

Name: _____

GCSE (1 – 9)

Compound and Inverse Functions

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 Given that $f(x) = x - 4$ find:

(a) $f(5)$

$$f(5) = 5 - 4$$

(b) $f(3)$

$$f(3) = 3 - 4$$

1

1

(Total for Question 2 is 2 marks)

2 Given that $g(x) = 2x^2 - 10$ find:

(a) $g(2)$

$$\begin{aligned} g(2) &= 2(2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

(b) $g(-2)$

$$\begin{aligned} g(-2) &= 2(-2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

(c) Solve: $g(x) = 8$

$$2x^2 - 10 = 8$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

3

(Total for Question 2 is 5 marks)

3 Given that $f(x) = 3x - 5$ find:

(a) $f(3)$

$$\begin{aligned} f(3) &= 3(3) - 5 \\ &= 9 - 5 \end{aligned}$$

4

(1)

(b) $f(-2)$

$$\begin{aligned} f(-2) &= 3(-2) - 5 \\ &= -6 - 5 \end{aligned}$$

- 11

(1)

^{solve}
(c) $f(x) = 1$

$$3x - 5 = 1$$

$$3x = 6$$

$$x = 2$$

x = 2

(2)

(Total for Question 3 is 4 marks)

4 Given that $f(x) = x^2 - 3$ find:

(a) $f(10)$

$$\begin{aligned} f(10) &= (10)^2 - 3 \\ &= 100 - 3 \end{aligned}$$

97

(1)

(b) $f(-1)$

$$\begin{aligned} f(-1) &= (-1)^2 - 3 \\ &= 1 - 3 \end{aligned}$$

- 2

(1)

(c) Find: $f^{-1}(x) = 8$

$$y = x^2 - 3$$

$$x = y^2 - 3$$

$$x + 3 = y^2$$

$$y = \sqrt{x + 3}$$

$$f^{-1}(x) = \sqrt{x + 3}$$

$$8 = \sqrt{x + 3}$$

$$64 = x + 3$$

$$61 = x$$

x = 61

(2)

(Total for Question 4 is 4 marks)

5 Given that $f(x) = 2x - 4$ and $g(x) = 3x + 5$

(a) Find $gf(3)$

$$\begin{aligned} f(3) &= 2(3) - 4 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} g(2) &= 3(2) + 5 \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

(b) Work out an expression for $f^{-1}(x)$

$$y = 2x - 4$$

$$x = 2y - 4$$

$$x + 4 = 2y$$

$$\frac{x + 4}{2} = y$$

(c) Solve $f(x) = g(x)$

$$\begin{array}{r} 2x - 4 = 3x + 5 \\ -2x \quad \quad -2x \end{array}$$

$$\begin{array}{r} -4 = x + 5 \\ -5 \quad \quad -5 \end{array}$$

$$-9 = x$$

$$x = -9$$

(2)

(Total for Question 5 is 6 marks)

6 Given that $f(x) = 3x + 1$ and $g(x) = x^2$

(a) Find $fg(x)$

$$fg(x) = 3x^2 + 1$$

(b) Work out an expression for $gf(x)$

$$\underline{gf(x) = 3x^2 + 1}$$

(2)

(c) Solve $fg(x) = gf(x)$

$$\underline{gf(x) = (3x + 1)^2}$$

(2)

$$3x^2 + 1 = (3x + 1)^2$$

$$3x^2 + 1 = (3x + 1)(3x + 1)$$

$$3x^2 + 1 = 9x^2 + 3x + 3x + 1$$

$$3x^2 + 1 = 9x^2 + 6x + 1$$

$$1 = 6x^2 + 6x + 1$$

$$0 = 6x^2 + 6x$$

$$0 = 6x(x + 1)$$

$$x = 0 \quad x = -1$$

$$\underline{x = 0 \text{ or } x = -1}$$

(3)

(Total for Question 6 is 7 marks)

7 Given that $f(x) = x^2 - 17$ and $g(x) = x + 3$

(a) Work out an expression for $g^{-1}(x)$

$$y = x + 3$$

$$x = y + 3$$

$$x - 3 = y$$

$$\underline{g^{-1}(x) = x - 3}$$

(2)

(b) Work out an expression for $f^{-1}(x)$

$$y = x^2 - 17$$

$$x = y^2 - 17$$

$$x + 17 = y^2$$

$$\sqrt{x + 17} = y$$

$$\underline{f^{-1}(x) = \sqrt{x + 17}}$$

(2)

(c) Solve $f^{-1}(x) = g^{-1}(x)$

$$\sqrt{x + 17} = x - 3$$

$$x + 17 = (x - 3)^2$$

$$x + 17 = (x - 3)(x - 3)$$

$$x + 17 = x^2 - 3x - 3x + 9$$

$$x + 17 = x^2 - 6x + 9$$

$$17 = x^2 - 7x + 9$$

$$0 = x^2 - 7x - 8$$

$$0 = (x - 8)(x + 1)$$

$$x = 8 \quad x = -1$$

$$\underline{x = 8 \text{ or } x = -1}$$

(4)

(Total for Question 7 is 8 marks)

8 The function f is defined such that

$$f(x) = x^2 - 1$$

(a) Find an expression for $f(x-2)$

$$\begin{aligned} f(x-2) &= (x-2)^2 - 1 \\ &= (x-2)(x-2) - 1 \\ &= x^2 - 2x - 2x + 4 - 1 \\ &= x^2 - 4x + 3 \end{aligned}$$

$$\underline{f(x-2) = x^2 - 4x + 3} \quad (2)$$

(b) Hence solve: $f(x-2) = 0$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1$$

$$\underline{x = 1 \text{ or } x = 3} \quad (2)$$

(Total for Question 8 is 4 marks)

9 The function f is defined such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$

$$\begin{aligned}y &= 4x - 1 \\x &= 4y - 1 \\x + 1 &= 4y \\ \frac{x + 1}{4} &= y\end{aligned}$$

$$\underline{f^{-1}(x) = \frac{x + 1}{4}} \quad (2)$$

The function g is defined such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant}$$

(b) Given that $fg(2) = 12$
Work out the value of k .

$$\begin{aligned}g(2) &= k(2)^2 \\ &= 4k\end{aligned}$$

$$\begin{aligned}f(4k) &= 4(4k) - 1 \\ &= 16k - 1\end{aligned}$$

$$16k - 1 = 12$$

$$16k = 13$$

$$k = \frac{13}{16}$$

$$\underline{k = \frac{13}{16}}$$

(2)

(Total for Question 9 is 4 marks)