$\mu = 20 \quad \sigma = 5$

1 Given that $X \sim N(20, 25)$ find the following probabilities

(a) $P(X > 25)$
(b) $P(X < 14.8)$
(c) $P(13 < X < 19)$

a) 0.159
b) 0.149
c) 0.340

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2 The distribution of the weights of coffee in a jar is normally distributed with a mean of 200 g and a standard deviation of 4.2 g.

Find the probability that the weight of the coffee is:

(a) More than 205 g (1)
(b) Less than 195 g (1)
(c) Between 195 g and 205 g (1)

a) 0.117
b) 0.117
c) 0.776

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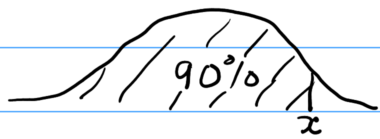
3 The times it takes students to complete a test are normally distributed with a mean of 58 minutes and a standard deviation of 12 minutes.

Find the time by which 90% of students complete the test.

Inverse normal



73.4 minutes

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4 The heights of a group of people are normally distributed with a mean of 164 cm and 10% of the people have a height of less than 150 cm.

Find the standard deviation of the heights of the people.

$$z = \frac{x - \mu}{\sigma}$$

For the standard normal distribution $\mu = 0 \quad \sigma = 1$

Inverse normal Area = 0.1 $\mu = 0 \quad \sigma = 1$

$z = -1.28$

$$-1.28 = \frac{150 - 164}{\sigma}$$

$\sigma = 10.9$ cm

5 A variable Y is normally distributed with a standard deviation of 2.8. Given $P(Y < 26) = 0.3$

Find the mean of Y.

standard normal $z = \frac{x - \mu}{\sigma}$

$z = -0.524$ (from inverse normal with $\mu = 0$ $\sigma = 1$)

$$-0.524 = \frac{26 - \mu}{2.8}$$

$$-1.47 = 26 - \mu$$

$$\mu = 27.5$$

---

6 The 100m race times an athlete records are normally distributed.
16% of the time the athlete is faster than 10 seconds.
23% of the time the athlete is slower than 11 seconds.

Find the mean and standard deviation of the times.

Area $= 0.16$ $z = -0.994$

Area $= 0.77$ $z = 0.739$

$$-0.994 = \frac{10 - \mu}{\sigma} \qquad 0.739 = \frac{11 - \mu}{\sigma}$$

$$-0.994\sigma = 10 - \mu \qquad 0.739\sigma = 11 - \mu$$

$$\mu = 10 + 0.994\sigma \qquad \mu = 11 - 0.739\sigma$$

$$10 + 0.994\sigma = 11 - 0.739\sigma$$

$$1.73\sigma = 1$$

$$\sigma = 0.577$$

$$\mu = 10.6$$

7 A variable X is normally distributed. Given $P(X < 52) = 0.2$ and $P(X > 60) = 0.3$

Find the mean and standard deviation of X.

Area $= 0.2$ $\quad z = -0.842$

Area $= 0.7$ $\quad z = 0.524$

$$-0.842 = \frac{52-\mu}{\sigma} \qquad 0.524 = \frac{60-\mu}{\sigma}$$

$$-0.842\sigma = 52-\mu \qquad 0.524\sigma = 60-\mu$$

$$\mu = 52 + 0.842\sigma \qquad \mu = 60 - 0.524\sigma$$

$$52 + 0.842\sigma = 60 - 0.524\sigma$$

$$1.37\sigma = 8$$

$$\underline{\sigma = 5.86}$$

$$\underline{\mu = 56.9}$$

8 The weight of beans in a tin produced by company A normally distributed with a mean of 415 g and a standard deviation of 3.4 g.

(a) Find the probability that a randomly selected tin of beans has a weight of less than 409 g. **(2)**

Company A suspects that the mean weight of tins from a machine is lower than it should be.

(b) Write down the company's null and alternative hypothesis. **(1)**

A sample of 20 tins is taken and the mean weight is found to be 413 g .

(c) Carry out a test at the 1% significance level to see if there is evidence that the machine is producing tins with a mean of less than 415 g. **(3)**

a/ 0.0388

b/ $H_0: \mu = 415$ $H_1: \mu < 415$

c/ $\bar{X} \sim N\left(415, \frac{3.4^2}{20}\right)$

$P(\bar{X} < 413) = 0.00426$

$0.00426 < 1$ There is evidence to support the claim that the weight is lower than it should be.

9 The length of the bus journey from Leeds to London is normally distributed with a mean of 220 minutes and a standard deviation of 8 minutes.

(a) Find the probability of a bus taking longer than 235 minutes.

The bus company suspect that the mean bus time has changed.

(b) Write down a null and alternative hypothesis for a two-tailed test.

They take a sample of 10 bus journeys and find a mean time of 230 minutes.

(c) Test at the 1% significance level whether there is evidence that the mean time has changed.

a/ 0.0304

b/ $H_0 : \mu = 220 \quad H_1 : \mu \neq 220$

$\bar{X} \sim N\left(220, \frac{8^2}{10}\right)$

c/ $P(X > 230) = 0.0000386$

$0.0000386 < 0.005$

There is evidence to suggest the mean time has changed.

10 A company advertised phone batteries with a mean standby time of 235 hours.

A phone shop manager feels that the batteries have a standby time of less than 235 hours.

The manager conducts an experiment and collects the following summary statistics:

$$n = 200 \qquad \Sigma x = 46313 \qquad \Sigma x^2 = 10861255$$

(a) Find the mean and standard deviation of the sample.

(b) Carry out a hypothesis test to test the manager's claim at the 1% significance level.
State your assumptions clearly.

a/ $\bar{x} = \dfrac{46313}{200} = \underline{\underline{231.565}}$

$\sigma = \sqrt{\dfrac{10861255}{200} - (231.565)^2} = \underline{\underline{26.2}}$ (3sf)

b/ $H_0: \mu = 235 \qquad H_1: \mu < 235$

$\bar{X} \sim N\left(235, \dfrac{26.2^2}{200}\right)$

$P(\bar{X} < 231.565) = 0.0319 \qquad 0.0319 > 0.01$

There is not enough evidence to support the manager's claim.

11 The battery life, in hours, for a fully charged wireless keyboard is normally distributed with mean 50 hours and standard deviation 6 hours.

Hannah has a fully charged wireless keyboard.

(a) Find the probability that the keyboard lasts longer than 45 hours. **(1)**

Hannah uses her keyboard for 45 hours.

(b) Find the probability that Hannah's keyboard lasts for another 10 hours. **(4)**

Hannah believes that the mean battery life of the wireless keyboards are longer than 50 hours. She took a random sample of 20 of these keyboards and found that their mean lifetime was 51.1 hours.

(c) Stating your hypotheses clearly and using a 5% level of significance, test Hannah's belief. **(5)**

a/ $\mu = 50 \quad \sigma = 6$

$P(X > 45) = \underline{\underline{0.798}}$

b/ $P(X > 55) = 0.202$

$P(X > 55 \mid x > 45) = \dfrac{0.202}{0.798} = \underline{\underline{0.254}}$

c/ $H_0 : \mu = 50 \qquad H_1 : \mu > 50$

$\bar{X} \sim N\left(50, \dfrac{6^2}{20}\right) \qquad \sigma = \dfrac{6}{\sqrt{20}}$

$P(\bar{X} > 51.1) = 0.206$

There is not enough evidence to support Hannah's belief.

12 A machine fills bottles with water. The amount of water in each bottle, W ml, is normally distributed with mean 500 ml.

Given that 30% of bottles contain more than 504 ml

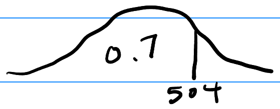
(a) Find the value of k such that $P(k < W < 505) = 0.45$ **(4)**

The machine is adjusted so that the standard deviation of the amount of water in each bottle is now 6 ml.

Following the adjustments the company manager now believes that the mean amount of water in each bottle is less than 500 ml.

She takes a random sample of 20 bottles and finds the mean amount of water to be 498.1 ml

(b) Test the company manager's belief at the 5% significance level.
You should state your hypotheses clearly. **(5)**

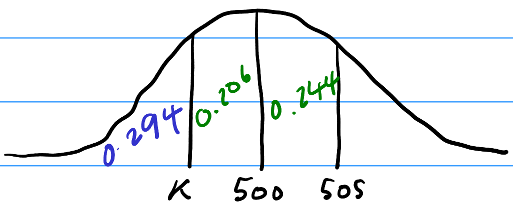


$z = 0.524$ (From inverse normal) $\mu = 0 \quad \sigma = 1$

$$0.524 = \frac{504 - 500}{\sigma}$$

$$0.524\sigma = 4$$

$$\sigma = 7.63$$



$P(500 < x < 505) = 0.244$

$0.45 - 0.244 = 0.206$

$0.5 - 0.206$
$= 0.294$

$\underline{k = 495.9}$

b) $H_0: \mu = 500 \qquad H_1: \mu < 500$

$\bar{X} \sim N\left(500, \frac{6^2}{20}\right) \qquad \sigma = 6 \div \sqrt{20}$

$P(\bar{X} < 498.1) = 0.0784 \qquad 0.0784 > 0.05$

there is not enough evidence to support the company manager's belief.

13 A call centre claims that the amount of time a worker spends on each phone call can be modelled by a normal distribution with a mean of 8 minutes and a standard deviation of 2 minutes.

(a) Using this model, find the probability that the time the worker spends on a randomly selected phone call is more than 11 minutes. **(1)**

The call centre manager thinks that the mean time for each phone call is more than 8 minutes.

The manager takes a random sample of 20 phone calls and finds that the average time is 8.8 minutes.

(b) Test the manager's belief at the 5% significance level.
You should state your hypotheses clearly. **(4)**

a/ 0.0668

b/ $H_0: \mu = 8 \qquad H_1: \mu > 8$

$\bar{X} \sim N\left(8, \frac{2^2}{20}\right) \qquad \sigma = \frac{2}{\sqrt{20}}$

$P(\bar{X} > 8.8) = 0.0368$

There is evidence to support the manager's belief.