Name: ______________________________

GCSE (1 – 9)

Proof of Circle Theorems

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided

– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

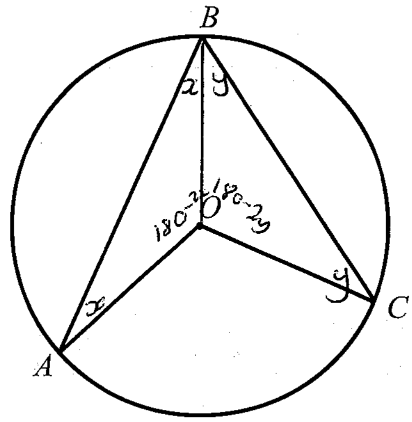
- The marks for each question are shown in brackets

– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1



Let $ABO = x$
let $CBO = y$

A, *B* and *C* are points on the circumference of a circle, centre *O*.

Prove that angle *AOC* is twice the size of angle *ABC*.
You must **not** use any circle theorems in your proof.

$ABO = BAO$ Angles at the base of an isosceles triangle are equal

$CBO = BCO$ ——— " ———

$AOB = 180 - 2x$ Angles in a triangle add to 180°

$BOC = 180 - 2y$ ——— " ———

$ABC = x + y$

$AOC = 360 - (180 - 2x) - (180 - 2y)$

$= 360 - 180 + 2x \cancel{+} - 180 + 2y$

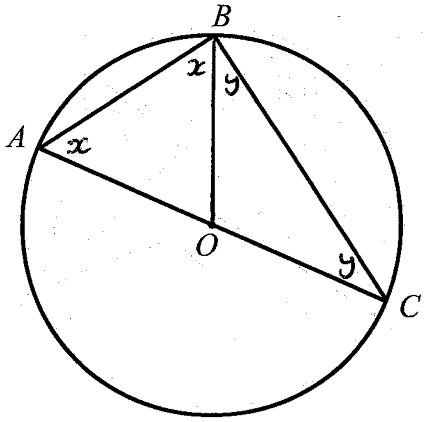
$= 2x + 2y$

$= 2(x + y)$

$= 2(ABC)$

(Total for Question 1 is 4 marks)

2



Let $OAB = x$

Let $OBC = y$

A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOC is a diameter of the circle.

Prove that angle *ABC* is 90°
You must **not** use any circle theorems in your proof.

$OAB = OBA$ Angles at the base of an isosceles triangle are equal

$OBC = OCB$ ——— " ———

$AOB = 180 - 2x$ Angles in a triangle add to $180°$

$BOC = 180 - 2y$

$180 - 2x + 180 - 2y = 180$

$360 - 2x - 2y = 180$

$180 = 2x + 2y$

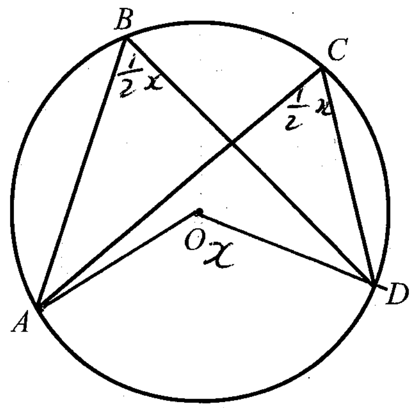
$\underline{90 = x + y}$

$ABC = x + y$

$= \underline{90°}$

(Total for Question 2 is 4 marks)

3



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABD* and angle *ACD* are equal.

$AOD = x$

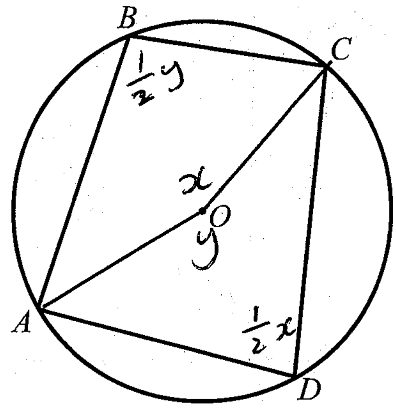
$ABD = \frac{1}{2}x$ Angle at the circumference is half the angle at the centre

$ACD = \frac{1}{2}x$ ——— " ———

$\underline{ABD = ACD} = \frac{1}{2}x$

(Total for Question 3 is 2 marks)

4



A, B, C and D are points on the circumference of a circle, centre O.

Prove that angle ABC and angle ADC add to 180°

Let AOC (minor) $= x$

Let AOC (Major) $= y$

$ACD = \frac{1}{2}x$ The angle at the centre is twice the angle at the circumference.

$ABC = \frac{1}{2}y$ ——— " ———

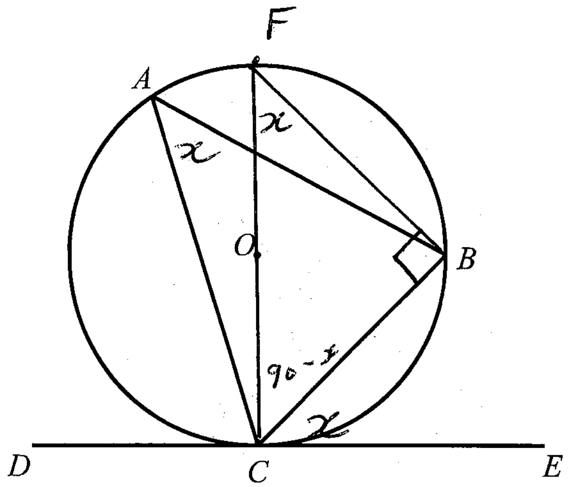
$x + y = 360$ Angles around a point add to 360°

$\therefore \frac{1}{2}x + \frac{1}{2}y = 180$

$ABC + ADC = 180$

(Total for Question 4 is 4 marks)

5



A, B and C are points on the circumference of a circle, centre O.
DCE is a tangent to the circle.

Prove that angle BCE and angle BAC are equal.

Let $BCE = x$

$OCE = 90°$ Tangent meets radius at $90°$

$OCB = 90 - x$

$FBC = 90°$ Angle in a semi-circle is $90°$

$BFC = 180 - 90 - (90 - x)$

$= x$

$BAC = x$

~~$BCE = x$ $BFC = x$~~

Angles from the same points (in same segment) are equal.

$BAC = BCE = x$

(Total for Question 5 is 4 marks)