Name:

## GCSE (1-9)

## Iteration

## Instructions

- Use black ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.


## Information

- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 The number of rabbits in a field $t$ days from now is $P_{t}$ where

$$
\begin{aligned}
& P_{0}=220 \\
& P_{t+1}=1.15\left(P_{t}-20\right)
\end{aligned}
$$

Work out the number of rabbits in the garden 3 days from now.

2 The number of people living in a town $t$ years from now is $P_{t}$ where

$$
\begin{aligned}
& P_{0}=55000 \\
& P_{t+1}=1.03\left(P_{t}-800\right)
\end{aligned}
$$

Work out the number of people in the town 3 years from now.

3 Using $x_{n+1}=3+\frac{9}{x_{n}^{2}}$
With $x_{0}=3$
Find the values of $x_{1}, x_{2}$ and $x_{3}$.

$$
\begin{aligned}
& x_{1}=. \\
& x_{2}=. \\
& x_{3}=.
\end{aligned}
$$

$\qquad$
$\qquad$

4 Using $x_{n+1}=\frac{5}{x_{n}^{2}+3}$
With $x_{0}=1$
Find the values of $x_{1}, x_{2}$ and $x_{3}$.

$$
\begin{aligned}
& x_{1}=. \\
& x_{2}=. \\
& x_{3}=.
\end{aligned}
$$

5 Starting with $x_{0}=3$, use the iteration formula $x_{n+1}=\frac{7}{x_{n}^{2}}+2$ three times to find an estimate for the solution to $x^{3}-2 x^{2}=7$

6 Starting with $x_{0}=0$, use the iteration formula $x_{n+1}=\frac{2}{x_{n}^{2}+3}$ three times to find an estimate for the
solution to $x^{3}+3 x=2$

7 Using $x_{n+1}=\frac{5}{x_{n}^{2}}+2$
With $x_{0}=2.5$
(a) Find the values of $x_{1}, x_{2}$ and $x_{3}$.

$$
\begin{align*}
& x_{1}=. \\
& x_{2}=. \\
& x_{3}=. \tag{3}
\end{align*}
$$

(b) Explain the relationship between the values of $x_{1}, x_{2}$ and $x_{3}$ and the equation $x^{3}-2 x^{2}-5=0$
$\qquad$
$\qquad$
$\qquad$

8 (a) Show that the equation $2 x^{3}-x^{2}-3=0$ has a solution between $x=1$ and $x=2$.
(b) Show that the equation $2 x^{3}-x^{2}-3=0$ can be rearranged to give: $x=\sqrt{\frac{3}{2 x-1}}$
(c) Starting with $x_{0}=1$, use the iteration formula $x_{n+1}=\sqrt{\frac{3}{2 x_{n}-1}}$ twice to find an estimate for the solution to $2 x^{3}-x^{2}-3=0$

9 Using $\quad x_{n+1}=1+\frac{1}{x_{n}^{2}}$
With $x_{0}=2$
(a) Find the values of $x_{1}, x_{2}$ and $x_{3}$.

$$
\begin{aligned}
& x_{1}=. \\
& x_{2}=. \\
& x_{3}=.
\end{aligned}
$$

$\qquad$
$\qquad$
$\qquad$
(b) Explain the relationship between the values of $x_{1}, x_{2}$ and $x_{3}$ and the equation $x^{3}-x^{2}-1=0$
$\qquad$
$\qquad$
$\qquad$

10 (a) Show that the equation $x^{3}+4 x=1$ has a solution between $x=0$ and $x=1$.
(b) Show that the equation $x^{3}+4 x=1$ can be rearranged to give: $x=\frac{1}{4}-\frac{x^{3}}{4}$
(c) Starting with $x_{0}=0$, use the iteration formula $x_{n+1}=\frac{1}{4}-\frac{x_{n}^{3}}{4}$ twice to find an estimate for the solution to $x^{3}+4 x=1$

