A Level Maths: Differentiation

1 $f(x) = \ln x + e^x$

(a) Find $f'(x)$ **(2)**

(b) Find $f''(x)$ **(2)**

(Total for question 1 is 4 marks)

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2 Differentiate with respect to x,

(a) $2x^3 + e^{4x}$ **(2)**

(b) $(x^2 + 5)^3$ **(2)**

(Total for question 2 is 4 marks)

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3 Differentiate with respect to x,

(a) $x + \dfrac{5}{x^2 + 4}$ **(2)**

(b) $8 + e^{x^2}$ **(2)**

(Total for question 3 is 4 marks)

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4 The point P lies on the curve with equation $y = 2 + \ln(3 - 2x)$ with x coordinate 1

Find an equation to the tangent to the curve at the point P.

(Total for question 4 is 5 marks)

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5 The point P lies on the curve with equation $y = \dfrac{3}{2x + 1}$ with x coordinate 1

(a) Find an equation to the normal to the curve at the point P. **(5)**

The normal intersects the curve again at the point Q.

(b) Find the exact coordinates of Q. **(2)**

(Total for question 5 is 7 marks)

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6 The point P lies on the curve with equation $y = \dfrac{2}{\sqrt{2x + 1}}$ with x coordinate 4

(a) Find an equation to the tangent to the curve at the point P. **(5)**

The tangent intersects the x axis at the point A and the y axis at the point B.

(b) Find the exact area of the triangle AOB, where O is the origin. **(4)**

(Total for question 6 is 9 marks)

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7 Differentiate with respect to x

(a) $2xe^x$ **(2)**

(b) $3x^2 \ln 2x$ **(3)**

(Total for question 7 is 5 marks)

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8 Differentiate with respect to x,

(a) $x^2\sqrt{2x+1}$ **(3)**

(b) $x \ln (x+1)$ **(3)**

(Total for question 8 is 6 marks)

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9 Differentiate with respect to x,

(a) $(x+5)(x+1)^3$ **(3)**

(b) $2 + x^3 \ln (2x+1)$ **(3)**

(Total for question 9 is 6 marks)

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10 The point P lies on the curve with equation $y = (3x - 1) \ln (2 - x)$ with x coordinate 1.

Find an equation of the tangent to the curve at the point P.

(Total for question 10 is 5 marks)

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11 Find the coordinates of the stationary points of the curve $y = x(x-3)^3$ and determine the nature of the stationary points.

(Total for question 11 is 8 marks)

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12 The point P lies on the curve with equation $y = x\sqrt{x-1}$ with x coordinate 5

Find an equation of the normal to the curve at the point P.

(Total for question 12 is 5 marks)

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13 The point P lies on the curve with equation $y = x e^{x^2}$ with x coordinate 1

(a) Find an equation to the tangent to the curve at the point P. **(5)**

The tangent intersects the x axis at the point A and the y axis at the point B.

(b) Find the exact area of the triangle AOB, where O is the origin. **(4)**

(Total for question 13 is 9 marks)

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14 Differentiate with respect to x

(a) $\dfrac{e^x}{2x+1}$ **(4)**

(b) $\dfrac{\ln(x^2+1)}{3x+2}$ **(4)**

(Total for question 14 is 8 marks)

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15 The point P lies on the curve with equation $y=\dfrac{3x}{2x-1}$ with x coordinate 1.

Find an equation to the tangent to the curve at the point P.

(Total for question 15 is 5 marks)

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16 The point P lies on the curve with equation $y=\dfrac{e^x+1}{e^x+3}$ with x coordinate 0.

Find an equation to the normal to the curve at the point P.

(Total for question 16 is 5 marks)

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17 Find the coordinates of the stationary points of the curve $y=\dfrac{x}{9+x^2}$ and determine the nature of the stationary points.

(Total for question 17 is 7 marks)

18

$$f(x)=\frac{x-5}{2x+3}+\frac{2x+4}{2x^2+7x+6}$$

(a) Express f(x) as a fraction in its simplest form. **(3)**

(b) Hence find f '(x) in its simplest form **(4)**

(Total for question 18 is 7 marks)

19

$$f(x)=\frac{4x+1}{2x+3}-\frac{4x-6}{4x^2-9}$$

(a) Express f(x) as a fraction in its simplest form. **(3)**

(b) Hence find f '(x) in its simplest form **(4)**

(Total for question 19 is 7 marks)

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26 A curve has the equation $2x^2 - 3xy + y^2 = 12$:

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(5)**

(b) Find an equation for the normal to the curve at the point (1, -2) **(3)**

(Total for question 26 is 8 marks)

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27 A curve has the equation $3x^2 + xy + y^2 = 20$

The gradient of the tangent to the curve is $\frac{4}{3}$ at the points P and Q.

(a) Show that $2x + y = 0$ at P and Q. **(6)**

(b) Find the coordinates of P and Q. **(3)**

(Total for question 27 is 9 marks)

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28 A curve has the equation $x^2 + 4xy - x + y^2 = 35$:

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(5)**

(b) Find an equation for the tangent to the curve at the point P (2, 3) **(3)**

(Total for question 28 is 8 marks)

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29 A curve has the equation $2 \sin x + 2 \cos y = 3$ $\quad 0 \leq x \leq \pi \quad 0 \leq y \leq \pi$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(3)**

(b) find the coordinates of the point where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ **(4)**

(Total for question 29 is 7 marks)

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30 (a) Given that $y = 2^x$ show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x \ln 2$ **(2)**

(b) Find the equation to the tangent of the curve $y = 3^{(x^2)}$ at the point (2, 81) **(4)**

(Total for question 30 is 6 marks)

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20 Use the derivatives of sin(x) and cos(x) to show that:

(a) $\frac{d}{dx}(\tan x) = \sec^2 x$ **(3)**

(b) $\frac{d}{dx}(\sec x) = \sec x \tan x$ **(3)**

(c) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ **(3)**

(d) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ **(3)**

(Total for question 20 is 12 marks)

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21 Differentiate with respect to x,

(a) $x^2 \cos 2x$ **(3)**
(b) $3 \sin (2x + 1)$ **(2)**

(Total for question 21 is 5 marks)

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22 Differentiate with respect to x,

(a) $e^{3x} (\cos 2x + \sin x)$ **(3)**
(b) $\ln (\sin x)$ **(3)**

(Total for question 22 is 6 marks)

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23 The curve C has the equation $x = 2 \tan y$

(a) Find $\frac{dx}{dy}$ in terms of y **(2)**
(b) Hence find $\frac{dy}{dx}$ in terms of x **(4)**

(Total for question 23 is 6 marks)

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24 The point P lies on the curve with equation $y = \operatorname{cosec} x + \cos 2x$ with x coordinate $\frac{\pi}{4}$

Find an equation to the tangent to the curve at the point P.

(Total for question 24 is 6 marks)

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25 The point P lies on the curve with equation $y = \sec 2x$ with x coordinate $\frac{\pi}{6}$

Find an equation to the normal to the curve at the point P.

(Total for question 25 is 6 marks)

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31 Given that x is measured in radians, prove, from first principles, that the derivative of sin (x) is cos (x)

You may assume the formula for sin (A± B) and that as h→ 0 $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(Total for question 31 is 5 marks)

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32 Given that x is measured in radians, prove, from first principles, that the derivative of cos (x) is –sin (x)

You may assume the formula for cos (A± B) and that as h→ 0 $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(Total for question 32 is 5 marks)

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33 A curve has the parametric equations

$$x = 2t + 1, \qquad y = t^2 - 1$$

(a) Find the points where the curve crosses the coordinate axes. **(2)**

(b) Find an expression for $\frac{dy}{dx}$ in terms of x. **(3)**

(Total for question 33 is 5 marks)

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34 A curve has the parametric equations

$$x = \tan^2 t, \qquad y = \cos t, \qquad 0 < t < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. **(3)**

(b) Find an equation of the tangent to the curve when t = $\frac{\pi}{4}$ **(5)**

(c) Find a cartesian equation for the curve. **(4)**

(Total for question 34 is 12 marks)

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35 A curve has the parametric equations

$$x = \sin^2 t, \qquad y = \sin 2t, \qquad 0 < t < \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. **(3)**

(b) Find an equation of the normal to the curve when t = $\frac{\pi}{6}$ **(5)**

(c) Find a cartesian equation for the curve. **(4)**

(Total for question 35 is 12 marks)

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36 Given that $y = \dfrac{\sin\theta}{\sin\theta + \cos\theta}$

Show that $\dfrac{dy}{d\theta} = \dfrac{1}{\sin 2\theta + 1}$

(Total for question 36 is 5 marks)

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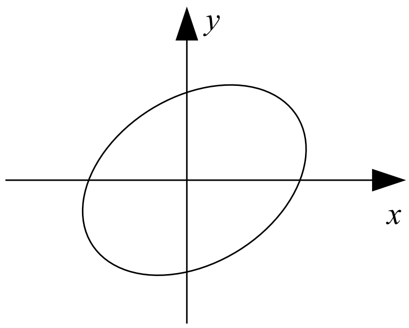
37 The diagram show a sketch of the curve with equation $x^2 - xy + 2y^2 = 20$



(a) Show that $\dfrac{dy}{dx} = \dfrac{2x - y}{x - 4y}$ **(4)**

(b) Find the points on the curve where $\dfrac{dy}{dx} = 0$ **(5)**

(Total for question 37 is 9 marks)

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38 (a) Sketch the graph of $y = (3 - x)^2 + 5, \quad 0 \leq x \leq 6$ **(3)**

The line with equation $x + y = k$, where k is a constant, intersects the curve at two distinct points.

(b) State the range of values of k, writing your answer in set notation. **(5)**

(Total for question 38 is 8 marks)

39

$$\frac{10x + 3 - 4x^2}{(1 - x)(2x + 1)} = A + \frac{B}{1 - x} + \frac{C}{2x + 1} \qquad x > 1$$

(a) Find the values of A, B and C. **(4)**

$$f(x) = \frac{10x + 3 - 4x^2}{(1 - x)(2x + 1)}$$

(b) Prove that f(x) is an increasing function. **(3)**

(Total for question 39 is 7 marks)

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40 The temperature, K°C, of water in a kettle is modelled by the formula $K = 20 + 75e^{-0.2t}$
Where t is the time in minutes since measurement began.

(a) State the starting temperature of the kettle. **(1)**

(b) Find $\frac{dK}{dt}$ **(3)**

(Total for question 40 is 4 marks)

41

$$y = \frac{x^2 + 4x}{(x+2)^2} \qquad x \neq -2$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+2)^n}$ where A and n are constants to be found. **(4)**

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$ **(1)**

(Total for question 41 is 5 marks)

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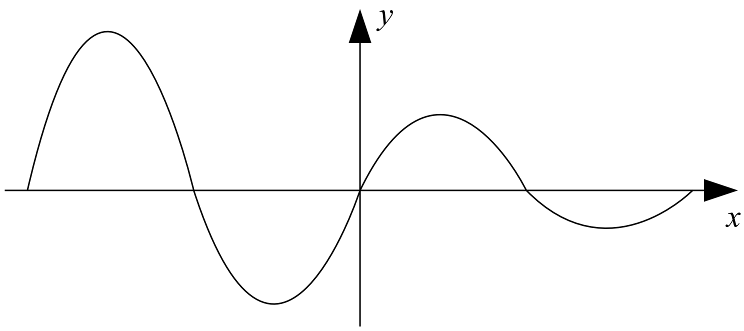
42 The curve C with equation $f(x) = \frac{\sin 2x}{e^x}$ $-\pi < x < \pi$ is shown in the diagram.



(a) Show that the x-coordinates of the turning points of C satisfy the equation $\tan 2x = 2$ **(6)**

(b) Hence find the coordinates of the turning points **(3)**

(Total for question 42 is 9 marks)

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43 The curve C has the equation $x = 2\tan 2y$ $\frac{-\pi}{4} < y < \frac{\pi}{4}$

Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are constants to be found

(Total for question 43 is 5 marks)

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44 The curve C has the equation $y = x^x$ $x > 0$

Find, by firstly taking logarithms, the x-coordinate of the turning point of C.

(Total for question 44 is 5 marks)

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45 A storage tank is modelled in the shape of a hollow cylinder.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres.

The volume of the tank is 12 m^3

(a) Show that the surface area of the tank, in m^2, is $2\pi r^2 + \frac{24}{r}$ **(4)**

(b) Use calculus to find the radius of the tank for which the surface area is a minimum. **(4)**

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer. **(2)**

(Total for question 45 is 10 marks)

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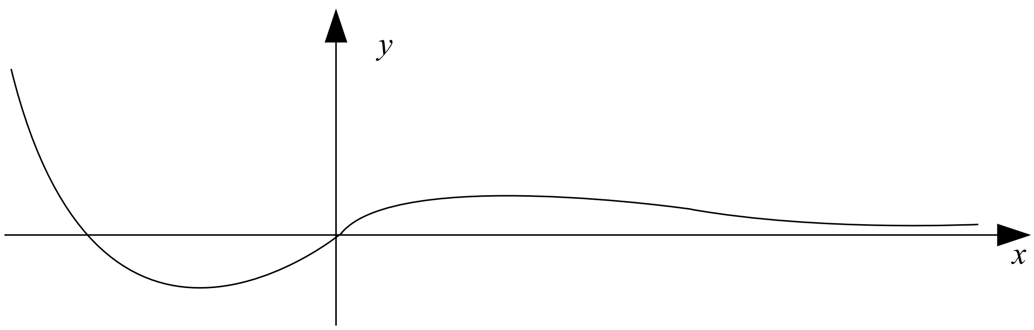
46 The diagram shows the graph with equation $y = f(x)$ where $f(x) = (2x^2 + 3x)e^{-x}$ $x \in \mathbb{R}$



(a) Show that $f'(x) = e^{-x}(x + 3 - 2x^2)$ **(3)**

(b) Hence, find, in simplest form, the exact coordinates of the stationary points of C **(3)**

The function g and the function h are defined by

$$g(x) = 2f(x) \qquad x \in \mathbb{R}$$

$$h(x) = f(x) + 2 \qquad x > 0$$

(c) Find (i) the range of g
(ii) the range of h **(3)**

(Total for question 46 is 9 marks)

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47 Curve C is given by $x + 5 = \sin x \cos x + 2y \qquad 0 < x < \pi$

(a) Show that $\dfrac{dy}{dx} = \sin^2 x$ **(4)**

(b) Find the coordinates of the point of inflection of C. **(3)**

(Total for question 47 is 7 marks)

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48 A function f is given by $f(x) = \dfrac{5 + \ln x}{3 + 2\ln x} \qquad x > 0$

Prove that f is a decreasing function.

(Total for question 48 is 3 marks)

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49 A curve C has the equation $y = \dfrac{x^2 + 5}{x} + \ln x \qquad x > 0$

(a) Show that $\dfrac{dy}{dx} = \dfrac{x^2 + x - 5}{x^2}$ **(4)**

(b) Hence find the coordinates of the turning point of C. **(3)**

(Total for question 49 is 7 marks)

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50 A curve is defined by the parametric equations

$$x = 3 \times 2^{-t} + 2$$

$$y = 5 \times 2^{t} - 3$$

(a) Show that $\dfrac{dy}{dx} = -\dfrac{5}{3} \times 2^{2t}$ **(3)**

(b) Find a Cartesian equation for the curve. **(2)**

(Total for question 50 is 5 marks)

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51 A six-sided box, in the shape of a cuboid, is made from a sheet metal. The base of the box is x cm by y cm and the height of the box is x cm.

The volume of the box is 5000 cm^2.

(a) Show that the the area of sheet metal, A cm^2, is given by $A = \dfrac{20000}{x} + 2x^2$ **(4)**

(b) Use calculus to show to find the value of x for which A is stationary. **(4)**

(c) Prove that this value of x gives a minimum value of A. **(2)**

(d) Calculate the minimum area of sheet metal needed to make the box. **(2)**

(Total for question 51 is 10 marks)

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52 Find the coordinates of the stationary point of the curve with equation

$$x^2 + 2x = e^y - 3$$

(Total for question 52 is 7 marks)

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53 A function f is defined by $f(x) = \dfrac{x}{\sqrt{2x-3}}$

(a) State the maximum possible domain of f. **(2)**

(b) Use the quotient rule to show that $f'(x) = \dfrac{x-3}{(2x-3)^{\frac{3}{2}}}$ **(3)**

(c) Show that the graph of $y = f(x)$ has exactly one point of inflection. **(7)**

(Total for question 53 is 12 marks)

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54 Given $y = e^{kx}$, where k is a constant, find $\dfrac{dy}{dx}$ in terms of k.

(Total for question 54 is 2 marks)

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55 The volume of a sphere is increasing at a constant rate of $2\ cm^3s^{-1}$

Show that the rate of increase of the radius when $r = 2$ cm is $\dfrac{a}{\pi}$, where a is a constant.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

(Total for question 55 is 4 marks)

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56 Given $y = 3^x$, show that $\dfrac{dy}{dx} = 3^x \ln 3$

(Total for question 56 is 2 marks)

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57 A curve, C, has the equation $y = \dfrac{e^{2x+1}}{x^2}$

Show that C has exactly one stationary point.

Fully justify your answer.

(Total for question 57 is 7 marks)

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58 A curve, C, has the equation $xy^2 + x^2y = 10$

(a) Prove that the curve does not intersect the coordinate axes. **(2)**

(b) Show that $\dfrac{dy}{dx} = -\dfrac{y(2x+y)}{x(x+2y)}$ **(5)**

(c) Find the exact coordinates of the stationary point. **(5)**

(Total for question 58 is 12 marks)

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59 A curve, C, has the equation $x^2y + xy = 6$

When $x > 0$, find the equation of the tangent to C when $y = 1$.

(Total for question 59 is 7 marks)

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60 A curve, C, has the equation $5\sin y + x\cos y = Ax$

where A is a constant.

C passes through the point $P\left(\sqrt{3}, \dfrac{\pi}{3}\right)$

(a) Show that $A = 3$ **(2)**

(b) Show that $\dfrac{dy}{dx} = \dfrac{3 - \cos y}{5\cos y - x\sin y}$ **(5)**

(c) Hence find the equation of the tangent to C at P. **(4)**

(Total for question 60 is 11 marks)

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61 A curve has parametric equations $x = 3t + \frac{1}{t}$ and $y = 3t - \frac{1}{t}$ for $t \neq 0$

(a) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. **(4)**

(b) Explain why the curve has no stationary points. **(2)**

(c) By considering $x + y$, or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets. **(4)**

(Total for question 61 is 10 marks)

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62 A curve has the equation $y = x \ln x$

(a) Find the coordinates of the turning point of the curve. **(4)**

(b) Determine whether this turning point is a maximum or a minimum. **(3)**

(Total for question 62 is 7 marks)

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63 A curve has the equation $x^3 - x^2y - 2y - 5 = 0$

(a) Show that $\frac{dy}{dx} = \frac{3x^2 - 2xy}{x^2 + 2}$ **(4)**

(b) Find the equation of the normal to the curve at the point (1, 2). **(4)**

(Total for question 63 is 8 marks)

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64 Let $f(x) = x^3 + 5x$. Use differentiation from first principles to show that $f'(x) = 3x^2 + 5$.

(Total for question 64 is 6 marks)

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65 A curve has the equation $y = a^{x^2}$, where a is a constant greater than 1.

(a) Show that $\frac{dy}{dx} = 2xa^{x^2} \ln a$ **(3)**

(b) The tangent at the point $(1, a)$ passes through the point $\left(\frac{1}{2}, 0\right)$. **(4)**

Find the value of a, giving your answer in an exact form.

(c) By considering $\frac{d^2y}{dx^2}$ show that the curve is convex for all values of x. **(5)**

(Total for question 65 is 12 marks)

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66 Differentiate with respect to x:

(a) $(2x + 5)^5$ **(2)**

(b) $3x^2 \ln x$ **(3)**

(Total for question 66 is 5 marks)

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67 A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by $V = 400r - \pi r^3$ **(4)**

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3. **(5)**

(c) Justify that the value of V you have found is a maximum. **(2)**

(Total for question 67 is 11 marks)

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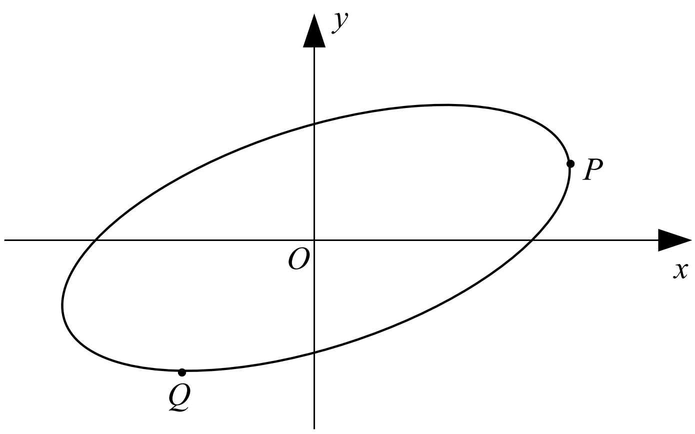
68 A curve has the equation $y = x^3 + kx^2 + 5$, where k is a constant.

Given that the curve has a point of inflection where $x = 2$, show that $k = -6$.

(Total for question 68 is 5 marks)

69



The diagram shows the curve with equation $x^2 - 2xy + 4y^2 - 12 = 0$

(a) Show that $\dfrac{dy}{dx} = \dfrac{x - y}{x - 4y}$ **(3)**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the exact distance PQ , is $k\sqrt{5}$ where k is a rational number to be determined. **(7)**

(Total for question 69 is 10 marks)

70 Differentiate with respect to x:

(a) $\sin(2x + 5)$ **(2)**

(b) $\dfrac{3x^2}{2x - 1}$ **(3)**

(c) $\tan(x^2)$ **(2)**

(Total for question 70 is 7 marks)

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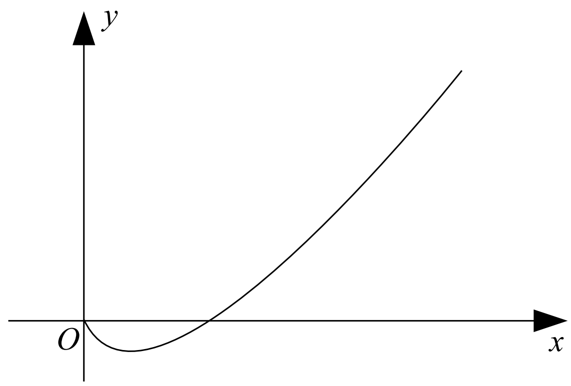
71 The curve C is defined by the parametric equations $x = 2\cos t$, $y = \sin t$.

The line L is a tangent to C at the point given by $t = \dfrac{\pi}{3}$

Find the point where L cuts the y-axis.

(Total for question 71 is 6 marks)

72



The diagram shows the curve given parametrically by the equations $x = 4t^2$, $y = t(t^2 - 8)$, for $t > 0$

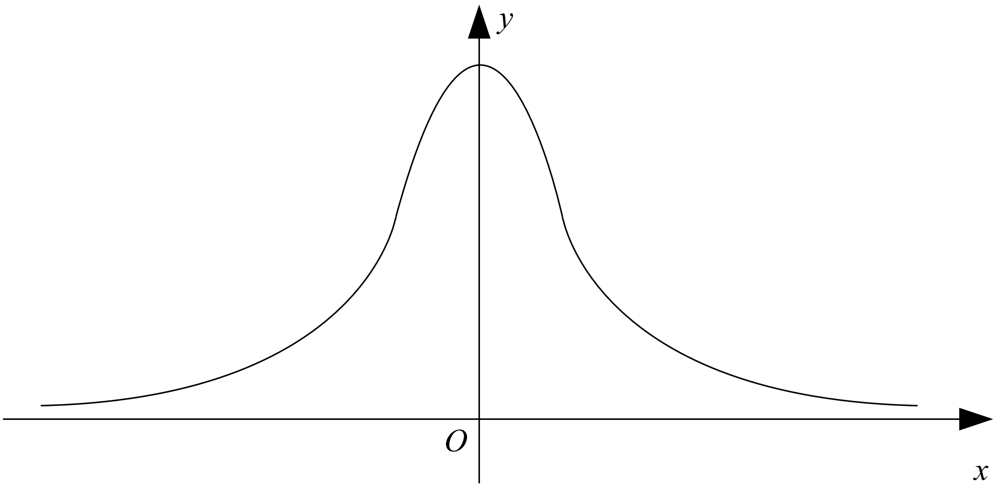
(a) Show that $\dfrac{dy}{dx} = \dfrac{3t^2 - 8}{8t}$ **(3)**

(b) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $3x - 12y + 8 = 0$ **(3)**

(c) Find the cartesian equation of the curve. **(3)**

(Total for question 72 is 9 marks)

73



The diagram shows the curve with the equation $y = \dfrac{1}{1+x^2}$

(a) Find $\dfrac{d^2y}{dx^2}$ **(5)**

(b) Hence find the set of values of x for which the curve is concave. **(2)**

(Total for question 73 is 7 marks)