AS Level Maths: The Binomial Expansion

1 (a) Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2 + \dfrac{x}{2}\right)^6$ **(4)**

(b) Use your expansion to find an estimate for the value of 2.05^6

(2)

(Total for question 1 is 6 marks)

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2 (a) Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2 - \dfrac{x}{8}\right)^7$

(4)

$f(x) = (ax + b)\left(2 - \dfrac{x}{8}\right)^7$ where a and b are constants

Given that the first two terms, in ascending powers of x, in the series expansion of
$f(x)$ are 384 and $-104x$

(b) Find the values of a and b **(4)**

(Total for question 2 is 8 marks)

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3 (a) Fully expand $(p + q)^5$ **(4)**

The probability of Dave being late for school on any day is 0.1. Let p represent the probability that Dave is late on a given day.

(b) Using the last two terms of the binomial expansion, or otherwise, find the probability that Dave is late no more than one time in a school week.

(3)

(Total for question 3 is 7 marks)

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4 (a) Expand $(1 + 4x)^8$ in ascending powers of x, up to and including x^3, simplifying each coefficient in the expansion. **(5)**

(b) Showing your working clearly, use your expansion to find, to 5 significant figures an approximation for 1.04^8. **(2)**

(Total for question 4 is 7 marks)

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5 (a) Find the first four terms, in ascending powers of x, of the binomial expansion $(2 + kx)^6$ **(4)**

Given that the coefficient of the x^3 term in the expansion is -20

(b) Find the value of k **(3)**

(Total for question 5 is 7 marks)

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6 (a) Find the first three terms, in ascending powers of x, of the binomial expansion $(1 - 2x)^5$ **(4)**

(b) Find the first three terms, in ascending powers of x, of the binomial expansion $(1 + x)(1 - 2x)^5$

(3)

(Total for question 6 is 7 marks)

7 (a) Find the first 3 terms, in ascending powers of *x,* of the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^8$$

Giving each term in its simplest form. **(4)**

$\mathrm{f}(x) = (ax + b) \left(2 + \dfrac{x}{8}\right)^8$, where a and b are constants.

Given the first two terms, in ascending powers of x, in the series expansion of f(*x*) are 28 and 62*x*

(b) Find the values of a and b. **(4)**

(Total for question 7 is 8 marks)

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8 (a) Find the first 3 terms, in ascending powers of *x,* of the binomial expansion of

$$\left(3 + \frac{2x}{5}\right)^6$$

Giving each term in its simplest form. **(4)**

(b) Explain how you could use your expansion to find an approximation for 2.92^6

You do not need to perform the calculation. **(1)**

(Total for question 8 is 5 marks)

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9 (a) Find the first 4 terms, in ascending powers of *x,* of the binomial expansion of

$$(1 + kx)^{10}$$

where *k* is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given the coefficient of x^3 is twice the coefficient of *x*.

(b) Find the possible values of *k*. **(3)**

(Total for question 9 is 6 marks)

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10 $\mathrm{f}(x) = (2 + kx)^6,$ where *k* is a constant.

Given that one of the terms in the binomial expansion of f(x) is $2500x^3$

(a) Find the value of *k*. **(4)**

(b) Using this value of a find the constant term in the expansion of $\left(2 + \dfrac{4}{x}\right) \cdot (2 + kx)^6,$

(3)

(Total for question 10 is 7 marks)

11 (a) Find the first 3 terms in the expansion of $(1 - 4x)^5$ in ascending powers of x. **(3)**

(b) Using your expansion, approximate $(0.992)^5$ **(2)**

(Total for question 11 is 5 marks)

12 In the expansion of $(1 + x)^n$ where $n > 4$ the coefficient x^4 is 7.5 times the coefficient of x^2

Find the value of n.

(Total for question 12 is 5 marks)

13 Prove that $(3 + 2x)^4 + (3 - 2x)^4 \geq 162$

Fully justify your answer.

(Total for question 13 is 6 marks)

14 In the binomial expansion of $\left(\sqrt{5} + \sqrt{3}\right)^4$ there are two irrational terms.

Find the difference between these two terms.

(Total for question 14 is 3 marks)

15 Find the first 4 terms in the expansion of $(2 - 5x)^7$ in ascending powers of x.

(Total for question 15 is 4 marks)

16 Find the coefficient of the x term in the binomial expansion of $(4 - x)^5$

(Total for question 16 is 2 marks)

17 Find the first 3 terms in the expansion of $(1 - 3x)^6$ in ascending powers of x.

(Total for question 17 is 3 marks)

18 (a) Find and simplify the first three terms in the expansion of $(2 + 3x)^5$ in ascending powers of x. **(3)**

(b) In the expansion of $(1 + ax)(2 + 3x)^5$ the coefficient of x^2 is 752.

Find the value of a. **(3)**

(Total for question 18 is 6 marks)

19 (a) Expand $(1 - 2x)^4$ in ascending powers of x. **(2)**

(b) **Using your expansion** find the exact value of $(0.98)^4$ **(3)**

(Total for question 19 is 5 marks)

20 Find the binomial expansion of $(5 - 2x)^3$

(Total for question 20 is 3 marks)