

A Level Maths: Trigonometric Functions

- 1 (a) Sketch the graphs of $\cos \theta$ and $\sec \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$ (4)
- (b) Sketch the graphs of $\sin \theta$ and $\operatorname{cosec} \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$ (4)
- (c) Sketch the graphs of $\tan \theta$ and $\cot \theta$, on the same set of axes, in the interval $0 \leq \theta \leq 2\pi$ (4)

(Total for question 1 is 12 marks)

- 2 Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$2 \operatorname{cosec} \theta = 5$$

Give your answers to 3 significant figures.

(Total for question 2 is 4 marks)

- 3 Solve, for $0 \leq x \leq 2\pi$, the equation,

$$2 \cot x = 3 \sec x$$

Give your answers in terms of π .

(Total for question 3 is 5 marks)

- 4 Solve, for $-\pi \leq x \leq \pi$, the equation,

$$5 \cos x + \cot x = 0$$

Give your answers to 2 decimal places where appropriate.

(Total for question 4 is 5 marks)

- 5 Solve, for $0 \leq x \leq 360$, the equation,

$$\sec^2 x + 5 \sec x + 6 = 0$$

Give your answers to 1 decimal place where appropriate.

(Total for question 5 is 5 marks)

- 6 Solve, for $0 \leq x \leq 360$, the equation,

$$\cot^2 x = 9$$

Give your answers to 1 decimal place.

(Total for question 6 is 5 marks)

- 7 Solve, for $-180 \leq \theta \leq 180$, the equation,

$$\sec (2\theta + 10) = -1.3$$

Give your answers to 1 decimal place.

(Total for question 7 is 4 marks)

8 (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$ (2)

(b) Solve, for $0 \leq \theta \leq 360$, the equation,

$$\tan^2\theta + \sec^2\theta + 5\sec\theta = 2$$

Give your answers to 1 decimal place. (5)

(Total for question 8 is 7 marks)

9 (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ (2)

(b) Solve, for $0 \leq \theta \leq 2\pi$, the equation,

$$\operatorname{cosec}^2\theta + \cot^2\theta = 3$$

Give your answers in terms of π . (5)

(Total for question 9 is 7 marks)

10 Solve, for $0 \leq x \leq 360$, the equation,

$$\tan^2x + 4\sec x - 2 = 0$$

Give your answers to 1 decimal place.

(Total for question 10 is 5 marks)

11 Solve, for $-180 \leq x \leq 180$, the equation,

$$2\cot^2x - \operatorname{cosec}^2x + \operatorname{cosec}x = 4$$

Give your answers to 1 decimal place where appropriate.

(Total for question 11 is 5 marks)

12 Prove the identities:

(a) $\sec^2x - \operatorname{cosec}^2x \equiv \tan^2x - \cot^2x$ (2)

(b) $(\sec x - \cos x)^2 \equiv \tan^2x - \sin^2x$ (2)

(Total for question 12 is 5 marks)

13 Prove that:

(a) $\sec^4x - \tan^4x \equiv 1 + 2\tan^2x$ (2)

(b) Hence solve, for $0 \leq x \leq 360$, the equation, (4)

$$\sec^4x - \tan^4x = 3$$

(Total for question 13 is 6 marks)

14 $f(x) = 3x^3 + 4x^2 + 13x + 4$

(a) Show that $(3x + 1)$ is a factor of $f(x)$ (2)

(b) Factorise $f(x)$ completely (3)

(c) Prove that there are no real solutions to $3\sec^2\theta + 4\sec\theta + 13 + 4\cos\theta = 0$ (5)

(Total for question 14 is 10 marks)

15 (a) Prove that $\cos\theta + \sin\theta \tan\theta \equiv \sec\theta$ (4)

(b) Hence find the exact roots of the equation $\cos\theta + \sin\theta \tan\theta = 4\cos\theta$, for $0 \leq \theta \leq 2\pi$ (4)

(Total for question 15 is 8 marks)

16 (a) Show that the equation $2\sec^2x + 4\tan^2x = 5\sec x$
can be written in the form $a\sec^2x + b\sec x + c = 0$ (2)

(b) Hence, given x is obtuse find the exact value of $\tan x$. (5)

(Total for question 16 is 7 marks)

17 (a) Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} \equiv \sec^2\theta$ (4)

(b) Hence solve, for $0 < \theta < 2\pi$, $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = 2\tan\theta$ (3)

(Total for question 17 is 7 marks)

18 Solve the equation $\operatorname{cosec}^2x - 2\cot x = 4$ for $0 < \theta < 360$

(Total for question 18 is 4 marks)