

# A Level Maths: The Binomial Expansion

- 1 Work out the first four terms, in ascending powers of  $x$ , in the binomial expansion of  $\frac{1}{(1+x)^2}$

**(Total for question 1 is 3 marks)**

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- 2 (a) Expand  $(4 + 6x)^{-\frac{1}{2}}$  in ascending powers of  $x$  up to and including  $x^3$  (4)  
(b) Write down the range of values of  $x$  for which the expansion is valid. (1)

**(Total for question 2 is 5 marks)**

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- 3 (a) Expand  $(8 + 12x)^{\frac{1}{3}}$  in ascending powers of  $x$  up to and including  $x^3$  (4)  
(b) Write down the range of values of  $x$  for which the expansion is valid. (1)

**(Total for question 3 is 5 marks)**

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- 4 (a) Expand  $\sqrt[3]{1 + 2x}$  in ascending powers of  $x$  up to and including  $x^3$  and state the set of values of  $x$  for which the expansion is valid. (5)  
(b) Use your expansion to find an approximation for  $\sqrt[3]{1.1}$  to 5 decimal places. (2)

**(Total for question 4 is 7 marks)**

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- 5 (a) Work out the first three terms in the binomial expansion of  $\frac{1}{(2 + 3x)^2}$  (4)  
(b) Write down the range of values of  $x$  for which the expansion is valid. (1)

**(Total for question 5 is 5 marks)**

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- 6 (a) Expand  $(3 + 2x)^{\frac{1}{2}}$  in ascending powers of  $x$  up to and including  $x^3$  (4)  
(b) Write down the range of values of  $x$  for which the expansion is valid. (1)  
(c) Use your expansion, with  $x = 0.1$ , to find an approximation of the value of  $\sqrt{5}$ . Give your answer to 3 decimal places. (3)

**(Total for question 6 is 8 marks)**

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- 7 (a) Expand  $(1 + 3x)^{-3}$  in ascending powers of  $x$  up to and including  $x^3$  (4)  
(b) Write down the range of values of  $x$  for which the expansion is valid. (1)  
(c) Use your expansion to find the expansion of  $\frac{2x + 1}{(1 + 3x)^3}$  up to and including  $x^3$  (3)

**(Total for question 7 is 8 marks)**

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- 8 (a) Expand  $(9 - 2x)^{\frac{1}{2}}$  in ascending powers of  $x$  up to and including  $x^3$  (4)
- (b) Write down the range of values of  $x$  for which the expansion is valid. (1)
- (c) Use your expansion, with a suitable value of  $x$ , to find the value of  $\sqrt{8.9}$  correct to 5 significant figures (2)

(Total for question 8 is 7 marks)

- 9 (a) Use binomial expansion to show that  $\sqrt{\frac{1+5x}{1-x}} \approx 1 + 3x - \frac{3}{2}x^2$  (6)

A student substitutes  $x = \frac{1}{2}$  into the expansion to find an estimate for  $\sqrt{7}$

- (b) Give a reason why the student should not use  $x = \frac{1}{2}$  (1)

- (c) Substitute  $x = \frac{1}{9}$  into  $\sqrt{\frac{1+5x}{1-x}} = 1 + 3x - \frac{3}{2}x^2$

to obtain an approximation for  $\sqrt{7}$ . Give your answer as a fraction in its simplest form. (3)

(Total for question 9 is 10 marks)

- 10 (a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$\frac{1}{\sqrt{9+x}}$$

giving each coefficient in its simplest form. (4)

The expansion can be used to find an approximation of  $\sqrt{3}$

Possible values of  $x$  that could be substituted into this expansion are

- $x = 18$  because  $\frac{1}{\sqrt{9+18}} = \frac{1}{\sqrt{27}} = \frac{\sqrt{3}}{9}$
- $x = 3$  because  $\frac{1}{\sqrt{9+3}} = \frac{1}{\sqrt{12}} = \frac{\sqrt{3}}{6}$
- $x = -\frac{2}{3}$  because  $\frac{1}{\sqrt{9-\frac{2}{3}}} = \frac{1}{\sqrt{\frac{25}{3}}} = \frac{\sqrt{3}}{5}$

- (b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of  $x$  should not be used (1)

(ii) state, giving a reason, which of the three values of  $x$  would lead to the most accurate approximation to  $\sqrt{3}$  (1)

(Total for question 10 is 6 marks)

- 11** (a) Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of  $\sqrt{1+6x}$  giving each term in its simplest form. (3)
- (b) Explain how you could use  $x = \frac{1}{12}$  in the expansion to find an approximation for  $\sqrt{6}$ .  
There is no need to carry out the calculation. (2)

**(Total for question 11 is 5 marks)**

- 12** (a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of  $\frac{1}{\sqrt{4+x}}$  giving each coefficient in its simplest form. (3)
- (b) Hence find first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$\frac{1}{\sqrt{4-x^2}} \quad (2)$$

**(Total for question 12 is 5 marks)**

- 13** (a) Find the first three terms of the expansion of  $(9+2x)^{-\frac{1}{2}}$  in ascending powers of  $x$ . (4)
- (b) The expansion of  $\frac{a+bx}{\sqrt{9+2x}}$  is  $3+x \dots$  Find the values of the constants  $a$  and  $b$ . (3)

**(Total for question 13 is 7 marks)**

- 14** (a) Find the coefficient of  $x^5$  in the expansion of  $(2+3x)^7$  (1)
- (b) (i) Expand  $\sqrt{1-2x}$  as far as the term  $x^3$ . (3)
- (ii) State the range of the values for which your expansion is valid. (1)
- (iii) Use your expansion to find an estimate for  $\sqrt{98}$ , correct to five decimal places, and compare this with the value of  $\sqrt{98}$  given by your calculator. (3)

**(Total for question 14 is 8 marks)**

- 15** In this question you should assume  $-\frac{1}{2} < x < \frac{1}{2}$
- (a) For the binomial expansion of  $(1-2x)^2$  find and simplify the first four terms. (2)
- (b) Write down the sum to infinity of the series  $1+2x+4x^2+8x^3+\dots$  (1)
- (c) Hence or otherwise find and simplify an expression for  $2+6x+16x^2+40x^3+\dots$  (3)

**(Total for question 15 is 6 marks)**

**16** Find the first three terms of the expansion of  $(8 - 4x^3)^{\frac{1}{3}}$  in ascending powers of  $x$ .

**(Total for question 16 is 3 marks)**

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**17** The function  $f(x)$  is given by  $f(x) = \sqrt{1 + ax}$  where  $a$  is a non-zero constant.

In the binomial expansion of  $f(x)$ , the coefficients of  $x$  and  $x^2$  are equal.

(a) Find the value of  $a$ . **(3)**

(b) Using this value of  $a$

(i) State the set of values for which the binomial expansion is valid, **(1)**

(ii) Write down the quadratic function that approximates  $f(x)$  when  $x$  is small. **(1)**

**(Total for question 17 is 5 marks)**

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**18** (a) Work out the first three terms in the binomial expansion of  $(1 + 3x)^{\frac{1}{2}}$  **(3)**

(b) Write down the range of values of  $x$  for which the expansion is valid. **(1)**

**(Total for question 18 is 4 marks)**

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