A Level Maths: Trigonometry and Modelling

1 (a) Use the identity for sin (A + B) to express sin 2A in terms of sin A and cos A. **(2)**

(b) Use the identity for cos (A + B) to express cos 2A in terms of sin A and cos A. **(2)**

(c) Hence, express cos 2A in terms of

i) cos A **(2)**

ii) sin A **(2)**

(d) Use the identity for tan (A + B) to express tan 2A in terms of tan A. **(2)**

(Total for question 1 is 10 marks)

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2 (a) By writing sin 3θ as sin $(2\theta + \theta)$ show that $\sin 3\theta = 3\sin\theta - 4\sin^3 \theta$ **(2)**

(b) Solve, for $0 \leq \theta \leq 180$, the equation, **(4)**

$$3\sin\theta - 4\sin^3 \theta = 0.4$$

Give your answers to 1 decimal place.

(Total for question 2 is 6 marks)

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3 (a) By writing cos 3θ as cos $(2\theta + \theta)$ show that $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ **(2)**

(b) Solve, for $0 \leq \theta \leq \pi$, the equation, **(4)**

$$4\cos^3 \theta - 3\cos \theta = 0.5$$

Give your answers in terms of π.

(Total for question 3 is 6 marks)

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4 Solve, for $-180 \leq x \leq 180$, the equation,

$$\cos 2x - 7 \sin x + 3 = 0$$

(Total for question 4 is 5 marks)

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5 Solve, for $0 \leq x \leq 360$, the equation,

$$8 \sin x \cos x = 3$$

Give your answers to 1 decimal place.

(Total for question 5 is 5 marks)

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6 Prove the identity:

$$\frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x$$

(Total for question 6 is 2 marks)

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7 $$f(x) = 3\cos x - 4\sin x$$

Given that $f(x) = R\cos(x + \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 90$,

(a) Find the value of R and the value of α **(4)**

(b) Hence solve, for $0 \leq \theta \leq 360$, the equation

$$3\cos x - 4\sin x = 1$$

Give your answers to 1 decimal place. **(5)**

(c) Write down the minimum value of $3\cos x - 4\sin x$ **(1)**

(d) Find, to 1 decimal place, the smallest positive value of x for which this minimum occurs **(2)**

(Total for question 7 is 12 marks)

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8 (a) Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 \leq \alpha \leq 90$ **(4)**

(b) Hence find the maximum value of $5\sin x + 12\cos x$ and find, the smallest positive value of x for which this maximum occurs **(3)**

(Total for question 8 is 7 marks)

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9 $$f(x) = 5\cos\theta + \sin\theta$$

Given that $f(x) = R\cos(\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$

(a) Find the value of R and the value of α to 3 decimal places **(4)**

(b) Hence, solve for $0 \leq \theta \leq 2\pi$, the equation

$$5\cos\theta + \sin\theta = 2$$ **(3)**

(c) Calculate the minimum value of

$$5\cos 4x + \sin 4x + 15$$ **(2)**

(d) Find the smallest positive value of x for which this minimum occurs **(3)**

(Total for question 9 is 12 marks)

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10 (a) Express $2\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ **(4)**

(b) Hence find the greatest value of $(2\sin x - 3\cos x)^2$ and find, the smallest positive value of x for which this maximum occurs **(2)**

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$2\sin x - 3\cos x = 1$$

Give your answers to 3 decimal places. **(5)**

(Total for question 10 is 11 marks)

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11 The temperature, $f(t)$ degrees Celsius, of a building is modelled by the formula

$f(t) = 16 + 4\sin(15t)°$ $\quad 0 \le t < 24$

where t is the number of hours after midday.

(a) State the maximum and minimum temperature of the building according to the model. **(1)**

(b) Find the times, to the nearest minute, when the temperature is equal to 17 Degrees Celsius. **(4)**

(Total for question 11 is 5 marks)

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12 Solve, for $0 \le \theta < \pi$,

$$4\cos\theta = \operatorname{cosec}\theta$$

(Total for question 12 is 5 marks)

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13 Solve, for $0 \le \theta < 360°$,

$$3\cos\theta - 4\sin\theta = 2$$

giving your answers to 1 decimal place.

(Total for question 13 is 5 marks)

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14 (a) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}2\theta$$

(4)

(b) Sketch the graph of $y = 2\operatorname{cosec}2\theta$ for $0° < \theta < 360°$. **(2)**

(c) Solve, $0 \le \theta < 360°$,

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5$$

giving your answers to 1 decimal place. **(6)**

(Total for question 14 is 12 marks)

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15 (a) Solve, for $-180 \le \theta < 180°$,

$$4\sin 2\theta = 3\tan\theta$$

giving your answers, where necessary, to 1 decimal place.

(Total for question 15 is 6 marks)

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16 (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence, solve, for $0 \le \theta < \pi$,

$$8\sin^3 x - 6\sin x + 1 = 0$$

(5)

(Total for question 16 is 9 marks)

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17 (a) Express $2 \sin x + 3 \cos x$ in the form $R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$

The temperature, θ°C, inside a warehouse is modelled by the equation

$$\theta = 8 + 2 \sin (15t - 160) + 3 \cos (15t - 160)$$

where t is the number of hours after midnight. **(3)**

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day, **(1)**

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute **(3)**

(Total for question 17 is 7 marks)

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18 (a) Show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$ **(4)**

(b) Hence, solve, for $-\pi \leq \theta \leq \pi$,

$$1 - \cos 3x = \sin^2 x$$ **(5)**

(Total for question 18 is 9 marks)

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19 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - \cos x + 4$ **(7)**

(b) Calculate the minimum value of $\dfrac{1}{\sqrt{3}\sin x - \cos x + 4}$ **(2)**

(Total for question 19 is 9 marks)

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20 (a) Show that $\dfrac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x$ **(4)**

(b) Hence solve, for $-\pi \leq \theta \leq \pi$,

$$\frac{1 - \cos 2x}{1 + \cos 2x} = 3$$ **(2)**

(Total for question 20 is 6 marks)

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21 (a) Prove the identity:

$$\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x$$ **(4)**

(b) Hence, solve, for $0 \leq \theta < 2\pi$,

$$1 - \cos 2\theta = \sin 2\theta$$ **(3)**

(Total for question 21 is 7 marks)

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22 Prove the identity:

$$\frac{2\sin x}{2\cos x - \sec x} \equiv \tan 2x$$

(Total for question 22 is 4 marks)

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23 A curve has the equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 6 and the curve passes through the point $\left(\frac{\pi}{4}, 3\sqrt{3}\right)$

Find, to 3 decimal places, the values of a and b.

(Total for question 23 is 6 marks)

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24 Solve, for $0 \leq \theta < 360°$,

$$\sin(\theta - 45) = \cos(\theta + 30)$$

(Total for question 24 is 6 marks)

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25 Given that θ satisfies the equation $\sin(2\theta - 45) = 3\cos(2\theta - 45)$.

(a) Show that $\tan 2\theta = -2$ **(3)**

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. **(5)**

(Total for question 25 is 8 marks)

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26 (a) Express $5\cos 2x - 3\sin 2x$ in the form $R\cos(2x + a)$ **(3)**

(b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ onto the curve $y = 5\cos 2x - 3\sin 2x$ **(4)**

(Total for question 26 is 7 marks)

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27 Solve, for $0 \leq \theta < 2\pi$,

$$\cos\left(\theta + \frac{\pi}{3}\right) = \sin\theta$$

(Total for question 27 is 4 marks)

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28 (a) Express $5\cos x + 12\sin x$ in the form $R\cos(x - a)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ **(3)**

(b) Write down the range of the function

$$f(x) = 2 + 5\cos x + 12\sin x \qquad 0 \leq x < 2\pi$$ **(2)**

(Total for question 28 is 5 marks)

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29 (a) Express $7\cos x - 4\sin x$ in the form $R\cos(x + a)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ **(4)**

(b) Give full details of a sequence of two transformations needed to transform the curve $y = \sec x$ onto the curve $y = \dfrac{1}{7\cos x - 4\sin x}$ **(3)**

(Total for question 29 is 7 marks)