AS Level Maths: Vectors

1 Given that the point A has position vector $3\mathbf{i} + 4\mathbf{j}$ and the point B has position vector $-4\mathbf{i} + 7\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$ **(2)**

(Total for question 1 is 4 marks)

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2 Given that $|3\mathbf{i} + k\mathbf{j}| = 3\sqrt{17}$

Find the value of k

(Total for question 2 is 2 marks)

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3 Given that the point A has position vector $-5\mathbf{i} + 7\mathbf{j}$ and the point B has position vector $-8\mathbf{i} + 2\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$ **(2)**

(Total for question 3 is 4 marks)

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4 $\mathbf{a} = -5\mathbf{i} + 7\mathbf{j}$ and $\mathrm{b} = x\mathbf{i} + y\mathbf{j}$
Given that the resultant force of $\mathbf{a}$ and $\mathbf{b}$ is $-2\mathbf{i} - 3\mathbf{j}$ find the values of x and y

(Total for question 4 is 2 marks)

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5 In triangle ABC, $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{AC} = 8\mathbf{i} - 5\mathbf{j}$

(a) Find the vector $\overrightarrow{BC}$ **(2)**

(b) Find the length of the line AB **(2)**

(Total for question 5 is 4 marks)

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6 Three forces act on an object $\mathbf{F}_1 = -5\mathbf{i} + 7\mathbf{j}$, $\mathbf{F}_2 = 4\mathbf{i} + 6\mathbf{j}$ and $\mathbf{F}_3 = 3\mathbf{i} - 5\mathbf{j}$
Find the resultant force.

(Total for question 6 is 2 marks)

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7 A car is driving with a velocity of $(7\mathbf{i} - 5\mathbf{j})$ ms^{-1}

(a) Find speed of the car **(2)**

(b) Find the bearing the car is travelling on. **(2)**

(Total for question 7 is 4 marks)

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8 Given that the point A has position vector $2\mathbf{i} - 6\mathbf{j}$ and the point B has position vector $-4\mathbf{i} + 7\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$

Give your answer as a surd. **(2)**

(Total for question 8 is 4 marks)

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9 (a) Two non-zero vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, are such that

$$|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a}| + |\boldsymbol{b}|$$

Explain geometrically the significance of this statement. **(1)**

(b) Two different vectors, $\boldsymbol{m}$ and $\boldsymbol{n}$, are such that $|\boldsymbol{m}| = 5$ and $|\boldsymbol{m}+\boldsymbol{n}| = 7$
The angle between vector $\boldsymbol{m}$ and vector $\boldsymbol{n}$ is 30°

Find the angle between vector $\boldsymbol{m}$ and vector $\boldsymbol{m} - \boldsymbol{n}$, giving your answer in degrees to one decimal place. **(4)**

(Total for question 9 is 5 marks)

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10 [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A coastguard station O monitors the movements of a small boat.

At 08:00 the boat is at the point $(3\mathbf{i} - 4\mathbf{j})$ km relative to O.
At 10:20 the boat is at the point $(-2\mathbf{i} - 7\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place. **(3)**

(b) Calculate the speed of the boat, giving your answer in km h^{-1} **(3)**

(Total for question 10 is 6 marks)

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11 [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

At time $t = 0$, a particle P is at position $(-2\mathbf{i} + 4\mathbf{j})$m relative to a fixed origin, O.
The particle moves with velocity $(4\mathbf{i} - 6\mathbf{j})$$ms^{-1}$

(a) Find the speed of P. **(3)**

(b) Show that P passes through the point A with position $(8\mathbf{i} - 11\mathbf{j})$m. **(3)**

(Total for question 11 is 6 marks)

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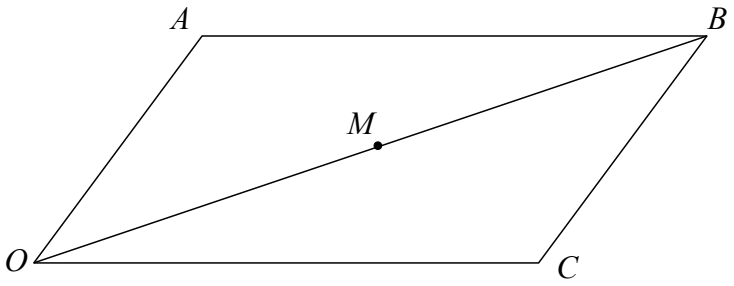
12 *OABC* is a parallelogram with $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$
M is the midpoint of *OB*.



(a) Find in terms of **a** and **c**, simplifying your answers.

(i) $\overrightarrow{AC}$ **(1)**

(ii) $\overrightarrow{OM}$ **(2)**

(b) Hence prove that the diagonals of a parallelogram bisect one another. **(4)**

(Total for question 12 is 7 marks)

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13 Points *A,B,C* and *D* have position vectors $\boldsymbol{a} = \begin{pmatrix}1\\3\end{pmatrix}, \boldsymbol{b} = \begin{pmatrix}3\\6\end{pmatrix}, \boldsymbol{c} = \begin{pmatrix}7\\5\end{pmatrix}, \boldsymbol{d} = \begin{pmatrix}5\\k\end{pmatrix}$

(a) Find the value of *k* for which *D* is the midpoint of *BC* **(1)**

(b) Find the two values of of *k* for which $|\overrightarrow{AD}| = 2\sqrt{5}$ **(3)**

(c) Find the value of *k* for which *ABCD* is a parallelogram. **(2)**

(Total for question 13 is 6 marks)

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14 Points *A* and *B* have position vectors $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}4\\-1\end{pmatrix}$ respectively

Point *C* has position vector $\begin{pmatrix}1\\p\end{pmatrix}$ and *ABC* is a straight line.

(a) Find *P* **(2)**

Point D has position vector $\begin{pmatrix}1\\q\end{pmatrix}$ and angle *ABD* = 90°

(b) Determine the value of *q*. **(3)**

(Total for question 14 is 5 marks)

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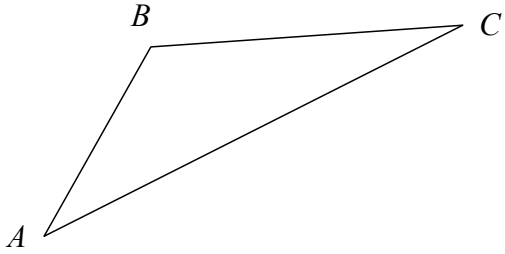
15 Points *ABC* is a triangle where $\overrightarrow{AB} = 2\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{AC} = 7\mathbf{i} + 7\mathbf{j}$



Show that *ABC* is an isosceles triangle.

(Total for question 15 is 3 marks)
