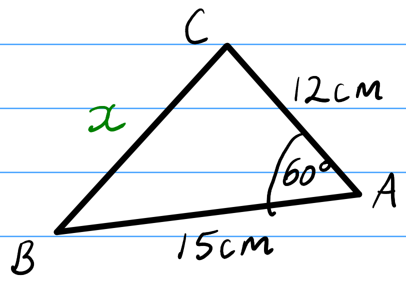
1 In triangle ABC, side AB has length 15cm, side AC has length 12cm and $\angle BAC = 60°$

(a) Find the length of side BC. **(3)**

(b) Find the area of triangle ABC. **(2)**



a/ $a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 12^2 + 15^2 - 2(12)(15)\cos(60)$

$= 189$

$x = \underline{13.7\text{cm}}$ (3sf)

b/ $\text{Area} = \frac{1}{2} ab \sin C$

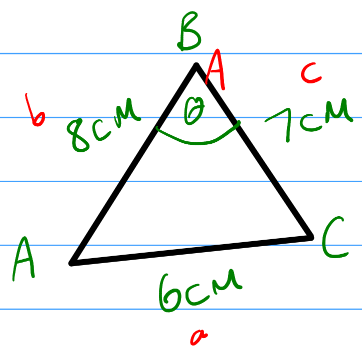
$= \frac{1}{2}(12)(15)\sin(60)$

$= \underline{77.9 \text{ cm}^2}$ (3sf)

2 In triangle ABC, side AB has length 8cm, side BC has length 7cm and side AC has length 6cm.

(a) Find the size of angle ABC. **(3)**

(b) Find the area of triangle ABC. **(2)**



a/ $\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{8^2 + 7^2 - 6^2}{2(8)(7)}$

$\theta = \cos^{-1}\left(\frac{11}{16}\right)$

$= \underline{46.6°}$ (3sf)

b/ $\text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(8)(7)\sin(46.567)$

$= \underline{20.3 \text{ cm}^2}$ (3sf)

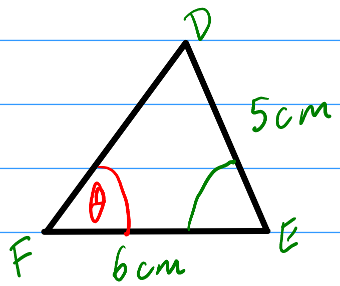
3 In triangle DEF, $ED = 5\text{cm}$ and $EF = 6\text{cm}$.

Given that $\sin(\angle DEF) = \frac{2}{3}$ and $\angle DEF$ is acute.

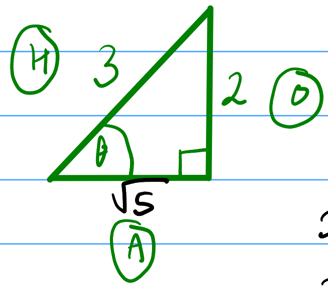
(a) Find the exact value of $\cos(\angle DEF)$ **(2)**

(b) Find the length of DF. **(4)**

(c) Find $\angle EFD$. **(3)**



a/ $\sin(DEF) = \frac{2}{3}$



$$x^2 + 2^2 = 3^2$$
$$x^2 + 4 = 9$$
$$x^2 = 5$$
$$x = \sqrt{5}$$

$$\cos(DEF) = \frac{\sqrt{5}}{3}$$

b/ $$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 5^2 + 6^2 - 2(5)(6)\left(\frac{\sqrt{5}}{3}\right)$$

$$= 16.2786\ldots$$

$$a = 4.03\text{ cm} \quad (3sf)$$

c/ $$\frac{\sin\theta}{5} = \frac{\sin(DEF)}{4.03} \qquad \left(\sin DEF = \frac{2}{3}\right)$$

$$\sin\theta = \frac{2/3}{4.03} \times 5$$

$$\sin\theta = 0.82617\ldots$$

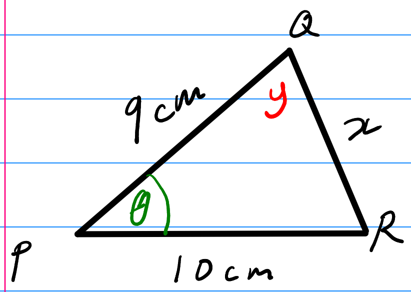
$$\theta = 55.7^\circ \quad (3sf)$$

4 In triangle PQR, side PQ has length 9cm and side PR has length 10cm.

Given the area of PQR is 30cm^2

(a) Find the length of side QR. **(5)**

(b) Find $\angle PQR$ **(3)**



a/ $\text{Area} = \frac{1}{2} ab \sin C$

$30 = \frac{1}{2}(9)(10) \sin \theta$

$\sin \theta = \frac{2}{3}$

$\underline{\underline{\theta = 41.8^\circ}}$

b/ $x^2 = 9^2 + 10^2 - 2(9)(10)\cos(41.8)$

$x^2 = 46.8359$

$x = \underline{\underline{6.84 \text{ cm}}}$

c/ $\frac{\sin y}{10} = \frac{\sin(41.8)}{6.84}$

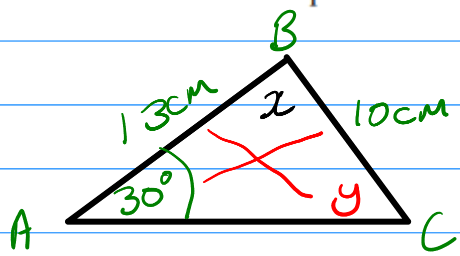
$\sin y = 0.974$

$y = \underline{\underline{76.9^\circ}}$

---

5 In the triangle ABC, $AB = 13$cm, $BC = 10$cm and angle $BAC = 30^\circ$

Find the two possible sizes of angle ABC, giving your answers to two decimal places.



$\frac{\sin y}{13} = \frac{\sin 30}{10}$

$\sin y = \frac{13}{20}$

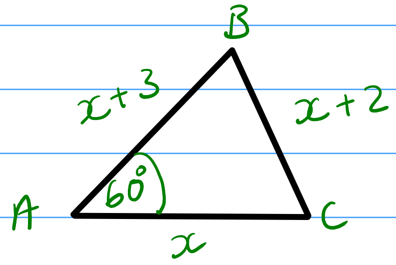
(second answer is 180 − first answer)

$y = 40.5, \ 139.5$

$\therefore x = \underline{\underline{10.5^\circ}}$ or $\underline{\underline{109.5^\circ}}$

6 In the triangle ABC, $AB = (x+3)$cm, $BC = (x+2)$ cm, $AC = x$ cm and angle $BAC = 60°$

Find the value of x.



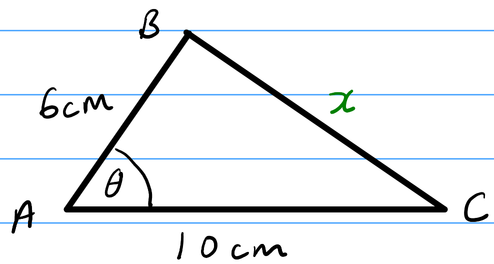
$$(x+2)^2 = (x+3)^2 + x^2 - 2(x+3)(x)\cos(60)$$
$$x^2+4x+4 = x^2+6x+9+x^2 - \tfrac{1}{2}(2)(x^2+3x)$$
$$x^2+4x+4 = 2x^2+6x+9 - x^2 - 3x$$
$$\cancel{x^2}+4x+4 = \cancel{x^2}+3x+9$$
$$\underline{x = 5}$$

---

7 In triangle ABC, side AB has length 6 cm, side AC has length 10 cm and $\angle BAC = \theta$, where θ is measured in degrees. The area of triangle ABC is 18 cm²

(a) Find the two possible values of $\cos\theta$. **(4)**

(b) Given that BC is the longest side of the triangle, find the exact length of BC. **(2)**



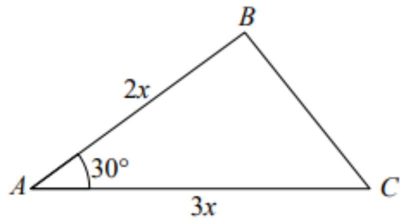
a) $\text{Area} = \frac{1}{2}ab\sin C$

$$18 = \frac{1}{2}(6)(10)\sin\theta$$
$$\sin\theta = \frac{3}{5}$$
$$\theta = \underline{36.9°},\ \underline{143.1°}$$

b) $\theta = \underline{143.1°}$

$$x^2 = 6^2 + 10^2 - 2(6)(10)\cos(143.1)$$
$$x^2 = 232$$
$$x = \underline{15.2\text{ cm}}$$

8



Given the area of triangle ABC is 12 cm^2

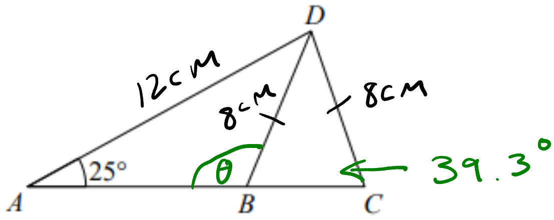
(a) Find the exact value of x. **(3)**

(b) Find the length of BC. **(3)**

a/
$$\frac{1}{2}(2x)(3x)\sin 30 = 12$$
$$3x^2\left(\frac{1}{2}\right) = 12$$
$$3x^2 = 24$$
$$x^2 = 8$$
$$x = \underline{\underline{\sqrt{8}}} \quad (\text{or } 2\sqrt{2})$$

b/
$$2x = 4\sqrt{2} \qquad 3x = 6\sqrt{2}$$
$$BC^2 = (4\sqrt{2})^2 + (6\sqrt{2})^2 - 2(4\sqrt{2})(6\sqrt{2})\cos(30)$$
$$= 20.86$$
$$BC = \underline{\underline{4.57 \text{ cm}}}$$

9



Given $AD = 12$ cm, $BD = CD = 8$ cm and angle $DAC = 25°$

(a) Find the size of angle ABD to one decimal place. **(3)**

(b) Find the length of AC **(3)**

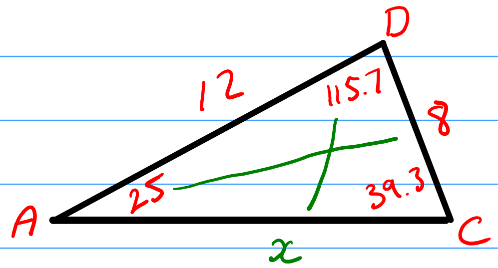
a) $\dfrac{\sin\theta}{12} = \dfrac{\sin 25}{8}$

$\sin\theta = \dfrac{\sin 25}{8} \times 12$

$\theta = 39.3°, \ \underline{\underline{140.7°}}$

$ABD = \underline{\underline{140.7°}}$

b)



$180 - 25 - 39.3 = 115.7°$

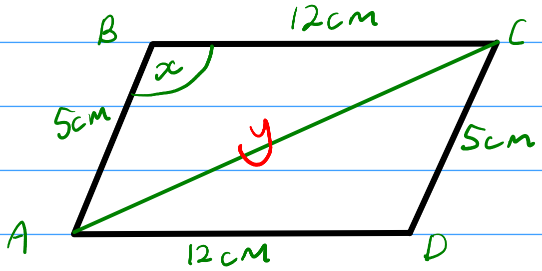
$\dfrac{x}{\sin 115.7} = \dfrac{8}{\sin 25}$

$x = \underline{\underline{17.1 \text{ cm}}}$

10 A parallelogram $ABCD$ has area 55 cm^2
Given AB has length 5 cm, BC has length 12 cm and angle ABC is obtuse.

(a) Find the size of angle ABC to 2 decimal places. **(3)**

(b) Find the length of the diagonal AC to 1 decimal place. **(2)**



a) $\frac{1}{2}(5)(12)\sin(x) = \frac{55}{2}$

$60 \sin x = 55$

$\sin x = \frac{11}{12}$

$x = \underline{\underline{66.4^\circ}}$

b) $a^2 = 5^2 + 12^2 - 2(5)(12)\cos(66.4)$

$a = \underline{\underline{11.0 \text{ cm}}}$

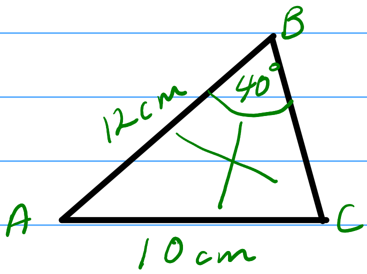
---

11 In triangle ABC, $AB = 12$ cm and angle $B = 40°$

(a) Given $AC = 10$ cm, find the two possible values for angle C, correct to 1 decimal place. **(4)**

(b) Given instead that the area of the triangle is $75\sqrt{2}$ cm^2, find BC. **(2)**

a)

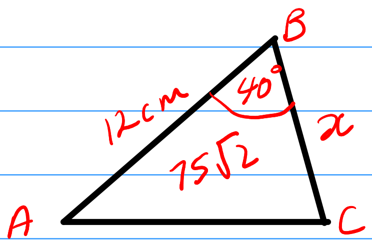


$\frac{\sin C}{12} = \frac{\sin 40}{10}$

$\sin C = \frac{\sin 40}{10} \times 12$

$C = \underline{\underline{50.5^\circ}}$

b)



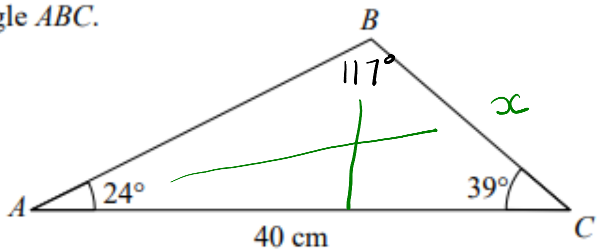
$\frac{1}{2}x(12)\sin 40 = 75\sqrt{2}$

$x = \frac{75\sqrt{2}}{6\sin(40)}$

$= \underline{\underline{27.5 \text{ cm}}}$

$180 - 24 - 39$
$= 117°$

12 Find the area of triangle ABC.

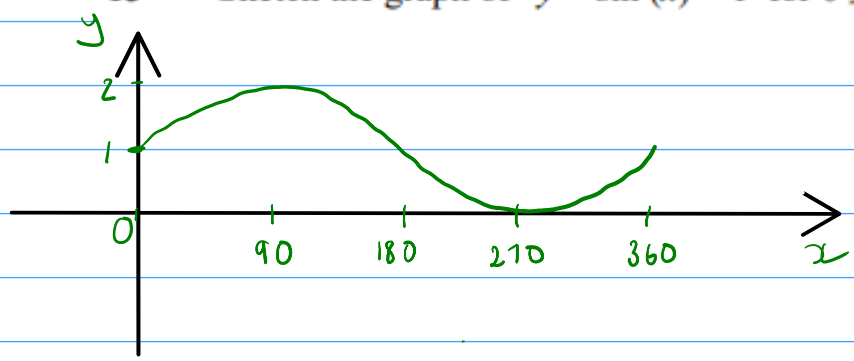


$$\frac{x}{\sin 24} = \frac{40}{\sin 117}$$

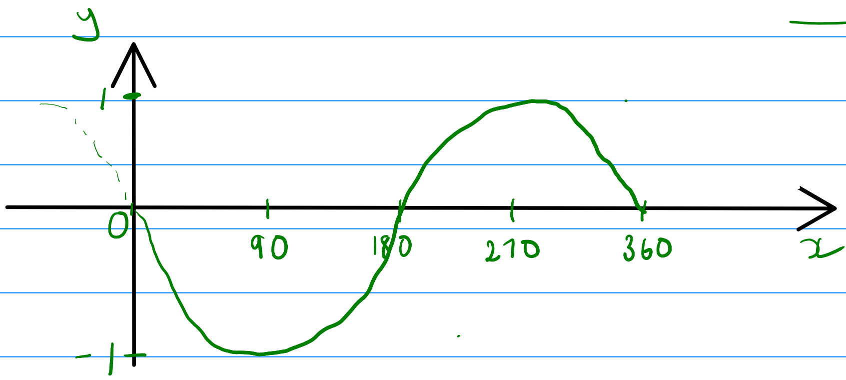
$$x = 18.2596...$$

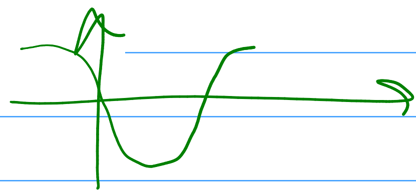
$$\text{Area} = \frac{1}{2}(40)(18.26)\sin(39)$$
$$= \underline{230} \text{ cm}^2 \quad (3sf)$$

13 Sketch the graph of $y = \sin(x) + 1$ for $0 \leq x \leq 360$

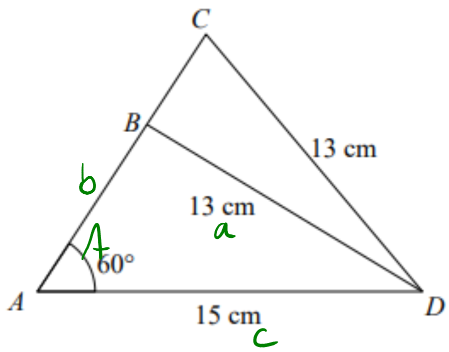


14 Sketch the graph of $y = \cos(x + 90)$ for $0 \leq x \leq 360$





15



ACD is a triangle and *B* lies on *AC*. Angle $CAD = 60°$, $AD = 15$ cm, $BD = CD = 13$ cm

(a) Find the length of *AC* **(3)**

(b) Hence, find the length of *AB* **(1)**

a/

$$13^2 = b^2 + 15^2 - 2b(15)\cos(60)$$

$$169 = b^2 + 225 - 15b$$

$$0 = b^2 - 15b + 56$$

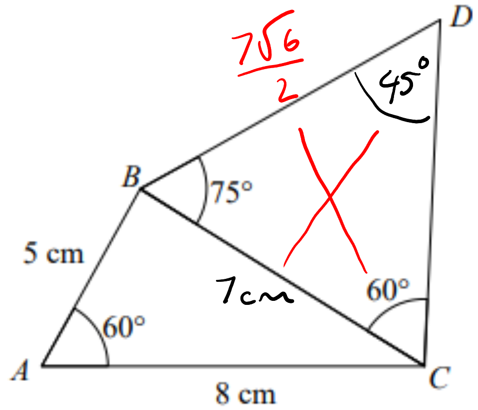
$$0 = (b-8)(b-7)$$

$$b = 8 \qquad b = 7$$

8 cm

b/ 7cm

16



Calculate the exact value of the length *BD*.

$BC^2 = 5^2 + 8^2 - 2(5)(8)\cos(60)$

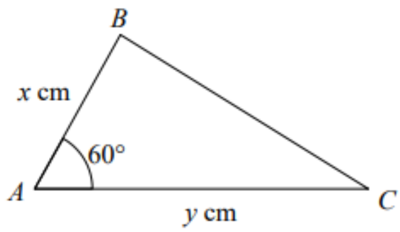
$BC^2 = 49$

$BC = 7$

$\frac{BD}{\sin 60} = \frac{7}{\sin 45}$

$BD = \frac{7\sqrt{6}}{2}$

17



In triangle ABC, $AB = x$, $AC = y$ and angle $A = 60°$. It is given the area of $ABC = 2\sqrt{3}(x+y)\text{cm}^2$.

(a) Show that $8x + 8y = xy$ **(2)**

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle. $\longrightarrow \angle ABC = 90°$

(b) Determine the value of x and the value of y. **(4)**

a/

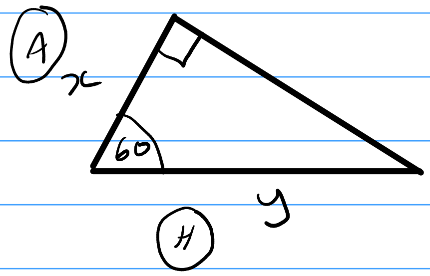
$$\frac{1}{2} xy \sin 60 = 2\sqrt{3}(x+y)$$

$$\frac{\sqrt{3}}{4} xy = 2\sqrt{3}(x+y)$$

$$xy = 8(x+y)$$

$$xy = 8x + 8y$$

b/



$$\cos 60 = \frac{x}{y}$$

$$\frac{1}{2} = \frac{x}{y}$$

$$y = 2x$$

$$x(2x) = 8x + 8(2x)$$

$$2x^2 = 8x + 16x$$

$$2x^2 = 24x$$

$$x^2 = 12x$$

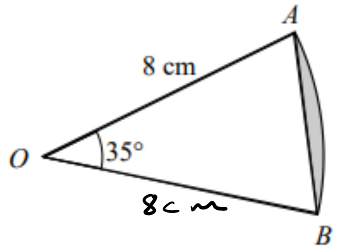
$$x^2 - 12x = 0$$

$$x(x-12) = 0$$

$x = 0 \qquad x = 12$

x cannot be zero. $\therefore$ $\underline{\underline{x = 12}}$ $\qquad$ $\underline{\underline{y = 24}}$

18



The diagram shows sector *AOB* of a circle with centre *O* and radius 8 cm. Angle *AOB* = 35°

(a) Calculate the length of the straight line *AB*. **(2)**

(b) Find the area of the shaded segment. **(3)**

a/ $a^2 = 8^2 + 8^2 - 2(8)(8)\cos(35)$

$a^2 = 23.1$

$a = \underline{\underline{4.81 \text{ cm}}}$ (3sf)

b/ Area of sector $= \frac{35}{360} \times \pi(8)^2$

Area of triangle $= \frac{1}{2}(8)(8)\sin(35)$

Shaded segment $= \frac{35}{360}\pi(8)^2 - \frac{1}{2}(8)(8)\sin(35)$

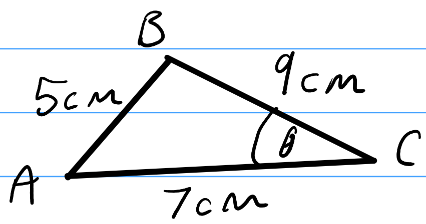
$= \underline{\underline{1.19 \text{ cm}^2}}$ (3sf)

19 In triangle ABC, side AB has length 5cm, side BC has length 9cm and side AC has length 7cm.

(a) Find the cosine of angle ACB, giving your answer as a fraction in its simplest form. **(2)**

(b) Find the exact area of the triangle. **(3)**

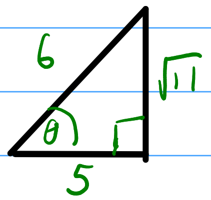
a)



$$\cos\theta = \frac{7^2 + 9^2 - 5^2}{2(7)(9)}$$

$$\cos\theta = \frac{5}{6}$$

b)

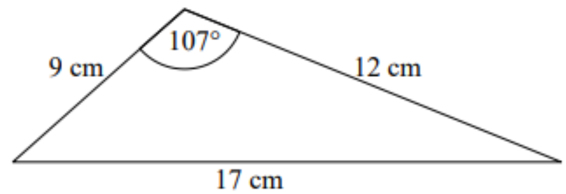


$$\sin\theta = \frac{\sqrt{11}}{6}$$

$$\text{Area} = \frac{1}{2}(7)(9)\frac{\sqrt{11}}{6}$$

$$= \frac{21\sqrt{11}}{4} \text{ cm}^2$$

20



Calculate the area of the triangle giving your answer correct to 3 significant figures.

$$\frac{1}{2}(9)(12)\sin(107) = 51.6 \text{ cm}^2$$