Name: _____

GCSE (1 – 9)

Circle Theorems

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

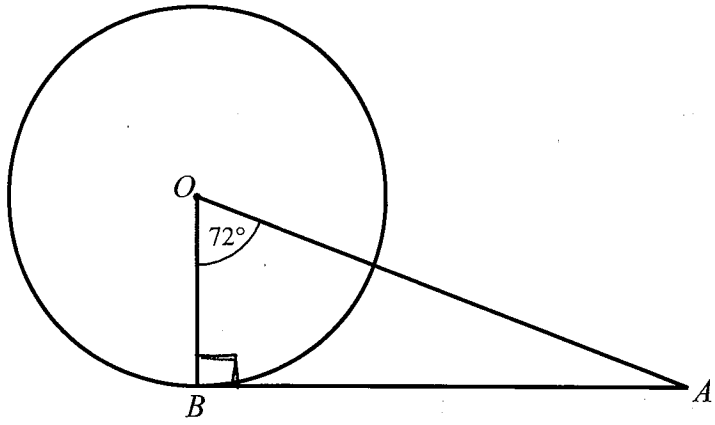
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1



B is a point on the circumference of a circle, centre *O*.
AB is a tangent to the circle.

Angle *BOA* = 72°

Work out the size of angle *BAO*.
You must show all your working.

OBA = 90° Tangent meets radius at 90°

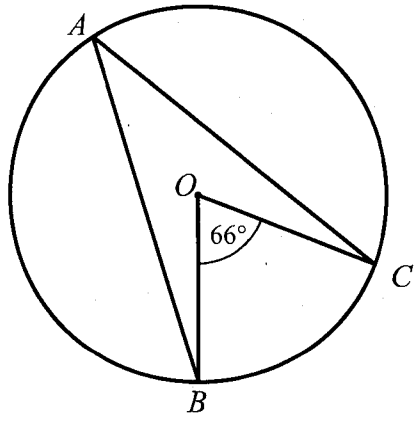
BAO = 180 − 90 − 72 Angles in a triangle add to 180°

= 18°

18 °

(Total for Question 1 is 2 marks)

2



A, *B*, *C* and *D* are points on the circumference of a circle.

Angle *BOC* = 66°

(i) Find the size of angle *BAC*.

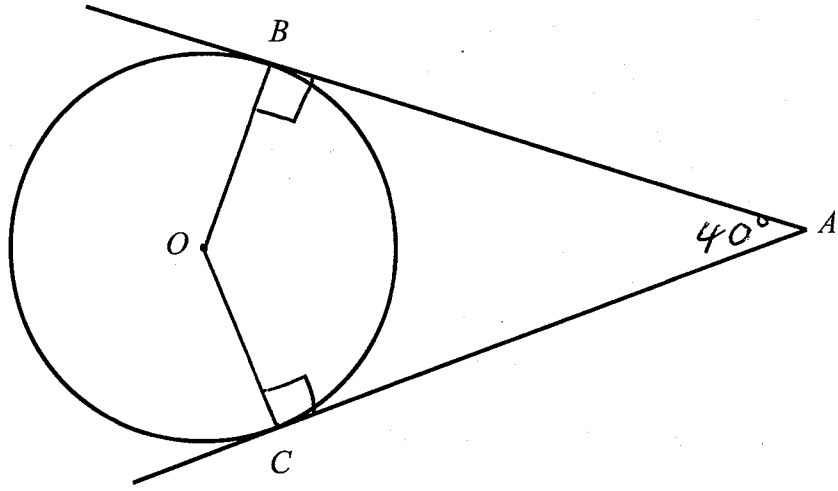
33 °

(ii) Give a reason for your answer.

The angle at the circumference is half the angle at the centre

(Total for Question 2 is 2 marks)

3



B and *C* are points on a circle, centre *O*.
AB and *AC* are tangents to the circle.

OBA and OCA = 90°

Angle *BAC* = 40°

Tangent meets radius at 90°

Work out the size of angle *BOC*.
You must show all your working.

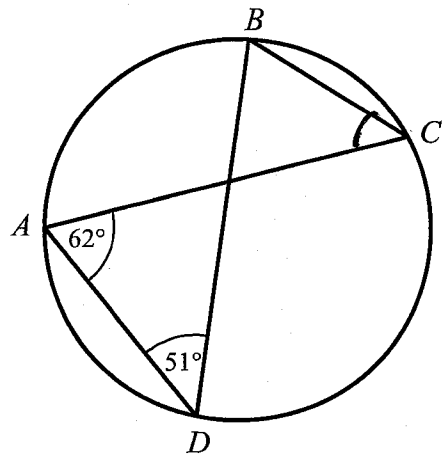
360 − 90 − 90 − 40 = 140°

Angles in a quadrilateral add to 360°

140 °

(Total for Question 3 is 3 marks)

4



A, *B*, *C* and *D* are points on the circumference of a circle.

Angle *CAD* = 62°
Angle *ADB* = 51°

(i) Find the size of angle *ACB*.

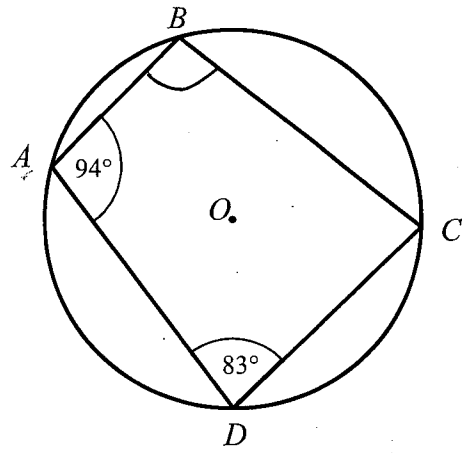
51 °

(ii) Give a reason for your answer.

Angles from the same points (to the circumference) are equal

(Total for Question 4 is 2 marks)

5



A, *B*, *C* and *D* are points on the circumference of a circle.

Angle $BAD = 94°$
Angle $ADC = 83°$

180 − 83

(i) Find the size of angle *ABC*.

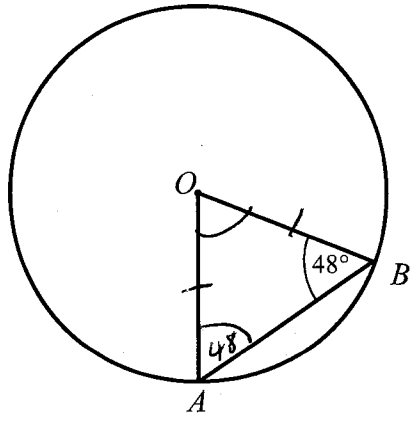
97 °

(ii) Give a reason for your answer.

opposite angles in a cyclic quadrilateral add to 180°

(Total for Question 5 is 2 marks)

6



A and *B* are points on the circumference of a circle, centre *O*.

Angle $ABO = 48°$

OAB = 48°
Angles at base of isosceles are equal

(i) Find the size of angle *AOB*.

180 − 48 − 48

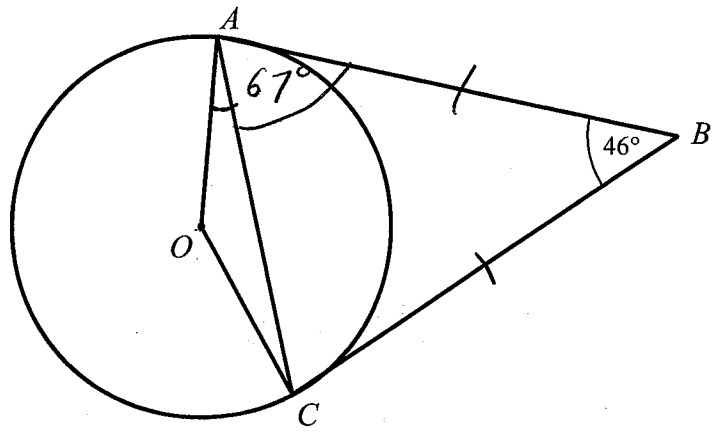
84 °

(ii) Give a reason for your answer.

Angles at the base of an isosceles triangle are equal + angles in a triangle add to 180°

(Total for Question 6 is 2 marks)

7



A and *C* are points on the circumference of a circle, centre *O*.
AB and *BC* are tangents to the circle.

Angle *ABC* = 46°

Find the size of angle *OAC*.
Give reasons for each stage of your working.

ABC is an isosceles triangle, 2 tangents from the same point are equal

$CAB = \frac{180 - 46}{2} = \frac{134}{2} = 67°$

Angles at the base of an isosceles triangle are equal

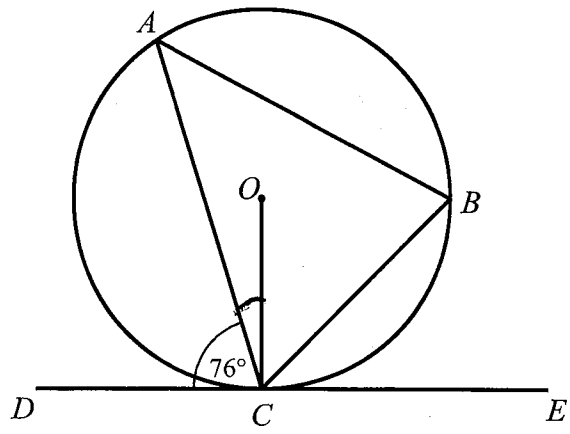
OAB = 90° Tangent meets radius at 90°

$OAC = 90 - 67 = 23°$

23 °

(Total for Question 7 is 3 marks)

8



A and B are points on the circumference of a circle, centre O.
DCE is a tangent to the circle.

Angle $ACD = 76°$

(a) Find the size of angle ACO.
You must ~~show~~ all your working.
Give reasons for

$OCD = 90°$ Tangent meets radius at 90°

$90 - 76 = 14°$

14 °

(2)

(b) Find the size of angle ABC.
You must ~~show~~ all your working.
give reasons for

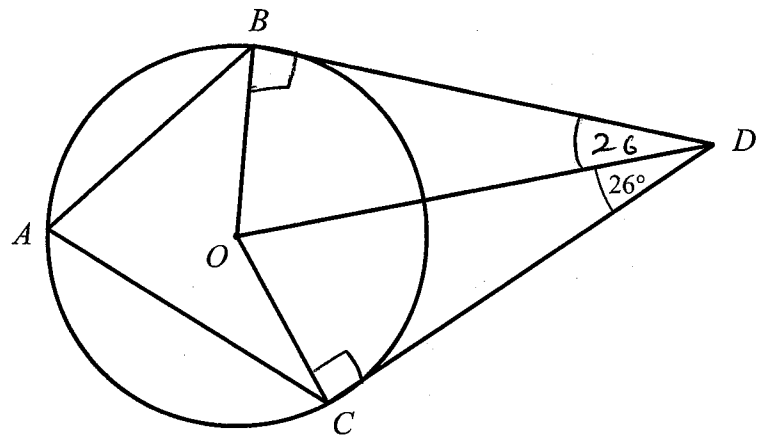
$ABC = 76°$ Alternate segment theorem

76 °

(2)

(Total for Question 8 is 4 marks)

9



A, *B* and *C* are points on the circumference of a circle, centre *O*.
BD and *CD* are tangents to the circle.

Angle *ODC* = 26°

Find the size of angle *BAC*.
Give reasons for each stage of your working.

Triangle BOD = Triangle DOC all sides are equal

OBD and OCD = 90° Tangent meets radius at 90°

BOD and COD = 180 − 90 − 26
= 64°

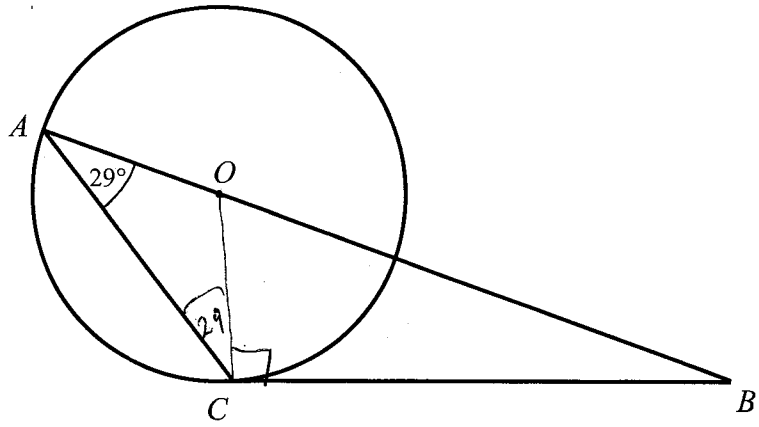
BOC = 2 × 64 = 128°

BAC = $\frac{128}{2}$ = 64° Angle at circumference is half the angle at the centre

64 °

(Total for Question 9 is 4 marks)

10



A and *C* are points on the circumference of a circle, centre *O*.
BC is a tangent to the circle.

Angle *CAB* = 29°

Find the size of angle *ABC*.
You must show all your working.

$OCA = 29°$ Angles at the base of an isosceles triangle are equal

$OCB = 90°$ Tangent meets radius at 90°

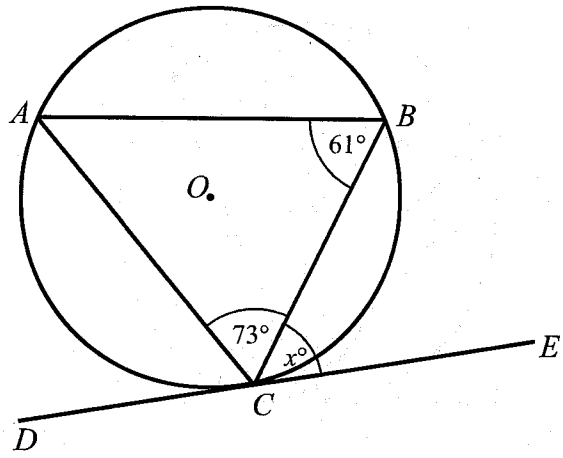
$ABC = 180 - 90 - 29 - 29$

$= 32°$ Angles in a triangle add to 180°

32 °

(Total for Question 10 is 4 marks)

11



A, *B* and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle $ABC = 61°$
Angle $ACB = 73°$
Angle $BCE = x°$

Find the value of x.
You must show all your working.

$BAC = 180 - 73 - 61$

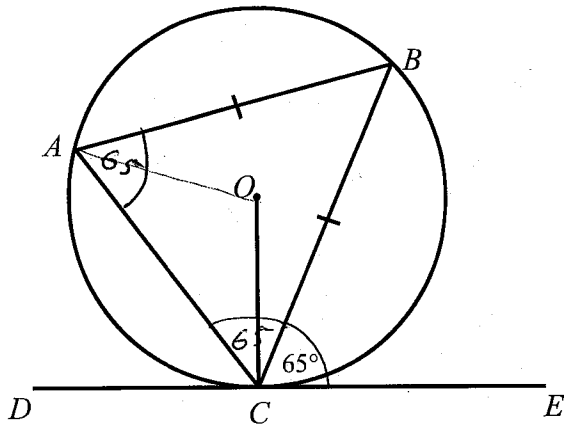
$= 46°$ Angles in a triangle add to 180°

$x = 46°$ Alternate segment theorem

46 °

(Total for Question 11 is 3 marks)

12



A, *B* and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

AB = *BC*
Angle *BCE* = 65°

Find the size of angle *AOC*.
You must show all your working.

$BAC = 65°$ Alternate segment theorem

$ACB = 65°$ Angles at the base of an isosceles triangle are equal

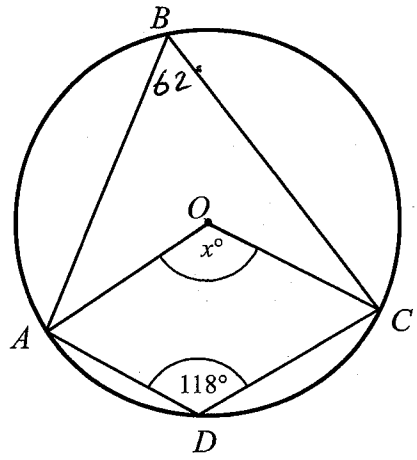
$ABC = 180 - 65 - 65$
$= 50°$ Angles in a triangle add to 180°

$AOC = 2 \times 50$ Angle at centre is twice angle at circumference
$= 100°$

100 °

(Total for Question 12 is 4 marks)

13



A, B, C and *D* are points on the circumference of a circle, centre *O*.

Angle $ADC = 118°$
Angle $AOC = x°$

Work out the value of x.
You must show all your working.

ABC = 180 − 118
= 62°

opposite angles in a cyclic quadrilateral add to 180°

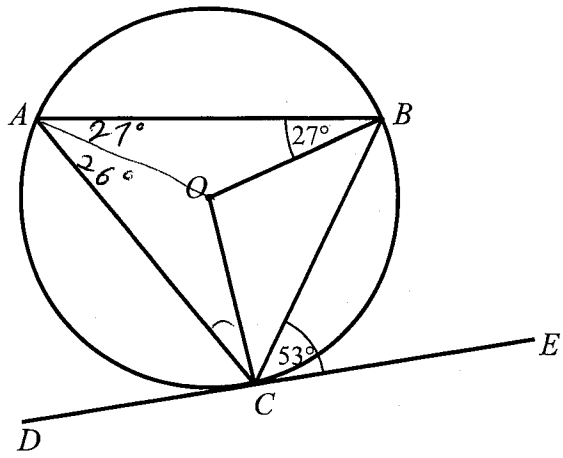
AOC = 2 × 62
= 124°

Angle at centre is twice angle at circumference

124 °

(Total for Question 13 is 3 marks)

14



A, *B* and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle *ABO* = 27°
Angle *BCE* = 53°

Find the size of angle *ACO*.
~~You must show all your working~~.
Give reasons for each stage of your working

BAO = 27° Angles at base of isosceles triangle are equal

BAC = 53° Alternate segment theorem

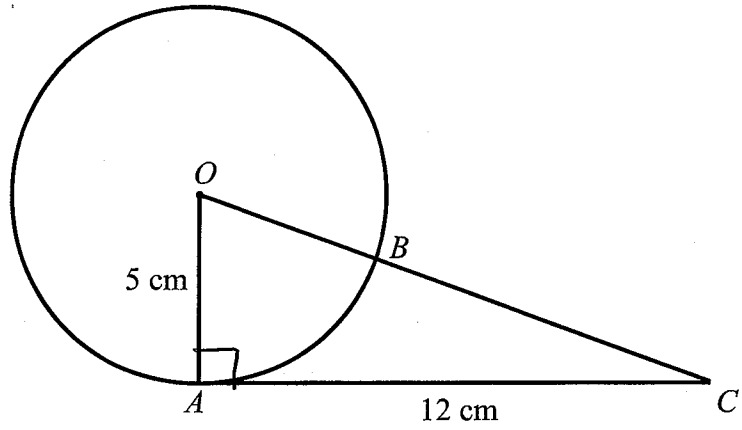
OAC = 53 − 27 = 26°

ACO = 26° Angles at base of isosceles triangle are equal

26 °

(Total for Question 14 is ~~3~~ 4 marks)

15



A and *B* is a point on the circumference of a circle, centre *O*.
AC is a tangent to the circle.
OBC is a straight line.

OA = 5 cm
AC = 12 cm

Find the length of *BC*.
You must show all your working.

$OAC = 90°$ Tangent meets radius at 90°

$5^2 + 12^2 = OC^2$

$169 = OC^2$

$OC = \sqrt{169}$

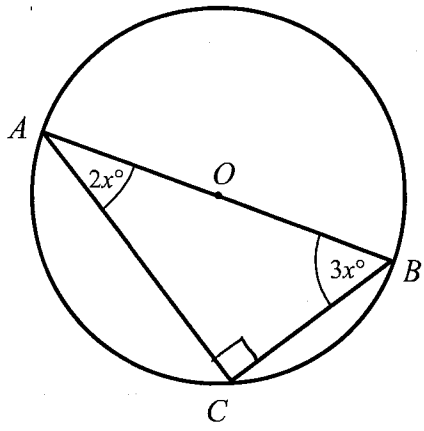
$= 13$ cm

$BC = 13 - 5 = 8$ cm (radius = 5cm)

8 cm

(Total for Question 15 is 4 marks)

16



A, *B* and *C* are points on the circumference of a circle, centre *O*.

Angle $CAB = 2x°$
Angle $ABC = 3x°$

Find the value of x.
You must show all your working.

$ACB = 90°$ Angle in a semi circle is $90°$

$2x + 3x + 90 = 180$

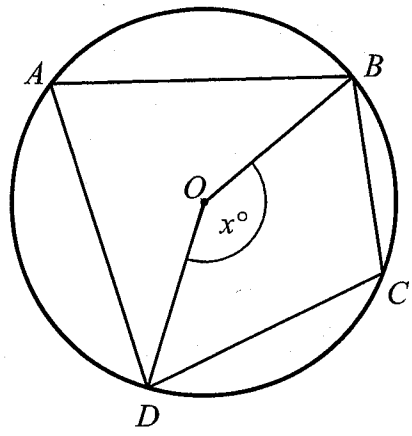
$5x = 90$

$x = 18°$

$x =$ 18

(Total for Question 16 is 3 marks)

17



A, B, C and *D* are points on the circumference of a circle, centre *O*.

Angle $BOD = x°$

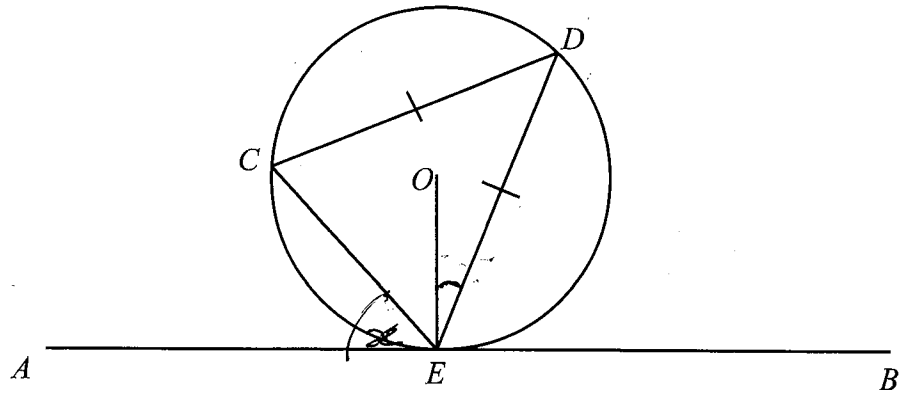
Find the size of angle *BCD*, in terms of *x*.
Give reasons for each stage of your working.

$BAD = \frac{1}{2}x$ Angle at circumference is half angle at centre

$BCD = 180 - \frac{1}{2}x$ opposite angles in a cyclic quadrilateral add to 180°

(Total for Question 17 is 3 marks)

18



C, D and E are points on a circle, centre O.
AEB is a tangent to the circle at E.

$CD = DE$
Angle $AEC = x°$

Find the size of angle OED, in terms of x.
Give reasons for each stage of your working.

$CDE = x$ Alternate segment theorem

$CED = \frac{180 - x}{2}$ Angles at the base of an isosceles triangle are equal

$OEC = 90 - x$ $OEA = 90°$ Tangent meets radius at 90°

$OED = \frac{180 - x}{2} - (90 - x)$

$= 90 - \frac{1}{2}x - 90 + x$

$= \frac{1}{2}x$

(Total for Question 18 is 5 marks)