AS Level Maths: Trigonometric Identities and Equations

1 Solve, for $0 \leq x < 180^{\circ}$, the equation,

$$\cos(2x + 15) = 0.3$$

Give your answers to one decimal place.

(Total for question 1 is 5 marks)

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2 Solve, for $0 \leq \theta < 180^{\circ}$, the equation,

$$\sin(3\theta - 15) = 0.7$$

Give your answers to two decimal places.

(Total for question 2 is 5 marks)

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3 Solve, for $-180 \leq \theta < 180^{\circ}$, the equation,

$$\tan(\theta + 30) = -2.5$$

Give your answers to one decimal place.

(Total for question 3 is 4 marks)

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4 Solve, for $0 \leq x < 360^{\circ}$, the equation,

$$5\cos(x - 40) = 2$$

Give your answers to two decimal places.

(Total for question 4 is 4 marks)

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5 Solve, for $0 \leq x < 360^{\circ}$, the equation,

$$\tan^2(x) = 3$$

(Total for question 5 is 5 marks)

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6 (a) Show that the equation

$$2\sin^2 x = 7\cos x + 5$$

Can be written in the form

$$2\cos^2 x + 7\cos x + 3 = 0$$

(3)

(b) Hence solve, for $0 \leq x < 360^{\circ}$, the equation,

$$2\sin^2 x = 7\cos x + 5$$

(5)

(Total for question 6 is 8 marks)

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7 (a) Show that the equation

$$6\cos^2 x = 4 - \sin x$$

Can be written in the form

$$6\sin^2 x - \sin x - 2 = 0$$

(3)

(b) Hence solve, for $0 \leq x < 360^{\circ}$, the equation,

$$6\cos^2 x = 4 - \sin x$$

Give your answers to one decimal place where appropriate.

(6)

(Total for question 7 is 9 marks)

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8 Find all values for x in the interval $0 \leq x < 360^{\circ}$, for which

$$2\cos^2 x - 3\sin^2 x = 14\cos x$$

Give your answers to one decimal place.

(Total for question 8 is 8 marks)

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9 (a) Sketch the graph of $y = \sin(x - 30)$ for x in the interval $0 \leq x < 360^{\circ}$ **(2)**

(b) Find all values for x in the interval $0 \leq x < 360^{\circ}$, for which

$$\sin(x - 30) = 0.3$$

Give your answers to one decimal place.

(4)

(Total for question 9 is 6 marks)

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10 Find all values for x in the interval $0 \leq x < 360^{\circ}$, for which

$$3\tan x = 4\sin x$$

Give your answers to one decimal place where appropriate.

(Total for question 10 is 7 marks)

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11 (a) Show that the equation

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Can be written in the form

$$4\cos^2 2x + 2\cos 2x - 3 = 0$$

(4)

(b) Find all values for x in the interval $0 \leq x < 180^{\circ}$, for which

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Give your answers to two decimal places.

(6)

(Total for question 11 is 10 marks)

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12 (a) Show that the equation

$$1 + \cos x = 3 \tan x \sin x$$

Can be written in the form

$$4\cos^2 x + \cos x - 3 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 360°$, the equation,

$$1 + \cos x = 3 \tan x \sin x$$

(5)

Give your answers to one decimal place where appropriate.

(Total for question 12 is 9 marks)

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13 (a) Show that

$$\frac{6\cos^2\theta + 7\sin\theta - 8}{1 - 2\sin\theta} \equiv 3\sin\theta - 2$$

(4)

(b) Hence solve, for $0 \leq \theta < 360°$, the equation,

$$\frac{6\cos^2\theta + 7\sin\theta - 8}{1 - 2\sin\theta} = 2\cos\theta - 2$$

(3)

(Total for question 13 is 7 marks)

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14 (a) Solve, for $360 \leq \theta < 720°$, the equation,

$$3 \cos \theta = 8 \tan \theta$$

(5)

The first four positive solutions, in order of size, of the equation

$$\cos(2a + 50) = 0.7$$

are a_1, a_2, a_3 and a_4

(b) To the nearest degree find the value of a_4.

(3)

(Total for question 14 is 8 marks)

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15 Solve the equation $\tan^2 2x - 3 = 0$ giving all the solutions for the interval $0 \leq x < 360°$

(Total for question 15 is 4 marks)

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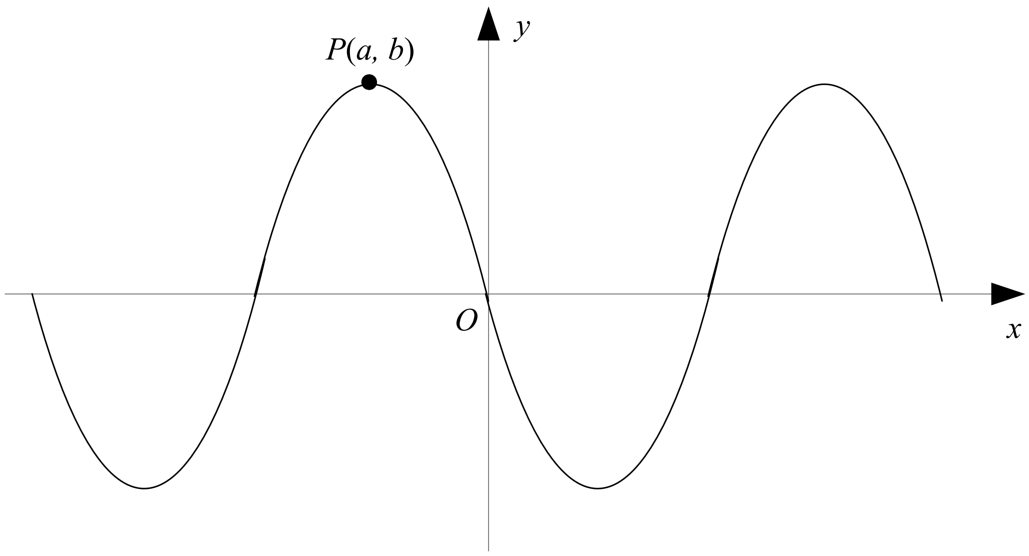
16 Given $\cos(75°) = \dfrac{\sqrt{6} - \sqrt{2}}{4}$ and $\sin(75°) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

Show that $\tan^2(75°)$ can be written in the form $a + b\sqrt{3}$

Fully justify your answer.

(Total for question 16 is 3 marks)

17



The graph shows part of the curve with equation $y = 4 \sin x°$

The point P is a maximum point on the curve with a being the smallest negative value of x that a maximum occurs.

(a) State the value of a and the value of b. **(1)**

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 4 \sin x°$ to the curve with equation

(i) $y = 4 \sin (x + 28)$

(ii) $y = 4 \sin (3x)$ **(2)**

(c) Solve, for $360 \leq \theta < 720°$,

$$4 \sin \theta = \tan \theta$$

Give your answers to one decimal place where appropriate. **(5)**

(Total for question 17 is 8 marks)

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18 Solve $\tan 2\theta - 1 = 0$ giving all the solutions for the interval $0 \leq \theta < 360°$

(Total for question 18 is 3 marks)

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19 (a) Solve $6\sin^2 \theta = \cos \theta + 4$ giving all the solutions for the interval $0 \leq \theta < 360°$ **(4)**

(b) Hence, hence solve $6\sin^2 2\theta = \cos 2\theta + 4$ giving all the solutions for the interval $0 \leq \theta < 360°$ **(2)**

(Total for question 19 is 6 marks)

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20 At 12 noon the temperature in Harry's house is 22°C
At 6 pm the temperature in Harry's house in 25°C

Harry models the temperature in his house, T, by the formula

$$T = A + B\sin(15h)$$

where h is the number of hours after 12 noon.

(a) State the value that Harry should use for A. **(1)**

(b) State the value that Harry should use for B. **(1)**

(c) Using this model, calculate the temperature in Harry's house at 9 pm. **(1)**

(d) Using the model find the number of hours in a day that the temperature will be above 23.5°C **(4)**

(Total for question 20 is 7 marks)

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21 It is given that $\sin y = -0.2$ and $180° < y < 270°$

Find the exact value of $\cos y$

(Total for question 21 is 2 marks)

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22 Jacob has to solve the equation

$$3 - \sin x = 1 + 2\cos^2 x$$

where $-180^{\circ} \leq x < 180^{\circ}$

Jacob's working is as follows:

$$3 - \sin x = 1 + 2\cos^2 x$$
$$2 - \sin x = 2\cos^2 x$$
$$2 - \sin x = 2(1 - \sin^2 x)$$
$$2 - \sin x = 2 - 2\sin^2 x$$
$$-\sin x = -2\sin^2 x$$
$$1 = 2\sin x$$
$$\sin x = 0.5$$
$$x = 30°$$

(a) Explain the two errors that Jacob has made. **(2)**

(b) Write down all the values of x that satisfy the equation

$$3 - \sin x = 1 + 2\cos^2 x$$

where $-180^{\circ} \leq x < 180^{\circ}$ **(2)**

(Total for question 22 is 4 marks)

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23 Find all solutions of

$$6\cos^2 x + 5\sin x - 7 = 0$$

where $0° \leq x < 360°$

Give your solutions to the nearest degree.

(Total for question 23 is 4 marks)

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24 (a) Show that the equation

$$2\sin^2 x = 4\cos^2 x - \cos x$$

can be expressed in the form

$$6\cos^2 x - \cos x - 2 = 0$$

(3)

(b) Hence, solve the equation

$$2\sin^2 2\theta = 4\cos^2 2\theta - \cos 2\theta$$

giving all values of θ between 0° and 180°, correct to 1 decimal place. **(5)**

(Total for question 24 is 8 marks)

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25 (a) Solve the equation $\sin^2 x = 0.25$ for $0° \leq x < 360°$ **(3)**

(b) Solve the equation $\tan 3x = 1$ for $0° \leq x < 180°$ **(3)**

(Total for question 25 is 6 marks)

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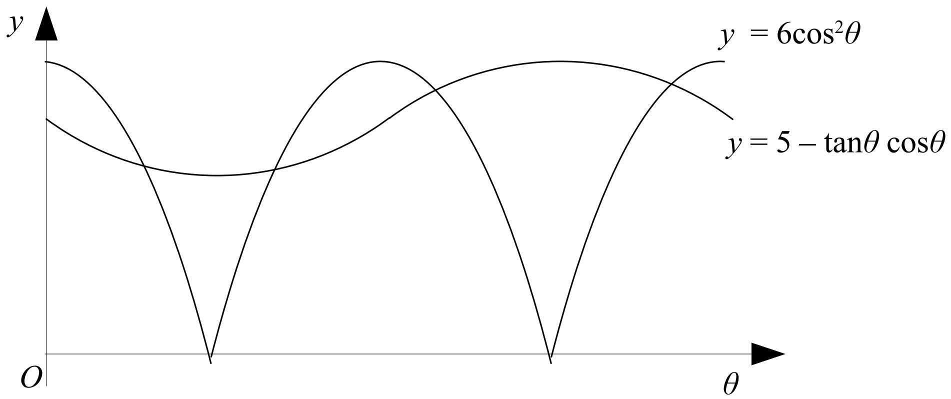
26 (a) Show that the equation $5 - \tan\theta\cos\theta = 6\cos^2\theta$
can be expressed in the form $6\sin^2 x - \sin x - 1 = 0$

(2)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = 5 - \tan\theta\cos\theta$, where is θ in degrees.

(b) Solve the inequality $5 - \tan\theta\cos\theta > 6\cos^2\theta$ for $0° \leq \theta < 360°$

(5)

(Total for question 26 is 7 marks)

27 (a) Solve the equation $\sin^2 x = \tan^2 x$ for $0^\circ \leq x \leq 180^\circ$ **(5)**

(b) Prove that $\dfrac{2\sin x - \cos^2 x + 1}{2 + \sin x} \equiv \sin x$ **(3)**

(Total for question 27 is 8 marks)

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28 (a) Sketch the graphs of $y = 3\cos x$ and $y = \sin x$ for $0^\circ \leq x \leq 180^\circ$ on the same axes. **(2)**

(b) Find the exact coordinates of the point of intersection of these graphs, giving the answer in the form ($\arctan a$, $k\sqrt{b}$), where a and b are integers and k is rational. **(4)**

(Total for question 28 is 6 marks)

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29 Solve the equation $5\sin x = 3\cos x$ for $0^\circ \leq x \leq 360^\circ$

(Total for question 29 is 3 marks)

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30 Solve the equation $24\tan x + 5\cos x = 0$ for $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest degree

(Total for question 30 is 6 marks)
