1 (a) Use the identity for sin (A + B) to express sin 2A in terms of sin A and cos A.

(b) Use the identity for cos (A + B) to express cos 2A in terms of sin A and cos A.

(c) Hence, express cos 2A in terms of
i) cos A
ii) sin A

(d) Use the identity for tan (A + B) to express tan 2A in terms of tan A.

a/

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A+A) = \sin A \cos A + \cos A \sin A$$
$$\sin 2A = 2 \sin A \cos A$$

b/

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A+A) = \cos A \cos A - \sin A \sin A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$

c/ i)

$$\sin^2 A = 1 - \cos^2 A$$

$$\cos 2A = \cos^2 A - (1 - \cos^2 A)$$
$$\cos 2A = 2\cos^2 A - 1$$

ii/

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos 2A = (1 - \sin^2 A) - \sin^2 A$$
$$\cos 2A = 1 - 2\sin^2 A$$

d/

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

2 (a) By writing sin 3θ as sin $(2\theta + \theta)$ show that sin $3\theta = 3\sin\theta - 4\sin^3\theta$

(b) Solve, for $0 \leq \theta \leq 180$, the equation,

$$3\sin\theta - 4\sin^3\theta = 0.4$$

Give your answers to 1 decimal place.

a/ $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$\sin 3\theta = 2\sin\theta\cos\theta\cos\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= \underline{\underline{3\sin\theta - 4\sin^3\theta}}$

b/ $\sin 3\theta = 0.4$

$3\theta = 23.6°, 156.4°, 383.6°, 516.4°$

$\theta = \underline{\underline{7.9°}}, \underline{\underline{52.1°}}, \underline{\underline{127.9°}}, \underline{\underline{172.1°}}$

3 (a) By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$ show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(b) Solve, for $0 \le \theta \le \pi$, the equation,

$$4\cos^3\theta - 3\cos\theta = 0.5$$

Give your answers in terms of π.

a/ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(2\theta+\theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

b/ $\cos 3\theta = 0.5$

$3\theta = \frac{1}{3}\pi, \frac{5}{3}\pi, \frac{7}{3}\pi$

$\theta = \frac{1}{9}\pi, \frac{5}{9}\pi, \frac{7}{9}\pi$

---

4 Solve, for $-180 \le x \le 180$, the equation,

$$\cos 2x - 7\sin x + 3 = 0$$

$\cos 2x = 1 - 2\sin^2 x$

$1 - 2\sin^2 x - 7\sin x + 3 = 0$

$2\sin^2 x + 7\sin x - 4 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -4$ $\times$

$x = 30°, 150°$

5 Solve, for $0 \le x \le 360$, the equation,

$$8 \sin x \cos x = 3$$

Give your answers to 1 decimal place.

$\sin 2A = 2 \sin A \cos A$

$4 \sin 2A = 8 \sin A \cos A$

$4 \sin 2x = 3$

$\sin 2x = 3/4$

$2x = 48.6°, 131.4°, 408.6°, 491.4°$

$x = \underline{24.3°}, \underline{65.7°}, \underline{204.3°}, \underline{245.7°}$

6 Prove the identity:

$$\frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x$$

$\cos 2x = \cos^2 x - \sin^2 x$

$$\frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$$

$$\frac{\cancel{(\cos x + \sin x)}(\cos x - \sin x)}{\cancel{(\cos x + \sin x)}}$$

$\underline{\cos x - \sin x}$

7 $$f(x) = 3\cos x - 4\sin x$$

Given that $f(x) = R\cos(x + \alpha)$, where $R > 0$ and $0 \le \alpha \le 90$,

(a) Find the value of R and the value of α

(b) Hence solve, for $0 \le \theta \le 360$, the equation

$$3\cos x - 4\sin x = 1$$

Give your answers to 1 decimal place.

(c) Write down the minimum value of $3\cos x - 4\sin x$

(d) Find, to 1 decimal place, the smallest positive value of x for which this minimum occurs

a/ $$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$R\cos(x + \alpha) = R\cos x \cos\alpha - R\sin x \sin\alpha$$

$$3\cos x \quad - 4\sin x$$

$$R\cos\alpha = 3 \qquad R\sin\alpha = 4$$

$$R = \sqrt{3^2 + 4^2} \qquad \tan\alpha = \frac{4}{3}$$

$$= \underline{\underline{5}} \qquad \alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$$= \underline{\underline{53.1^\circ}}$$

b/ $$5\cos(x + 53.1) = 1$$

$$\cos(x + 53.1) = \frac{1}{5}$$

$$x + 53.1 = 78.5^\circ, 281.5^\circ$$

$$x = \underline{\underline{25.4^\circ}}, \underline{\underline{228.4^\circ}}$$

$$(\text{or } \underline{\underline{25.3^\circ}})$$

c/ $$5\cos(x + 53.1)$$

Min value $\underline{\underline{-5}}$

d/ $$180 - 53.1 = \underline{\underline{126.9^\circ}}$$

8 (a) Express 5 sin x + 12 cos x in the form R sin (x + α), where $R > 0$ and $0 \leq \alpha \leq 90$

(b) Hence find the maximum value of 5 sin x + 12 cos x and find, the smallest positive value of x for which this maximum occurs

a/ $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$

$5\sin x + 12\cos x$

$5 = R\cos\alpha \qquad 12 = R\sin\alpha$

$R = \sqrt{5^2+12^2} \qquad \tan\alpha = \frac{12}{5}$

$= 13 \qquad \alpha = 67.4°$

$\underline{\underline{13\sin(x+67.4)}}$

b/ Max value $= 13$

occurs at $90 - 67.4 = \underline{\underline{22.6°}}$

9 $$f(x) = 5\cos\theta + \sin\theta$$

Given that $f(x) = R\cos(\theta - \alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$

(a) Find the value of R and the value of α to 3 decimal places

(b) Hence, solve for $0 \le \theta \le 2\pi$, the equation

$$5\cos\theta + \sin\theta = 2$$

(c) Calculate the minimum value of

$$5\cos 4x + \sin 4x + 15$$

(d) Find the smallest positive value of x for which this minimum occurs

a/ $$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$5\cos\theta + \sin\theta$$

$R\cos\alpha = 5$ $\quad$ $R\sin\alpha = 1$

$R = \sqrt{5^2 + 1^2}$ $\quad$ $\tan\alpha = \frac{1}{5}$

$= \sqrt{26}$ $\quad$ $\alpha = 0.197$

b/ $$\sqrt{26}\cos(x - 0.197) = 2$$

$$\cos(x - 0.197) = \frac{2}{\sqrt{26}}$$

$$x - 0.197 = 1.17,\ 5.12$$

$$x = \underline{\underline{1.37}},\ \underline{\underline{5.31}}$$

c/ $$\sqrt{26}\cos(4x - 0.197) + 15$$

Min value $= \underline{\underline{15 - \sqrt{26}}}$ or $\underline{\underline{9.90}}$

d/ $$4x - 0.197 = \pi$$

$$x = \underline{\underline{0.835}}$$

10 (a) Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$

(b) Hence find the greatest value of $(2 \sin x - 3 \cos x)^2$ and find, the smallest positive value of x for which this maximum occurs

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$2 \sin x - 3 \cos x = 1$$

Give your answers to 3 decimal places.

a/

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$R \sin(x - \alpha) = R \sin x \cos \alpha - R \cos x \sin \alpha$$

$$2 \sin x \quad - \quad 3 \cos x$$

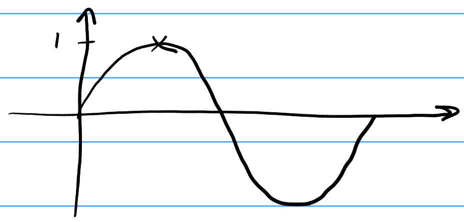
$R \cos \alpha = 2 \qquad R \sin \alpha = 3$

$R = \sqrt{2^2 + 3^2}$
$= \sqrt{13}$

$\tan \alpha = 3/2$
$\alpha = 0.983$

$$\underline{\underline{\sqrt{13} \sin(x - 0.983)}}$$

b/ Greatest value $\underline{\underline{13}}$



$$\sin(x - 0.983) = 1$$

$$\underline{\underline{x = 2.55}}$$

c/

$$\sqrt{13} \sin(x - 0.983) = 1$$

$$\sin(x - 0.983) = \frac{1}{\sqrt{13}}$$

$$x - 0.983 = 0.281, \ 2.861$$

$$x = \underline{\underline{1.264}}, \ \underline{\underline{3.843}}$$

11 The temperature, f(t) degrees Celsius, of a building is modelled by the formula

$$f(t) = 16 + 4\sin(15t)° \qquad 0 \le t < 24$$

where t is the number of hours after midday.

(a) State the maximum and minimum temperature of the building according to the model.

(b) Find the times, to the nearest minute, when the temperature is equal to 17 Degrees Celsius.

a/ max temp = 20°C
min temp = 12°C

b/ $16 + 4\sin 15t = 17$

$4\sin 15t = 1$

$\sin 15t = \frac{1}{4}$

$15t = 14.48, \ 165.52$

$t = 0.965, \ 11.03$

58mins, 11hrs 2mins

$\underline{\underline{12:58}}, \ \underline{\underline{23:02}}$

---

12 Solve, for $0 \le \theta < \pi$,

$$4\cos\theta = \operatorname{cosec}\theta$$

$4\cos\theta = \frac{1}{\sin\theta}$

$\sin 2A = 2\sin A\cos A$

$2\sin 2A = 4\sin A\cos A$

$4\cos\theta\sin\theta = 1$

$2\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{1}{6}\pi, \ \frac{5}{6}\pi$

$\theta = \underline{\underline{\frac{1}{12}\pi}}, \ \underline{\underline{\frac{5}{12}\pi}}$

13 Solve, for $0 \le \theta < 360°$,

$$3\cos\theta - 4\sin\theta = 2$$

giving your answers to 1 decimal place.

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$R\cos(\theta+\alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$3\cos\theta - 4\sin\theta$

$R\cos\alpha = 3 \qquad R\sin\alpha = 4$

$R = \sqrt{3^2+4^2} = 5 \qquad \tan\alpha = \frac{4}{3}$, $\alpha = 53.1°$

$5\cos(\theta+53.1) = 2$

$\cos(\theta+53.1) = \frac{2}{5}$

$\theta + 53.1 = 66.4°, 293.6°$

$\theta = \underline{\underline{13.3°}}, \underline{\underline{240.4°}}$

---

14 (a) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta$$

(b) Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

c ~~(b)~~ Solve, $0 \le \theta < 360°$,

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5$$

giving your answers to 1 decimal place.

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$\frac{1}{\cos\theta\sin\theta}$

$\frac{2}{\sin 2\theta}$

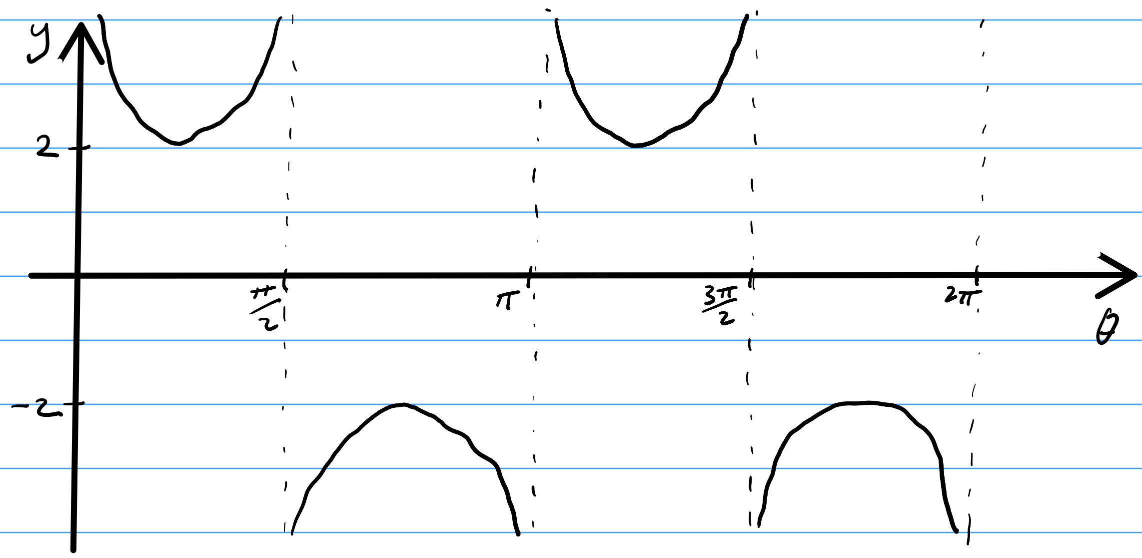
$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{\sin 2\theta} = \frac{1}{2\sin\theta\cos\theta}$

$\frac{2}{\sin 2\theta} = \frac{1}{\sin\theta\cos\theta}$

$2\operatorname{cosec} 2\theta$

b/



c/

$$2\operatorname{cosec} 2\theta = 5$$
$$\operatorname{cosec} 2\theta = \frac{5}{2}$$
$$\sin 2\theta = \frac{2}{5}$$
$$2\theta = 23.6,\ 156.4,\ 383.6,\ 516.4$$
$$\theta = \underline{\underline{11.8^\circ}},\ \underline{\underline{78.2^\circ}},\ \underline{\underline{191.8^\circ}},\ \underline{\underline{258.2^\circ}}$$

15 (a) Solve, for $-180 \leq \theta < 180°$,

$$4 \sin 2\theta = 3 \tan \theta$$

giving your answers, where necessary, to 1 decimal place.

$\sin 2\theta = 2\sin\theta\cos\theta$
$4\sin 2\theta = 8\sin\theta\cos\theta$

$$8\sin\theta\cos\theta = 3\tan\theta$$
$$8\sin\theta\cos\theta = \frac{3\sin\theta}{\cos\theta}$$
$$8\sin\theta\cos^2\theta = 3\sin\theta$$
$$8\sin\theta(1 - \sin^2\theta) = 3\sin\theta$$
$$8\sin\theta - 8\sin^3\theta = 3\sin\theta$$
$$5\sin\theta - 8\sin^3\theta = 0$$
$$\sin\theta(5 - 8\sin^2\theta) = 0$$

$\sin\theta = 0$ $\quad$ $5 - 8\sin^2\theta = 0$

$\sin^2\theta = \frac{5}{8}$

$\sin\theta = \frac{\sqrt{10}}{4}$ $\quad$ $\sin\theta = -\frac{\sqrt{10}}{4}$

$\theta = 0, -180°$ $\quad$ $\theta = 52.2°, 127.8°$ $\quad$ $\theta = -52.2°, -127.8°$

---

16 (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(b) Hence, solve, for $0 \leq \theta < \pi$,

$$8\sin^3 x - 6\sin x + 1 = 0$$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 1 - 2\sin^2 x$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(2x + x) = \sin 2x\cos x + \cos 2x\sin x$$
$$= 2\sin x\cos x\cos x + (1 - 2\sin^2 x)\sin x$$
$$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$$
$$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$$
$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$
$$= 3\sin x - 4\sin^3 x$$

b/

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$-\sin 3x = 4\sin^3 x - 3\sin x$$
$$-2\sin 3x = 8\sin^3 x - 6\sin x$$

$$-2\sin 3x + 1 = 0$$
$$2\sin 3x = 1$$
$$\sin 3x = \frac{1}{2}$$
$$3x = \frac{1}{6}\pi, \frac{5}{6}\pi, \frac{13}{6}\pi, \frac{17}{6}\pi$$

$$x = \frac{1}{18}\pi, \frac{5}{18}\pi, \frac{13}{18}\pi, \frac{17}{18}\pi$$

17 (a) Express $2 \sin x + 3 \cos x$ in the form $R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$

The temperature, θ°C, inside a warehouse is modelled by the equation

$$\theta = 8 + 2 \sin (15t - 160) + 3 \cos (15t - 160)$$

where t is the number of hours after midnight. $x = 15t - 160$ **(3)**

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day, **(1)**

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute **(3)**

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$R \sin (x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$

$2 \sin x + 3 \cos x$

$R \cos \alpha = 2$ $\quad$ $R \sin \alpha = 3$

$R = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

$\tan \alpha = \frac{3}{2}$

$\alpha = 56.3°$

$\underline{\underline{\sqrt{13} \sin (x + 56.3)}}$

b/ $\theta = 8 + \sqrt{13} \sin (15t - 160 + 56.3)$

$= 8 + \sqrt{13} \sin (15t - 103.7)$

Max temp $8 + \sqrt{13} = \underline{11.6°C}$

c/ $\sin (15t - 103.7) = 1$

$15t - 103.7 = 90$

$15t = 193.7$

$t = 12.9$

$= \underline{\underline{12:55}}$

18 (a) Show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

(b) Hence, solve, for $-\pi \leq \theta \leq \pi$,

$$1 - \cos 3x = \sin^2 x$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(2\theta+\theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$
$$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$
$$= \underline{4\cos^3\theta - 3\cos\theta}$$

b)

$$1 - (4\cos^3 x - 3\cos x) = \sin^2 x$$
$$1 - 4\cos^3 x + 3\cos x = 1 - \cos^2 x$$
$$0 = 4\cos^3 x - \cos^2 x - 3\cos x$$
$$\cos x(4\cos^2 x - \cos x - 3) = 0$$
$$\cos x(4\cos x + 3)(\cos x - 1) = 0$$

$\cos x = 0$ $\qquad \cos x = -\frac{3}{4}$ $\qquad \cos x = 1$

$x = \frac{1}{2}\pi, -\frac{1}{2}\pi$ $\qquad x = 2.42, -2.42$ $\qquad x = 0$

---

19 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - \cos x + 4$ **(7)**

(b) Calculate the minimum value of $\dfrac{1}{\sqrt{3}\sin x - \cos x + 4}$ **(2)**

$$\sin(A-B) = \sin A\cos B - \cos A \sin B$$
$$R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$$
$$\sqrt{3}\sin x - \cos x$$

$\sqrt{3} = R\cos\alpha$ $\qquad 1 = R\sin\alpha$

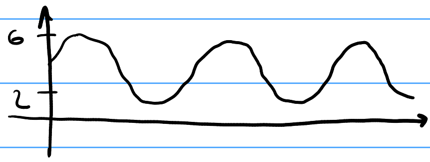
$R = \sqrt{3+1}$ $\qquad \tan\alpha = \frac{1}{\sqrt{3}}$

$= 2$ $\qquad \alpha = \frac{1}{6}\pi \ (30°)$

$$y = 2\sin\left(x - \frac{1}{6}\pi\right) + 4$$

- translation $\frac{1}{6}\pi$ or $30°$ in the positive x direction
- stretch $\times 2$ in the y direction
- translation 4 in positive y direction.

b/



Max value of $2\sin\left(x - \frac{1}{6}\pi\right) + 4 \quad = 6$

$\frac{1}{6}$

---

20 (a) Show that $\dfrac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x$

(b) Hence solve, for $-\pi \le \theta \le \pi$,

$$\frac{1 - \cos 2x}{1 + \cos 2x} = 3$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

a/

$$\frac{1 - (1 - 2\sin^2 x)}{1 + 2\cos^2 x - 1}$$

$$\frac{2\sin^2 x}{2\cos^2 x}$$

$$\tan^2 x$$

b/

$\tan^2 x = 3$

$\tan x = \sqrt{3}$ $\qquad\qquad$ $\tan x = -\sqrt{3}$

$x = \frac{1}{3}\pi, -\frac{2}{3}\pi$ $\qquad\qquad$ $x = -\frac{1}{3}\pi, \frac{2}{3}\pi$

21 (a) Prove the identity:

$$\frac{1-\cos 2x}{\sin 2x} \equiv \tan x$$

(b) Hence, solve, for $0 \le \theta < 2\pi$,

$$1-\cos 2\theta = \sin 2\theta$$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\sin 2x = 2\sin x \cos x$

a/ $\dfrac{1-(1-2\sin^2 x)}{2\sin x\cos x}$

$\dfrac{\cancel{2}\sin^2 x}{\cancel{2}\sin x \cos x}$

$\dfrac{\sin x}{\cos x} = \tan x$

b/ $1-\cos 2\theta = \sin 2\theta$

$\dfrac{1-\cos 2\theta}{\sin 2\theta} = 1$

$\tan\theta = 1$

$\theta = \frac{1}{4}\pi, \ \frac{5}{4}\pi$

---

22 Prove the identity:

$$\frac{2\sin x}{2\cos x - \sec x} \equiv \tan 2x$$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\dfrac{2\sin x}{2\cos x - \frac{1}{\cos x}}$ $\div \cos x$ $\div \cos x$

$\dfrac{2\tan x}{2-\frac{1}{\cos^2 x}}$

$\dfrac{2\tan x}{2-\sec^2 x}$

$1+\tan^2 x = \sec^2 x$

$$\frac{2\tan x}{2-(1+\tan^2 x)}$$

$$\frac{2\tan x}{1-\tan^2 x} = \tan 2x$$

23 A curve has the equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 6 and the curve passes through the point $\left(\frac{\pi}{4}, 3\sqrt{3}\right)$

Find, to 3 decimal places, the values of a and b.

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

$$R\cos\alpha = a \qquad R\sin\alpha = b$$

$$R = \sqrt{a^2+b^2} \qquad \tan\alpha = \frac{b}{a}$$

$$R\sin(x+\alpha) \qquad R = 6$$

$$y = 6\sin(x+\alpha) \qquad \frac{\pi}{4}, 3\sqrt{3}$$

$$3\sqrt{3} = 6\sin\left(\frac{\pi}{4}+\alpha\right)$$

$$\sin\left(\frac{\pi}{4}+\alpha\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{4}+\alpha = \frac{1}{3}\pi$$

$$\alpha = \frac{1}{12}\pi$$

$$6 = \sqrt{a^2+b^2} \qquad \tan\frac{1}{12}\pi = \frac{b}{a}$$

$$36 = a^2+b^2 \qquad 2-\sqrt{3} = \frac{b}{a}$$

$$b = (2-\sqrt{3})\,a$$

$$36 = a^2 + \left((2-\sqrt{3})a\right)^2$$

$$36 = a^2 + (7-4\sqrt{3})a^2$$

$$36 = (8-4\sqrt{3})\,a^2$$

$$a^2 = 18 + 9\sqrt{3}$$

$$a = \underline{\underline{5.796}} \qquad b = (2-\sqrt{3})(5.796)$$

$$= \underline{\underline{1.553}}$$

---

24 Solve, for $0 \le \theta < 360°$,

$$\sin(\theta - 45) = \cos(\theta + 30)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(\theta - 45) = \sin\theta \cos 45 - \cos\theta \sin 45$$

$$= \frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(\theta + 30) = \cos\theta \cos 30 - \sin\theta \sin 30$$

$$= \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$$

$$\frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta = \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$$

$$\frac{1+\sqrt{2}}{2}\sin\theta = \frac{\sqrt{3}+\sqrt{2}}{2}\cos\theta$$

$$(1+\sqrt{2})\sin\theta = (\sqrt{3}+\sqrt{2})\cos\theta$$

$$\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1+\sqrt{2}}$$

$$\theta = \underline{52.5°}, \ \underline{\underline{232.5°}}$$

25 Given that θ satisfies the equation $\sin(2\theta - 45) = 3\cos(2\theta - 45)$.

(a) Show that $\tan 2\theta = -2$

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle.

a/

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\sin(2\theta-45) = \sin 2\theta\cos 45 - \cos 2\theta\sin 45$$
$$= \frac{\sqrt{2}}{2}\sin 2\theta - \frac{\sqrt{2}}{2}\cos 2\theta$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\cos(2\theta-45) = \cos 2\theta\cos 45 + \sin 2\theta\sin 45$$
$$= \frac{\sqrt{2}}{2}\cos 2\theta + \frac{\sqrt{2}}{2}\sin 2\theta$$

$$\frac{\sqrt{2}}{2}\sin 2\theta - \frac{\sqrt{2}}{2}\cos 2\theta = \frac{3\sqrt{2}}{2}\cos 2\theta + \frac{3\sqrt{2}}{2}\sin 2\theta$$

$$\sin 2\theta - \cos 2\theta = 3\cos 2\theta + 3\sin 2\theta$$

$$-2\sin 2\theta = 4\cos 2\theta$$

$$\underline{\underline{\tan 2\theta = -2}}$$

b/

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$-2 = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$-2(1-\tan^2\theta) = 2\tan\theta$$
$$-2 + 2\tan^2\theta = 2\tan\theta$$
$$2\tan^2\theta - 2\tan\theta - 2 = 0$$
$$\tan^2\theta - \tan\theta - 1 = 0$$

$$\tan\theta = \frac{1+\sqrt{5}}{2} \qquad \tan\theta = \frac{1-\sqrt{5}}{2}$$

$\tan\theta$ is negative $\therefore$ $\underline{\underline{\tan\theta = \dfrac{1-\sqrt{5}}{2}}}$

26 (a) Express $5\cos 2x - 3\sin 2x$ in the form $R\cos(2x + a)$

(b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ onto the curve $y = 5\cos 2x - 3\sin 2x$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$R\cos(2x+a) = R\cos 2x\cos a - R\sin 2x\sin a$$
$$5\cos 2x - 3\sin 2x$$

$R\cos a = 5$ $\quad$ $R\sin a = 3$

$R = \sqrt{5^2+3^2}$
$= \sqrt{34}$

$\tan a = \frac{3}{5}$

$a = 0.540$ radians

$$\underline{\underline{\sqrt{34}\cos(2x + 0.540)}}$$

b/ 1. translation 0.540 in the negative x direction
2. stretch SF 0.5 in the x direction
3. stretch SF $\sqrt{34}$ in the y direction

---

27 Solve, for $0 \le \theta < 2\pi$,

$$\cos\left(\theta + \frac{\pi}{3}\right) = \sin\theta$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3} = \sin\theta$$

$$\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = \sin\theta$$

$$\frac{1}{2}\cos\theta = \frac{2+\sqrt{3}}{2}\sin\theta$$

$$2 - \sqrt{3} = \tan\theta$$

$$\theta = \underline{\underline{\frac{1}{12}\pi}}, \ \underline{\underline{\frac{13}{12}\pi}}$$

28 (a) Express $5\cos x + 12\sin x$ in the form $R\cos(x - a)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$

(b) Write down the range of the function

$$f(x) = 2 + 5\cos x + 12\sin x \qquad 0 \le x < 2\pi$$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$R\cos(x - a) = R\cos x \cos a + R\sin x \sin a$

$5\cos x + 12\sin x$

$R\cos a = 5 \qquad R\sin a = 12$

$R = \sqrt{5^2 + 12^2}$

$= 13$

$\tan a = \frac{12}{5}$

$a = 1.176$

$13\cos(x - 1.176)$

b/ $2 + 13\cos(x - 1.176)$

$-11 \le f(x) \le 15$

---

29 (a) Express $7\cos x - 4\sin x$ in the form $R\cos(x + a)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$

(b) Give full details of a sequence of two transformations needed to transform the curve $y = \sec x$ onto the curve $y = \dfrac{1}{7\cos x - 4\sin x}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$R\cos(x + \alpha) = R\cos x \cos\alpha - R\sin x \sin\alpha$

$7\cos x - 4\sin x$

$R\cos\alpha = 7 \qquad R\sin\alpha = 4$

$R = \sqrt{7^2 + 4^2}$

$= \sqrt{65}$

$\tan\alpha = \frac{4}{7}$

$\alpha = 0.519$

$\sqrt{65}\cos(x+0.519)$

b/ $\frac{1}{\sqrt{65}}\sec(x+0.519)$

- translation 0.519 units in the negative x direction
- stretch in y direction S.F $\frac{1}{\sqrt{65}}$