1 Given that the point A has position vector $3\mathbf{i} + 4\mathbf{j}$ and the point B has position vector $-4\mathbf{i} + 7\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$ **(2)**

a/ $\begin{pmatrix} -4 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ 3 \end{pmatrix}$

$-7\underline{i} + 3\underline{j}$

b/ $\sqrt{3^2 + 4^2} = 5$

2 Given that $|3\mathbf{i} + k\mathbf{j}| = 3\sqrt{17}$

Find the value of k

$\sqrt{3^2 + k^2} = 3\sqrt{17}$

$3^2 + k^2 = 153$

$k^2 = 144$

$K = \pm 12$

3 Given that the point A has position vector $-5\mathbf{i} + 7\mathbf{j}$ and the point B has position vector $-8\mathbf{i} + 2\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$ **(2)**

a/ $\begin{pmatrix} -8 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ $\quad -3\underline{i} - 5\underline{j}$

b/ $\sqrt{3^2 + 5^2} = \sqrt{34}$

4 $\mathbf{a} = -5\mathbf{i} + 7\mathbf{j}$ and $\mathbf{b} = x\mathbf{i} + y\mathbf{j}$
Given that the resultant force of **a** and **b** is $-2\mathbf{i} - 3\mathbf{j}$ find the values of x and y

$$\begin{pmatrix}-5\\7\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2\\-3\end{pmatrix}$$

$x = 3$ $\quad y = -10$

5 In triangle ABC, $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{AC} = 8\mathbf{i} - 5\mathbf{j}$

(a) Find the vector $\overrightarrow{BC}$ **(2)**

(b) Find the length of the line AB **(2)**

a/ $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= \begin{pmatrix}-6\\-2\end{pmatrix} + \begin{pmatrix}8\\-5\end{pmatrix} = \begin{pmatrix}2\\-7\end{pmatrix}$$

$2\underline{i} - 7\underline{j}$

b/ $\sqrt{6^2 + 2^2} = 2\sqrt{10}$

6 Three forces act on an object $\mathbf{F}_1 = -5\mathbf{i} + 7\mathbf{j}$, $\mathbf{F}_2 = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{F}_3 = 3\mathbf{i} - 5\mathbf{j}$
Find the resultant force.

$$\begin{pmatrix}-5\\7\end{pmatrix} + \begin{pmatrix}4\\6\end{pmatrix} + \begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}2\\8\end{pmatrix}$$

$2\underline{i} + 8\underline{j}$

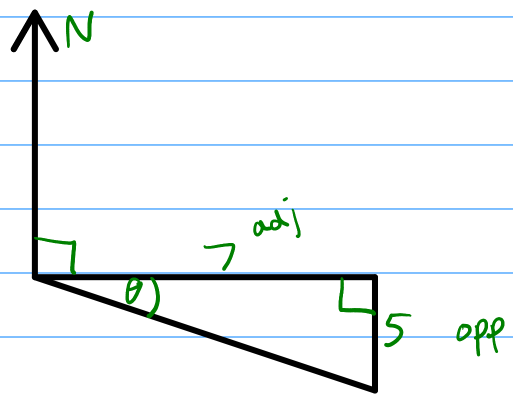
7 A car is driving with a velocity of $(7\mathbf{i} - 5\mathbf{j})\ \text{ms}^{-1}$

(a) Find speed of the car **(2)**

(b) Find the bearing the car is travelling on. **(2)**

a/ $\sqrt{7^2 + 5^2} = \underline{\underline{\sqrt{74}}}$ or $\underline{\underline{8.60\ \text{m/s}}}$ (3sf)

b/



$\tan\theta = \frac{5}{7}$

$\theta = 35.5$

$90 + 35.5 = \underline{\underline{126°}}$ (3sf)

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8 Given that the point A has position vector $2\mathbf{i} - 6\mathbf{j}$ and the point B has position vector $-4\mathbf{i} + 7\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$

Give your answer as a surd. **(2)**

a/ $\begin{pmatrix} -4 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ -6 \end{pmatrix} = \begin{pmatrix} -6 \\ 13 \end{pmatrix}$

$\underline{\underline{-6\underline{i} + 13\underline{j}}}$

b/ $\sqrt{6^2 + 13^2} = \underline{\underline{\sqrt{205}}}$

9 (a) Two non-zero vectors, ***a*** and ***b***, are such that

$$|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a}| + |\boldsymbol{b}|$$

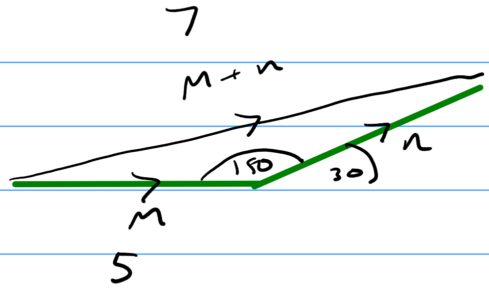
Explain geometrically the significance of this statement. **(1)**

(b) Two different vectors, ***m*** and ***n***, are such that $|\boldsymbol{m}| = 5$ and $|\boldsymbol{m}+\boldsymbol{n}| = 7$
The angle between vector ***m*** and vector ***n*** is 30°

Find the angle between vector ***m*** and vector ***m*** – ***n***, giving your answer in degrees to one decimal place. **(4)**

a/ $\underline{a}$ and $\underline{b}$ must be parallel (same direction)

b/

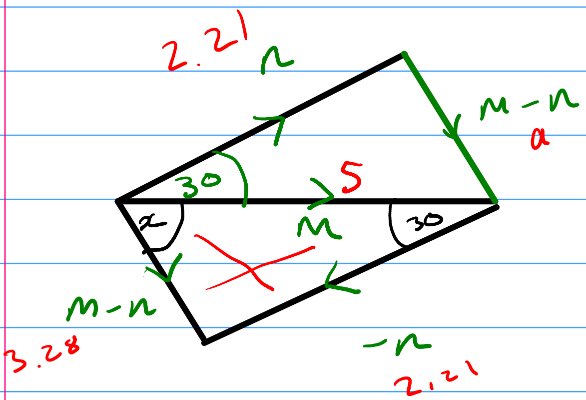


$7^2 = 5^2 + n^2 - 2(5)(n)\cos(150)$

$49 = 25 + n^2 + 5\sqrt{3}\,n$

$0 = n^2 + 5\sqrt{3}\,n - 24$

$\underline{\underline{n = 2.21}}$ or $\cancel{-10.9}$



$a^2 = 5^2 + 2.21^2 - 2(5)(2.21)\cos(30)$

$a^2 = 10.75$

$a = 3.28$

$$\frac{\sin x}{2.21} = \frac{\sin 30}{3.28}$$

$x = \sin^{-1}(0.3367\ldots)$

$= \underline{\underline{19.7^\circ}}$

10 *[In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.]*

A coastguard station O monitors the movements of a small boat.

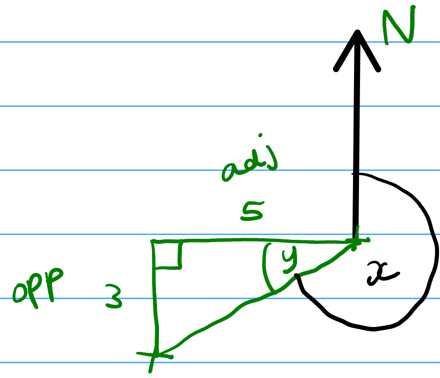
At 08:00 the boat is at the point (3i – 4j) km relative to O.
At 10:20 the boat is at the point (–2i – 7j) km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place. **(3)**

(b) Calculate the speed of the boat, giving your answer in km h^{-1} **(3)**

a) $\begin{pmatrix} -2 \\ -7 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$



$\tan y = \frac{3}{5}$

$y = 31.0°$

$270 - 31.0 = \underline{\underline{239.0°}}$

b) 2 hours 20 mins $= 2\frac{1}{3}$ hours

distance $= \sqrt{3^2 + 5^2}$
$= \sqrt{34}$

speed $= \sqrt{34} \div 2\frac{1}{3}$

$= \underline{\underline{2.50 \text{ km/h}}}$ (3sf)

11 [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

At time $t = 0$, a particle P is at position $(-2\mathbf{i} + 4\mathbf{j})$m relative to a fixed origin, O. The particle moves with velocity $(4\mathbf{i} - 6\mathbf{j})\text{ms}^{-1}$

(a) Find the speed of P. **(3)**

(b) Show that P passes through the point A with position $(8\mathbf{i} - 11\mathbf{j})$m. **(3)**

a/ $\text{speed} = \sqrt{4^2 + 6^2}$

$= \underline{\underline{7.21}} \text{ ms}^{-1}$ (3 sf)

b/ $r = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -6 \end{pmatrix} t$

$\begin{pmatrix} 8 \\ -11 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -6 \end{pmatrix} t$

$8 = -2 + 4t$

$10 = 4t$

$t = \underline{\underline{\frac{5}{2}}}$

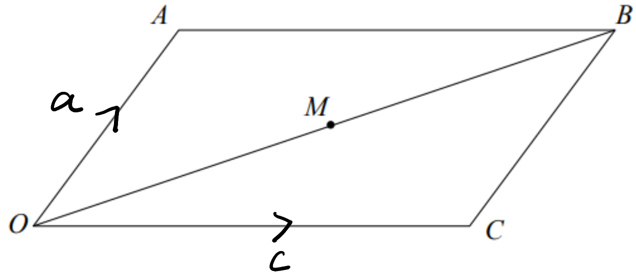
$-11 = 4 - 6t$

$-15 = -6t$

$t = \frac{15}{6} = \underline{\underline{\frac{5}{2}}}$

P passes through A after 2.5 seconds.

12 $OABC$ is a parallelogram with $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$
M is the midpoint of OB.



(a) Find in terms of **a** and **c**, simplifying your answers.

(i) $\overrightarrow{AC}$ **(1)**

(ii) $\overrightarrow{OM}$ **(2)**

(b) Hence prove that the diagonals of a parallelogram bisect one another. **(4)**

a/ $\overrightarrow{AC} = -a + c$

$= \underline{c} - \underline{a}$

b/ $\overrightarrow{OB} = a + c$

$\overrightarrow{OM} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{c}$

c/ $\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM}$

$= -a + \frac{1}{2}a + \frac{1}{2}c$

$= -\frac{1}{2}a + \frac{1}{2}c$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$ and $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OB}$

$\therefore$ the diagonals bisect eachother at M

13 Points A, B, C and D have position vectors $\boldsymbol{a} = \begin{pmatrix}1\\3\end{pmatrix}, \boldsymbol{b} = \begin{pmatrix}3\\6\end{pmatrix}, \boldsymbol{c} = \begin{pmatrix}7\\5\end{pmatrix}, \boldsymbol{d} = \begin{pmatrix}5\\k\end{pmatrix}$

(a) Find the value of k for which D is the midpoint of BC **(1)**

(b) Find the two values of of k for which $|\overrightarrow{AD}| = 2\sqrt{5}$ **(3)**

(c) Find the value of k for which $ABCD$ is a parallelogram. **(2)**

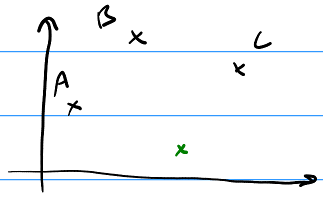
a/ $\frac{3+7}{2} = 5 \qquad \frac{6+5}{2} = 5.5$

Midpoint at $\begin{pmatrix}5\\5.5\end{pmatrix} \quad \therefore \quad \underline{\underline{k = 5.5}}$

b/ $\overrightarrow{AD} = \begin{pmatrix}5\\k\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}4\\k-3\end{pmatrix}$

$$\sqrt{4^2 + (k-3)^2} = 2\sqrt{5}$$
$$16 + (k-3)^2 = 20$$
$$(k-3)^2 = 4$$
$$k - 3 = \pm 2$$
$$k = \underline{\underline{5}} \quad \text{or} \quad \underline{\underline{1}}$$

c/



AB $\quad m = \frac{6-3}{3-1} = \frac{3}{2}$

CD $\quad m = \frac{5-k}{7-5} = \frac{5-k}{2} \qquad \underline{\underline{k = 2}}$

14 Points A and B have position vectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ respectively

Point C has position vector $\begin{pmatrix} 1 \\ p \end{pmatrix}$ and ABC is a straight line.

(a) Find P **(2)**

Point D has position vector $\begin{pmatrix} 1 \\ q \end{pmatrix}$ and angle $ABD = 90°$

(b) Determine the value of q. **(3)**

$$\vec{AB} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\frac{1}{2}\vec{AB} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 5 \end{pmatrix}}}$$

$$\underline{\underline{p = 5}}$$

b) AB $\quad m = \frac{3 - -1}{2 - 4} = \frac{4}{-2} = -2$

$\therefore$ gradient of BD $= \frac{1}{2}$ (perpendicular)

BD: $\quad m = \frac{q - -1}{1 - 4} = \frac{q+1}{-3}$

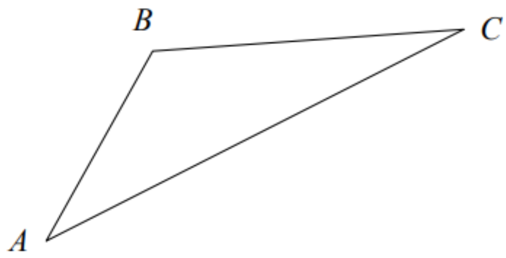
$$\frac{q+1}{-3} = \frac{1}{2}$$

$$2q + 2 = -3$$

$$2q = -5$$

$$\underline{\underline{q = -\frac{5}{2}}}$$

15 Points ABC is a triangle where $\overrightarrow{AB} = 2\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{AC} = 7\mathbf{i} + 7\mathbf{j}$



Show that ABC is an isosceles triangle.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= -2i - 5j + 7i + 7j$$
$$= 5i + 2j$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$|\overrightarrow{BC}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$|\overrightarrow{AC}| = \sqrt{7^2 + 7^2} = \sqrt{98}$$

Two sides equal $\therefore$ isosceles triangle.