF* is a triangle.



P is the point on *FD* such that *FP* : *PD* = 3 : 1
Q is the point on *FE* such that *FQ* : *QE* = 1 : 2

$\overrightarrow{FD} = 6\mathbf{a}$ an $\overrightarrow{FE} = 6\mathbf{b}$

Use a vector method to find $\overrightarrow{QR}$ in terms of **a** and **b**.

$\overrightarrow{QD} = -2\mathbf{b} + 6\mathbf{a}$

$\overrightarrow{QR} = x(-2\mathbf{b} + 6\mathbf{a})$

$\overrightarrow{EP} = -6\mathbf{b} + 4.5\mathbf{a}$

$\overrightarrow{ER} = y(-6\mathbf{b} + 4.5\mathbf{a})$

$\overrightarrow{QR} = 4\mathbf{b} + y(-6\mathbf{b} + 4.5\mathbf{a})$

$-2x\mathbf{b} + 6x\mathbf{a} = 4\mathbf{b} - 6y\mathbf{b} + 4.5y\mathbf{a}$

a// $6x = 4.5y$

$12x = 9y$

$y = \frac{4}{3}x$

b// $-2x = 4 - 6y$

$-2x = 4 - 6(\frac{4}{3}x)$

$-2x = 4 - 8x$

$6x = 4$

$x = \frac{2}{3}$

$\overrightarrow{QR} = \frac{2}{3}(-2\mathbf{b} + 6\mathbf{a})$

$= -\frac{4}{3}\mathbf{b} + 4\mathbf{a}$

$4\mathbf{a} - \frac{4}{3}\mathbf{b}$

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS