1 (a) Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2+\frac{x}{2}\right)^6$ **(4)**

(b) Use your expansion to find an estimate for the value of 2.05^6 **(2)**

a/ $(2)^6 + {}^6C_1(2)^5\left(\frac{x}{2}\right) + {}^6C_2(2)^4\left(\frac{x}{2}\right)^2$

$64 + 6(32)\left(\frac{x}{2}\right) + 15(16)\left(\frac{x^2}{4}\right)$

$64 + 96x + 60x^2$

b/ $2 + \frac{x}{2} = 2.05$

$\frac{x}{2} = 0.05$

$x = 0.1$

$64 + 96(0.1) + 60(0.1)^2 = \underline{\underline{74.2}}$

2 (a) Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2 - \frac{x}{8}\right)^7$ **(4)**

$f(x) = (ax + b)\left(2 - \frac{x}{8}\right)^7$ where a and b are constants

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 384 and $-104x$

(b) Find the values of a and b **(4)**

a/ $2^7 + {}^7C_1 (2)^6 \left(-\frac{x}{8}\right) + {}^7C_2 (2)^5 \left(-\frac{x}{8}\right)^2$

$128 - 56x + \frac{21}{2}x^2$

b/ $(ax + b)\left(128 - 56x + \frac{21}{2}x^2\right)$

$128ax + 128b - 56bx + \ldots$

$128b = 384$

$b = 3$

$128a - 56b = -104$

$128a - 56(3) = -104$

$128a = 64$

$a = \frac{1}{2}$

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3 (a) Fully expand $(p + q)^5$ **(4)**

The probability of Dave being late for school on any day is 0.1. Let p represent the probability that Dave is late on a given day.

(b) Using the last two terms of the binomial expansion, or otherwise, find the probability that Dave is late no more than one time in a school week.

(3)

a/ $p^5 + {}^5C_1 p^4 q + {}^5C_2 p^3 q^2 + {}^5C_3 p^2 q^3 + {}^5C_4 p q^4 + q^5$

$p^5 + 5p^4 q + 10p^3 q^2 + 10p^2 q^3 + 5pq^4 + q^5$

b/ $p = 0.1 \quad q = 0.9$

$5(0.1)(0.9)^4 + 0.9^5 = 0.91854$

4 (a) Expand $(1 + 4x)^8$ in ascending powers of x, up to and including x^3, simplifying each coefficient in the expansion. **(5)**

(b) Showing your working clearly, use your expansion to find, to 5 significant figures an approximation for 1.04^8. **(2)**

a/ $1^8 + {}^8C_1(1)^7(4x) + {}^8C_2(1)^6(4x)^2 + {}^8C_3(1)^5(4x)^3$

$1 + 32x + 448x^2 + 3584x^3$

b/ $1 + 4x = 1.04$

$4x = 0.04$

$x = 0.01$

$1 + 32(0.01) + 448(0.01)^2 + 3584(0.01)^3$

1.368384

$\underline{1.3684}$

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5 (a) Find the first four terms, in ascending powers of x, of the binomial expansion $(2 + kx)^6$ **(4)**

Given that the coefficient of the x^3 term in the expansion is -20

(b) Find the value of k **(3)**

a/ $(2)^6 + {}^6C_1(2)^5(kx) + {}^6C_2(2)^4(kx)^2 + {}^6C_3(2)^3(kx)^3$

$64 + 192kx + 240k^2x^2 + \underline{160k^3x^3}$

b/ $160k^3 = -20$

$k^3 = -\frac{1}{8}$

$\underline{k = -\frac{1}{2}}$

6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion $(1-2x)^5$ **(4)**

(b) Find the first three terms, in ascending powers of x, of the binomial expansion $(1+x)(1-2x)^5$ **(3)**

a/ $1 + 5(-2x) + 10(-2x)^2$

$1 - 10x + 40x^2$

b/ $(1+x)(1-10x+40x^2)$

$1 + x - 10x - 10x^2 + 40x^2 + \cancel{40x^3}$

$1 - 9x + 30x^2$

7 (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^8$$

Giving each term in its simplest form. **(4)**

$f(x) = (ax + b)\left(2 + \frac{x}{8}\right)^8$, where a and b are constants.

Given the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 28 and $62x$

(b) Find the values of a and b. **(4)**

a/ $2^8 + {}^8C_1(2)^7\left(\frac{x}{8}\right) + {}^8C_2(2)^6\left(\frac{x}{8}\right)^2$

$256 + 128x + 28x^2$

b/ $(ax+b)(256 + 128x + 28x^2)$

$256ax + 256b + 128bx + \ldots$

$256b = 28$

$b = \frac{7}{64}$

$256a + 128b = 62$

$256a + 128\left(\frac{7}{64}\right) = 62$

$256a = 48$

$a = \frac{3}{16}$

8 (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 + \frac{2x}{5}\right)^6$$

Giving each term in its simplest form. **(4)**

(b) Explain how you could use your expansion to find an approximation for 2.92^6

You do not need to perform the calculation. **(1)**

a/ $3^6 + {}^6C_1(3)^5\left(\frac{2x}{5}\right) + {}^6C_2(3)^4\left(\frac{2x}{5}\right)^2$

$729 + \frac{2916}{5}x + \frac{972}{5}x^2$

b/ $3 + \frac{2x}{5} = 2.92$

$\frac{2x}{5} = -\frac{2}{25}$

$x = -\frac{1}{5}$

substitute $-\frac{1}{5}$ into the expansion.

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9 (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible. **(3)**

Given the coefficient of x^3 is twice the coefficient of x.

(b) Find the possible values of k. **(3)**

a/ $1 + {}^{10}C_1 kx + {}^{10}C_2(kx)^2 + {}^{10}C_3(kx)^3$

$1 + 10kx + 45k^2x^2 + 120k^3x^3$

b/ $120k^3 = 2(10k)$

$120k^3 = 20k$

$6k^3 = k$

$6k^3 - k = 0$

$k(6k^2 - 1) = 0$

k cannot equal zero $\therefore$

$6k^2 = 1$

$k^2 = \frac{1}{6}$

$k = \pm\sqrt{\frac{1}{6}}$

10 $f(x) = (2 + kx)^6$ where k is a constant.

Given that one of the terms in the binomial expansion of f(x) is $2500x^3$

(a) Find the value of k. **(4)**

(b) Using this value of a find the constant term in the expansion of $\left(2 + \frac{4}{x}\right)(2 + kx)^6$. **(3)**

$$2^6 + {}^6C_1(2)^5(kx) + {}^6C_2(2)^4(kx)^2 + {}^6C_3(2)^3(kx)^3$$

$$64 + 192kx + 240k^2x^2 + 160k^3x^3$$

$$160k^3 = 2500$$

$$k^3 = \frac{125}{8}$$

$$k = \frac{5}{2}$$

b/ $\left(2 + \frac{4}{x}\right)\left(64 + 192kx \ldots\right)$

$128 + 192k$ (constant term

$128 + 192\left(\frac{5}{2}\right)$

608

11 (a) Find the first 3 terms in the expansion of $(1-4x)^5$ in ascending powers of x. **(3)**

(b) Using your expansion, approximate $(0.992)^5$ **(2)**

a) $1 + {}^5C_1 (1)^5 (-4x) + {}^5C_2 (1)^4 (-4x)^2$

$1 - 20x + 160x^2$

b) $1 - 4x = 0.992$

$\frac{1}{125} = 4x$

$x = \frac{1}{500}$

$1 - 20\left(\frac{1}{500}\right) + 160\left(\frac{1}{500}\right)^2$

0.96064

---

12 In the expansion of $(1+x)^n$ where $n > 4$ the coefficient x^4 is 7.5 times the coefficient of x^2

Find the value of n.

${}^nC_2\, x^2$ ${}^nC_4\, x^4$

$\frac{n!}{2!(n-2)!} x^2$ $\frac{n!}{4!(n-4)!} x^4$

$$\frac{n!}{4!(n-4)!} = 7.5 \frac{n!}{2!(n-2)!}$$

$$\frac{\cancel{n}\cancel{(n-1)}(n-2)(n-3)}{24} = 7.5 \frac{\cancel{n}\cancel{(n-1)}}{2}$$

$(n-2)(n-3) = 90$

$n^2 - 5n + 6 = 90$

$n^2 - 5n - 84 = 0$

$(n-12)(n+7) = 0$

$n = 12$ $\cancel{n = -7}$

n is greater than 4 $\therefore$ $n = 12$

13 Prove that $(3+2x)^4 + (3-2x)^4 \geq 162$

Fully justify your answer.

$3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2 + 4(3)(2x)^3 + (2x)^4$

$81 + 216x + 216x^2 + 96x^3 + 16x^4$

$3^4 + 4(3)^3(-2x) + 6(3)^2(-2x)^2 + 4(3)(-2x)^3 + (-2x)^4$

$81 - 216x + 216x^2 - 96x^3 + 16x^4$

$81 + \cancel{216x} + 216x^2 + \cancel{96x^3} + 16x^4 + 81 - \cancel{216x} + 216x^2 - \cancel{96x^3} + 16x^4$

$162 + \underbrace{432x^2 + 32x^4}$

These terms must be greater than or equal to zero (Any no. squared or to the power of 4 is +ve)

$\therefore \quad (3+2x)^4 + (3-2x)^4 \geqslant 162$

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14 In the binomial expansion of $\left(\sqrt{5}+\sqrt{3}\right)^4$ there are two irrational terms.

Find the difference between these two terms.

$(\sqrt{5})^4 + {}^4C_1(\sqrt{5})^3(\sqrt{3}) + {}^4C_2(\sqrt{5})^2(\sqrt{3})^2 + {}^4C_3(\sqrt{5})(\sqrt{3})^3 + (\sqrt{3})^4$

$25 + \underline{20\sqrt{15}} + 90 + \underline{12\sqrt{15}} + 9$

$20\sqrt{15} - 12\sqrt{15} = \underline{\underline{8\sqrt{15}}}$

15 Find the first 4 terms in the expansion of $(2-5x)^7$ in ascending powers of x.

$2^7 + {}^7C_1(2)^6(-5x) + {}^7C_2(2)^5(-5x)^2 + {}^7C_3(2)^4(-5x)^3$

$\underline{\underline{128 - 2240x + 16800x^2 - 70000x^3}}$

16 Find the coefficient of the x term in the binomial expansion of $(4-x)^5$

${}^5C_1(4)^4(-x)$

$-1280x$

$\underline{\underline{-1280}}$

17 Find the first 3 terms in the expansion of $(1-3x)^6$ in ascending powers of x.

$(1)^6 + {}^6C_1(1)^5(-3x) + {}^6C_2(1)^4(-3x)^2 + {}^6C_3(1)^3(-3x)^3$

$\underline{\underline{1 - 18x + 135x^2 - 540x^3}}$

18 (a) Find and simplify the first three terms in the expansion of $(2+3x)^5$ in ascending powers of x. **(3)**

(b) In the expansion of $(1+ax)(2+3x)^5$ the coefficient of x^2 is 752.

Find the value of a. **(3)**

a/ $2^5 + {}^5C_1(2)^4(3x) + {}^5C_2(2)^3(3x)^2$

$32 + 240x + 720x^2$

b/ $(1+ax)(32+240x+720x^2)$

$240ax^2 + 720x^2 = 752x^2$

$240a + 720 = 752$

$\underline{\underline{a = \frac{2}{15}}}$

19 (a) Expand $(1 - 2x)^4$ in ascending powers of x. **(2)**

(b) **Using your expansion** find the exact value of $(0.98)^4$ **(3)**

a/ $1^4 + {}^4C_1(1)^3(-2x) + {}^4C_2(1)^2(-2x)^2 + {}^4C_3(1)(-2x)^3 + (-2x)^4$

$1 - 8x + 24x^2 - 32x^3 + 16x^4$

b/ $1 - 2x = 0.98$

$0.02 = 2x$

$x = 0.01$

$1 - 8(0.01) + 24(0.01)^2 - 32(0.01)^3 + 16(0.01)^4$

$\underline{\underline{0.92236816}}$

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20 Find the binomial expansion of $(5 - 2x)^3$

$5^3 + 3(5)^2(-2x) + 3(5)(-2x)^2 + (-2x)^3$

$\underline{\underline{125 - 150x + 60x^2 - 8x^3}}$