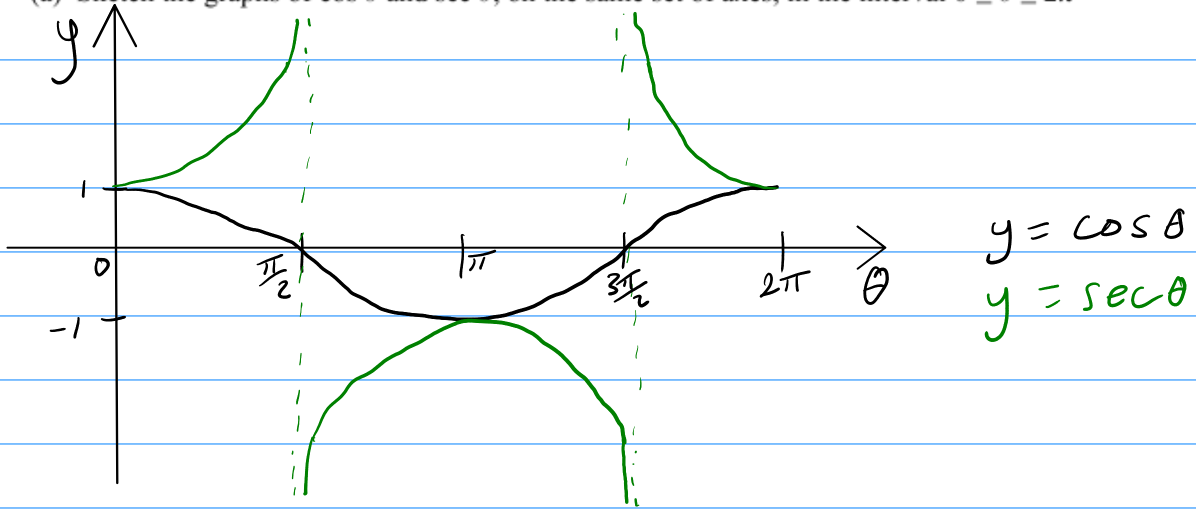
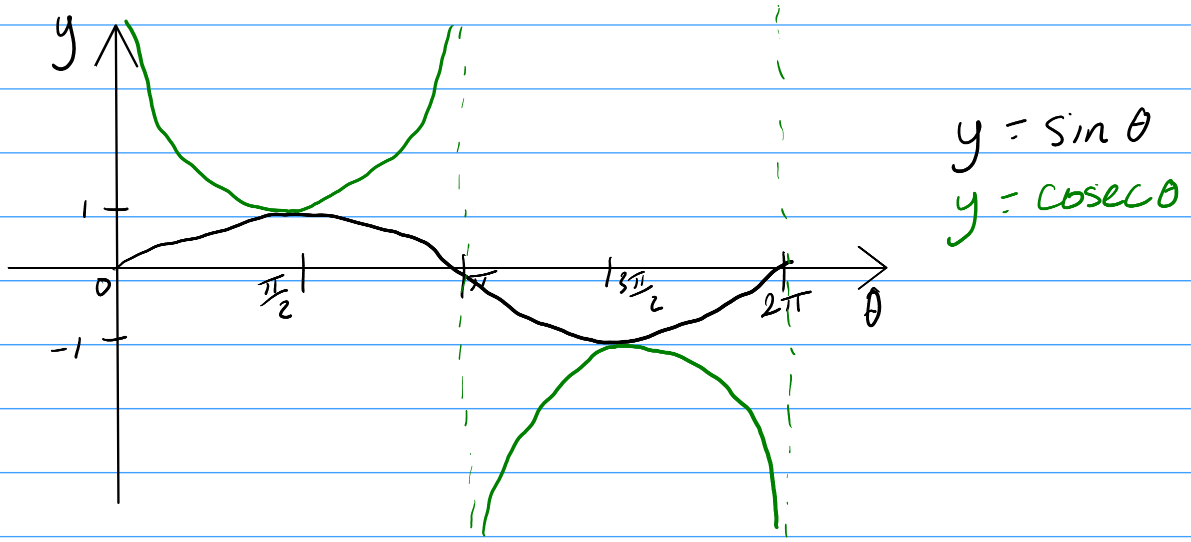
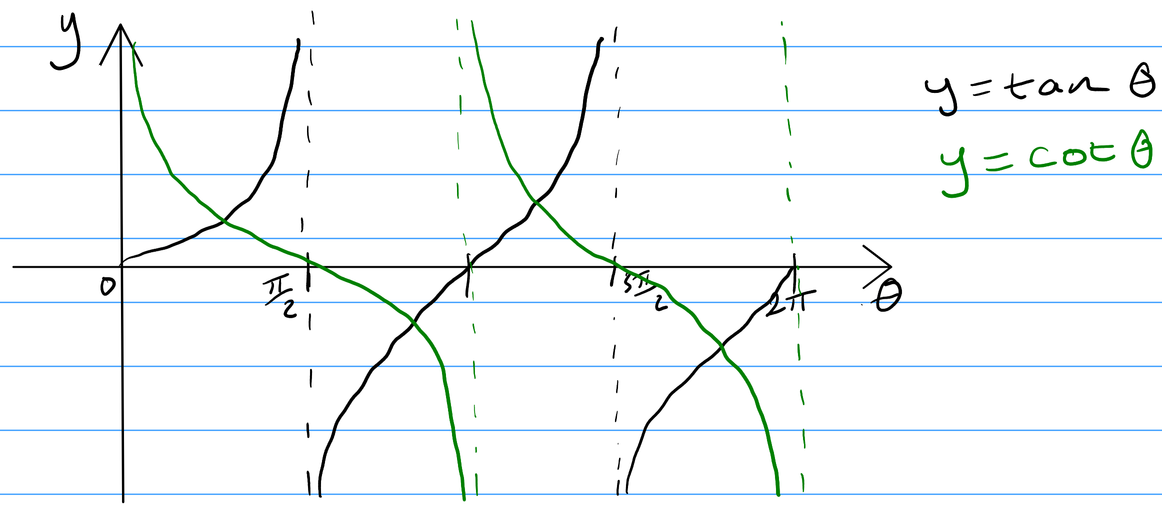
1 (a) Sketch the graphs of cos θ and sec θ, on the same set of axes, in the interval $0 \le \theta \le 2\pi$



(b) Sketch the graphs of sin θ and cosec θ, on the same set of axes, in the interval $0 \le \theta \le 2\pi$



(c) Sketch the graphs of tan θ and cot θ, on the same set of axes, in the interval $0 \le \theta \le 2\pi$



2 Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$2 \operatorname{cosec} \theta = 5$$

Give your answers to 3 significant figures.

$$\operatorname{cosec} \theta = \frac{5}{2}$$

$$\sin \theta = \frac{2}{5}$$

$$\theta = \underline{\underline{0.412}}, \quad \underline{\underline{2.73}}$$

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3 Solve, for $0 \leq x \leq 2\pi$, the equation,

$$2 \cot x = 3 \sec x$$

Give your answers in terms of π.

$$\frac{2 \cos x}{\sin x} = \frac{3}{\cos x}$$

$$2 \cos^2 x = 3 \sin x$$

$$2(1 - \sin^2 x) = 3 \sin x$$

$$2 - 2\sin^2 x = 3 \sin x$$

$$0 = 2\sin^2 x + 3\sin x - 2$$

$$\sin x = \frac{1}{2} \qquad \sin x = -2$$

$\times$ no sol.

$$x = \underline{\underline{\frac{1}{6}\pi}}, \quad \underline{\underline{\frac{5}{6}\pi}}$$

4 Solve, for $-\pi \le x \le \pi$, the equation,

$$5 \cos x + \cot x = 0$$

Give your answers to 2 decimal places where appropriate.

$$5 \cos x + \frac{\cos x}{\sin x} = 0$$

$$5 \cos x \sin x + \cos x = 0$$

$$\cos x (5 \sin x + 1) = 0$$

$$\cos x = 0 \qquad \sin x = -\frac{1}{5}$$

$$x = \frac{1}{2}\pi, \ -\frac{1}{2}\pi \qquad x = -0.20, \ \cancel{3.34}, \ -2.94$$

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5 Solve, for $0 \le x \le 360$, the equation,

$$\sec^2 x + 5 \sec x + 6 = 0$$

Give your answers to 1 decimal place where appropriate.

$$(\sec x + 2)(\sec x + 3) = 0$$

$$\sec x = -2 \qquad \sec x = -3$$

$$\cos x = -\frac{1}{2} \qquad \cos x = -\frac{1}{3}$$

$$x = 120^\circ, 240^\circ \qquad x = 109.5^\circ, 250.5^\circ$$

6 Solve, for $0 \leq x \leq 360$, the equation,

$$\cot^2 x = 9$$

Give your answers to 1 decimal place.

$\cot x = \pm 3$

$\tan x = \pm \frac{1}{3}$

$x = 18.4°, 198.4°, \cancel{-18.4}, 161.6°, 341.6°$

7 Solve, for $-180 \leq \theta \leq 180$, the equation,

$$\sec(2\theta + 10) = -1.3$$

Give your answers to 1 decimal place.

$\cos(2\theta + 10) = -\frac{1}{1.3}$

$2\theta + 10 = 140.28, 219.72, -219.72, -140.28$

$\theta = 65.1°, 104.9°, -114.9°, -75.1°$

8 (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$

(b) Solve, for $0 \leq \theta \leq 360$, the equation,

$$\tan^2\theta + \sec^2\theta + 5\sec\theta = 2$$

Give your answers to 1 decimal place.

a/ $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

b/ $\sec^2\theta - 1 + \sec^2\theta + 5\sec\theta = 2$

$2\sec^2\theta + 5\sec\theta - 3 = 0$

$(2\sec\theta - 1)(\sec\theta + 3) = 0$

$\sec\theta = \frac{1}{2}$ $\quad \sec\theta = -3$

$\cos\theta = 2$ $\quad \cos\theta = -\frac{1}{3}$

$\times$ $\quad \theta = 109.5°, 250.5°$

9 (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\text{cosec}^2\theta = 1 + \cot^2\theta$

(b) Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$\text{cosec}^2\theta + \cot^2\theta = 3$$

Give your answers in terms of π.

a/

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \text{cosec}^2\theta$$

b/

$$\cot^2\theta + 1 + \cot^2\theta = 3$$

$$2\cot^2\theta = 2$$

$$\cot^2\theta = 1$$

$$\tan^2\theta = 1$$

$$\tan\theta = 1 \qquad \tan\theta = -1$$

$$\theta = \frac{1}{4}\pi, \frac{5}{4}\pi \qquad \theta = \cancel{-\frac{1}{4}\pi}, \frac{3}{4}\pi, \frac{7}{4}\pi$$

---

10 Solve, for $0 \leq x \leq 360$, the equation,

$$\tan^2 x + 4\sec x - 2 = 0$$

Give your answers to 1 decimal place.

$$1 + \tan^2 x = \sec^2 x$$

$$\sec^2 x - 1 + 4\sec x - 2 = 0$$

$$\sec^2 x + 4\sec x - 3 = 0$$

$$\sec x = -2 + \sqrt{7}, \quad -2 - \sqrt{7}$$

$$\cos x = \cancel{\frac{2+\sqrt{7}}{3}}, \quad \frac{2-\sqrt{7}}{3}$$

$$x = 102.4^\circ, \quad 257.6^\circ$$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

11 Solve, for $-180 \leq x \leq 180$, the equation,

$$2\cot^2 x - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$$

Give your answers to 1 decimal place where appropriate.

$$2(\operatorname{cosec}^2 x - 1) - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$$
$$2\operatorname{cosec}^2 x - 2 - \operatorname{cosec}^2 x + \operatorname{cosec} x = 4$$
$$\operatorname{cosec}^2 x + \operatorname{cosec} x - 6 = 0$$
$$(\operatorname{cosec} x + 3)(\operatorname{cosec} x - 2) = 0$$
$$\operatorname{cosec} x = -3 \qquad \operatorname{cosec} x = 2$$
$$\sin x = -\frac{1}{3} \qquad \sin x = \frac{1}{2}$$

$x = -19.5°$, $\cancel{199.5°}$ $-160.5°$ $\qquad x = 30°, 150°$

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12 Prove the identities:

(a) $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$

(b) $(\sec x - \cos x)^2 \equiv \tan^2 x - \sin^2 x$

a/
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$\sec^2 x - \operatorname{cosec}^2 x$$
$$(1 + \tan^2 x) - (1 + \cot^2 x)$$
$$1 + \tan^2 x - 1 - \cot^2 x$$
$$\tan^2 x - \cot^2 x$$

b/
$$(\sec x - \cos x)^2$$
$$\sec^2 x - 2\sec x \cos x + \cos^2 x$$
$$\sec^2 x - 2 + \cos^2 x$$
$$1 + \tan^2 x - 2 + (1 - \sin^2 x)$$
$$1 + \tan^2 x - 2 + 1 - \sin^2 x$$
$$\tan^2 x - \sin^2 x$$

13 Prove that:

(a) $\sec^4 x - \tan^4 x \equiv 1 + 2\tan^2 x$

(b) Hence solve, for $0 \le x \le 360$, the equation,

$$\sec^4 x - \tan^4 x = 3$$

$1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

a/ $\sec^4 x - \tan^4 x$

$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$\sec^2 x + \tan^2 x$

$1 + \tan^2 x + \tan^2 x$

$1 + 2\tan^2 x$

b/ $1 + 2\tan^2 x = 3$

$2\tan^2 x = 2$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = 45^\circ, 225^\circ, \cancel{-45^\circ}, 135^\circ, 315^\circ$

---

14 $f(x) = 3x^3 + 4x^2 + 13x + 4$

(a) Show that $(3x + 1)$ is a factor of $f(x)$

(b) Factorise $f(x)$ completely

(c) Prove that there are no real solutions to $3\sec^2\theta + 4\sec\theta + 13 + 4\cos\theta = 0$

a/ $f(-\frac{1}{3}) = 3(-\frac{1}{3})^3 + 4(-\frac{1}{3})^2 + 13(-\frac{1}{3}) + 4$

$= 0$

$\therefore (3x+1)$ is a factor.

b/

$$\begin{array}{r}
x^2 + x + 4 \\
3x + 1 \overline{\big)\, 3x^3 + 4x^2 + 13x + 4} \\
\underline{3x^3 + x^2} \\
3x^2 + 13x \\
\underline{3x^2 + x} \\
12x + 4
\end{array}$$

$$(3x+1)(x^2+x+4)$$

c/

$$3\sec^2\theta + 4\sec\theta + 13 + 4\cos\theta = 0$$

$$3\sec^3\theta + 4\sec^2\theta + 13\sec\theta + 4 = 0$$

$$(3\sec\theta+1)(\sec^2\theta+\sec\theta+4) = 0$$

$$\sec\theta = -\tfrac{1}{3}$$

$$\cos\theta = -3$$

$\therefore$ no sols

$$b^2-4ac$$

$$1^2-4(1)(4) = -15$$

$\therefore$ no sols

---

15 (a) Prove that $\cos\theta + \sin\theta\tan\theta \equiv \sec\theta$

(b) Hence find the exact roots of the equation $\cos\theta + \sin\theta\tan\theta = 4\cos\theta$, for $0 \le \theta \le 2\pi$

$$\cos\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta\cos\theta}{\cos\theta} + \frac{\sin\theta\sin\theta}{\cos\theta}$$

$$\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}$$

$$\frac{1}{\cos\theta} = \sec\theta$$

b/

$$\sec\theta = 4\cos\theta$$

$$\frac{1}{\cos\theta} = 4\cos\theta$$

$$1 = 4\cos^2\theta$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \pm\frac{1}{2}$$

$$\theta = \frac{1}{3}\pi, \frac{5}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi$$

$1 + \tan^2 x = \sec^2 x$

16 (a) Show that the equation $2\sec^2 x + 4\tan^2 x = 5\sec x$

can be written in the form $a\sec^2 x + b\sec x + c = 0$

(b) Hence, given x is obtuse find the exact value of $\tan x$.

a/ $2\sec^2 x + 4(\sec^2 x - 1) = 5\sec x$

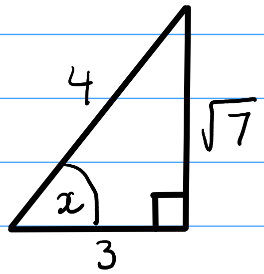
$2\sec^2 x + 4\sec^2 x - 4 = 5\sec x$

$6\sec^2 x - 5\sec x - 4 = 0$

b/ $\sec x = \frac{4}{3}$ $\quad$ $\sec x = -\frac{1}{2}$

$\cos x = \frac{3}{4}$ $\quad$ $\cos x = -2$

X



$\tan x = \frac{\sqrt{7}}{3}$

obtuse $\therefore$ $\tan x$ is negative

$\tan x = -\frac{\sqrt{7}}{3}$

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17 (a) Prove that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} \equiv sec^2\theta$

(b) Hence solve, for $0 < \theta < 2\pi$, $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = 2\tan\theta$

a/ $\operatorname{cosec} x = \frac{1}{\sin x}$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta} \qquad \begin{matrix} \times \sin\theta \\ \times \sin\theta \end{matrix}$$

$$\frac{1}{1 - \sin^2\theta}$$

$$\frac{1}{\cos^2\theta} = \underline{\sec^2\theta}$$

b/

$$\sec^2\theta = 2\tan\theta$$

$$1 + \tan^2\theta = 2\tan\theta$$

$$\tan^2\theta - 2\tan\theta + 1 = 0$$

$$(\tan\theta - 1)(\tan\theta - 1) = 0$$

$$\boxed{1 + \tan^2 x = \sec^2 x}$$

$$\tan\theta = 1$$

$$\theta = \frac{1}{4}\pi, \frac{5}{4}\pi$$

---

18 Solve the equation $\operatorname{cosec}^2 x - 2\cot x = 4$ for $0 < \theta < 360$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$1 + \cot^2 x - 2\cot x = 4$$

$$\cot^2 x - 2\cot x - 3 = 0$$

$$(\cot x + 1)(\cot x - 3) = 0$$

$$\cot x = -1 \qquad \cot x = 3$$

$$\tan x = -1 \qquad \tan x = \frac{1}{3}$$

$$x = \cancel{-45}, 135°, 315°, 18.4°, 198.4°$$