1 The line l passes through the coordinates (2, 1) and (4, –5).

Find an equation for l.

$x_1 \; y_1 \quad x_2 \; y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 1}{4 - 2}$$

$$= \frac{-6}{2} = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x - 2)$$

$$y - 1 = -3x + 6$$

$$y = -3x + 7$$

2 The line l_1 has the equation $2x + 3y + 5 = 0$
The line l_2 passes through the coordinates (1, 7) and (5, 1).

$x_1 \; y_1 \quad x_2 \; y_2$

Determine, giving full reasons for your answer, whether l_1 and l_2 are parallel, perpendicular or neither.

l_1: $3y = -2x - 5$

$$y = -\tfrac{2}{3}x - \tfrac{5}{3} \qquad m = -\tfrac{2}{3}$$

l_2: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{1 - 7}{5 - 1}$$

$$= \frac{-6}{4} = -\frac{3}{2}$$

Neither

not parallel as not the same gradient

not perpendicular as $-\frac{3}{2} \times -\frac{2}{3} \neq -1$

$x_1 \; y_1 \quad x_2 \; y_2$

3 (a) Find an equation of the straight line passing through the points (–2, 5) and (5, –1). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The line crosses the x axis at point A, the y axis at point B and O is the origin.

(b) Find the area of triangle AOB. **(3)**

a/

$$m = \frac{-1 - 5}{5 - -2} = \frac{-6}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{-6}{7}(x + 2)$$

$$7(y - 5) = -6(x + 2)$$

$$7y - 35 = -6x - 12$$

$$\underline{\underline{6x + 7y - 23 = 0}}$$

b/ crosses x when $y = 0$ $\quad 6x - 23 = 0$

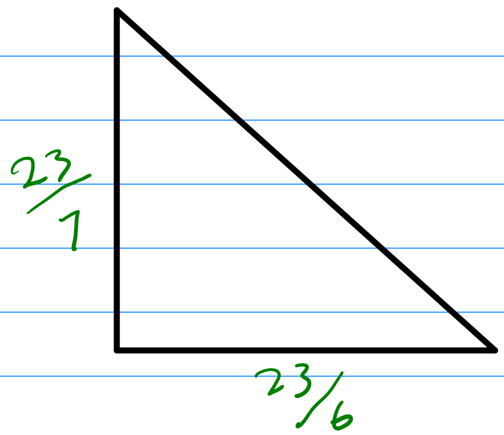
$$6x = 23$$

$$x = 23/6$$

crosses y when $x = 0$ $\quad 7y - 23 = 0$

$$7y = 23$$

$$y = 23/7$$



$$\text{Area} = \frac{1}{2} \cdot \frac{23}{6} \cdot \frac{23}{7}$$

$$= \underline{\underline{\frac{529}{84} \text{ units}^2}}$$

4 The points A and B have coordinates $(-1, k+2)$ and $(2k-3, 8)$ where k is a constant.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

Given the gradient of AB is $\frac{1}{3}$

(a) Show that $k = 4$ **(2)**

(b) Find the equation of the line the passes through A and B. **(3)**

(c) Find the equation of the perpendicular bisector of A and B. **(4)**
Give your answer in the form $ax + by + c = 0$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{3} = \frac{8 - (k+2)}{2k - 3 + 1}$$

$$\frac{1}{3} = \frac{8 - k - 2}{2k - 2}$$

$$2k - 2 = 3(6 - k)$$

$$2k - 2 = 18 - 3k$$

$$5k = 20$$

$$k = 4$$

b/ $(-1, 6)$ $\quad m = \frac{1}{3}$

$x_1 \quad y_1$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{1}{3}(x + 1)$$

$$y - 6 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{19}{3}$$

c/ $(-1, 6)$ and $(5, 8)$

midpoint $(2, 7)$

$x_1 \quad y_1$

perp $m = -3$

$$y - 7 = -3(x - 2)$$

$$y - 7 = -3x + 6$$

$$3x + y - 13 = 0$$

5 The straight line l has equation $2x - 3y + 24 = 0$ and meets the coordinate axis at the points A and B.

Find the distance of the midpoint of AB from the origin.

Give your answer in the form $k\sqrt{13}$

crosses x when $y = 0$ $\quad 2x + 24 = 0$

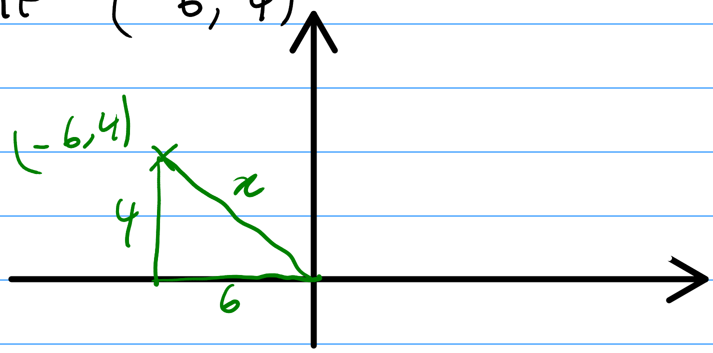
$x = -12$

crosses y when $x = 0$ $\quad -3y + 24 = 0$

$y = 8$

$(-12, 0)$ and $(0, 8)$

Midpoint $(-6, 4)$



$x^2 = 4^2 + 6^2$

$x^2 = 52$

$x = 2\sqrt{13}$

6 The line l_1 has gradient 2 and passes through (5, 7).

$x_1 \quad y_1$

(a) Find an equation for l_1 in the form $y = mx + c$

l_2 is perpendicular to l_1 and passes through (0, 1) **(2)**

(b) Find an equation for l_2. **(2)**

a/ $y - 7 = 2(x - 5)$

$y - 7 = 2x - 10$

$y = 2x - 3$

b/ perp $m = -\frac{1}{2}$ $\quad c = 1$

$y = -\frac{1}{2}x + 1$

7 The line l_1 has the equation $5x + 2y - 4 = 0$
The line l_2 has the equation $x - 4y + 1 = 0$

Find the coordinates of the point where l_1 and l_2 intersect.

$$5x + 2y = 4$$
$$x - 4y = -1 \qquad \text{(use calculator)}$$

$$\underline{\underline{\left(\frac{7}{11}, \frac{9}{22}\right)}}$$

---

8 The line l_1 has the equation $2x - 3y - 4 = 0$
The line l_2 is perpendicular to l_1 and passes through the point $(4, -1)$

Find an equation for l_2 in the form $ax + by + c = 0$

x_1, y_1

$$2x - 4 = 3y$$
$$y = \frac{2}{3}x - \frac{4}{3}$$

$$m = \frac{2}{3} \qquad \text{perp } \underline{m = -\frac{3}{2}}$$

$$y + 1 = -\frac{3}{2}(x - 4)$$

$$2(y + 1) = -3(x - 4)$$

$$2y + 2 = -3x + 12$$

$$\underline{\underline{3x + 2y - 10 = 0}}$$

9 The line l passes through the points $A(1, 4)$ and $B(-2, 13)$.

$x_1\ y_1 \quad x_2\ y_2$

(a) Find an equation for l. **(3)**

(b) Find the exact length of AB **(2)**

a/

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{13 - 4}{-2 - 1}$$

$$= \frac{9}{-3}$$

$$= \underline{-3}$$

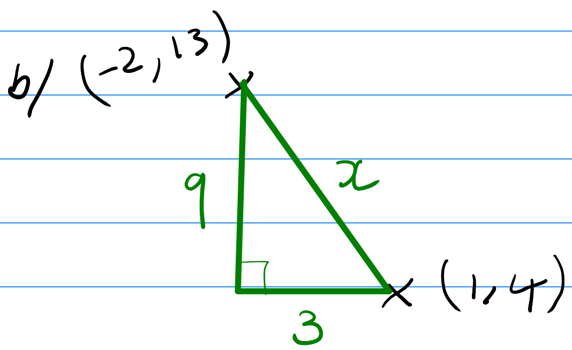
$$y - y_1 = m(x - x_1)$$

$$y - 4 = -3(x - 1)$$

$$y - 4 = -3x + 3$$

$$\underline{\underline{y = -3x + 7}}$$

b/



$$3^2 + 9^2 = x^2$$

$$90 = x^2$$

$$x = \sqrt{90}$$

$$= \underline{\underline{3\sqrt{10}}}$$

10 The line l_1 has gradient 3 and passes through (-2, 5).

(a) Find an equation for l_1 in the form $y = mx + c$ **(2)**

l_2 is perpendicular to l_1 and passes through (0, 4)

(b) Find an equation for l_2. **(2)**

(c) Find the coordinates of the point where l_1 and l_2 intersect. **(3)**

a/
$$y - 5 = 3(x+2)$$
$$y - 5 = 3x + 6$$
$$y = 3x + 11$$

b/ perp $m = -\frac{1}{3}$
$$y = -\frac{1}{3}x + 4$$

c/ Intersection where:
$$3x + 11 = -\frac{1}{3}x + 4$$
$$\frac{10}{3}x = -7$$
$$x = \frac{-21}{10}$$
$$y = 3\left(\frac{-21}{10}\right) + 11$$
$$= \frac{47}{10}$$
$$\left(\frac{-21}{10}, \frac{47}{10}\right)$$

11 The line l_1 has the equation $5y - 10 = 2x$
The point P with x coordinate 4 lies on l_1.
The line l_2 is perpendicular to l_1 and passes through the point P.

(a) Find an equation for l_2 in the form $ax + by + c = 0$

(4)

The lines l_1 and l_2 cross the x axis at the points Q and R respectively.

(b) Calculate the area of the triangle QPR.

(4)

l_1 : $5y - 10 = 2x$

$5y = 2x + 10$

$y = \frac{2}{5}x + 2$ $\left(m = \frac{2}{5}\right)$

when $x = 4$ $y = \frac{2}{5}(4) + 2$

$= \frac{18}{5}$

perp $m = -\frac{5}{2}$ $\left(4, \frac{18}{5}\right)$
x_1 y_1

$y - \frac{18}{5} = -\frac{5}{2}(x - 4)$

$2\left(y - \frac{18}{5}\right) = -5(x - 4)$

$2y - \frac{36}{5} = -5x + 20$

$10y - 36 = -25x + 100$

$\underline{25x + 10y - 136 = 0}$

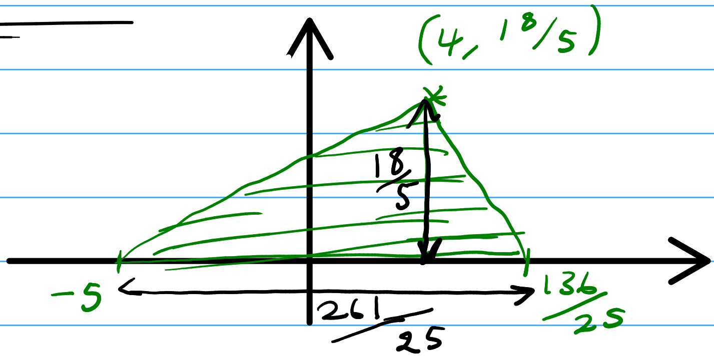
b/ cross x when $y = 0$

$25x - 136 = 0$

$x = \frac{136}{25}$

and $-10 = 2x$

$x = -5$



Area $= \frac{1}{2} \cdot \frac{261}{25} \cdot \frac{18}{5}$

$= \frac{2349}{125} = \underline{18.792}$

12 Three of the following points lie on the same straight line.

Which point does **not** lie on this line?

Tick **one** box.

(2, -1)	☐
(-2, 11)	☐
(-1, 7)	☑
(1, 2)	☐

$(2, -1)$ $(-2, 11)$

$x_1 \; y_1 \quad x_2 \; y_2$

$m = \dfrac{11+1}{-2-2}$

$= \dfrac{12}{-4} = -3$

$y + 1 = -3(x - 2)$

$y + 1 = -3x + 6$

$\underline{y = -3x + 5}$

$2 = -3(1) + 5$

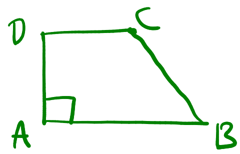
$2 = 2$ ✓ ✓$(1, 2)$

---

13 *ABCD* is a trapezium with point *A*(-2, 5), point *B*(4, 2) and point *C*(6, -4).

$x_1 \; y_1 \quad x_2 \; y_2$

AB is parallel to *DC*
AB is perpendicular to *AD*



(a) Find the equation of *CD* **(2)**

(b) Find the coordinates of *D*. **(3)**

AB : $m = \dfrac{2 - 5}{4 + 2}$

$= -3/6 = \underline{-\tfrac{1}{2}}$ (gradient of CD)

passes through $(6, -4)$

$x_1 \; y_1$

$y + 4 = -\frac{1}{2}(x - 6)$

$y + 4 = -\frac{1}{2}x + 3$

$\underline{y = -\frac{1}{2}x - 1}$

b/ perp $m = 2$ AD: $y - 5 = 2(x + 2)$ $(-2, 5)$

$x_1 \; y_1$

$\underline{y = 2x + 9}$

$-\frac{1}{2}x - 1 = 2x + 9$

$-\frac{5}{2}x = 10$

$x = -4$

when $x = -4$ $y = 2(-4) + 9$

$= 1$

$\underline{(-4, 1)}$

14 The point A has the coordinates (-3, -4), point B has the coordinates (7, 2).

x_1 y_1 x_2 y_2

Find the equation the perpendicular bisector of AB

$$\text{Midpoint} = \left(\frac{-3+7}{2}, \frac{-4+2}{2}\right)$$

$$= (2, -1)$$

x_1 y_1

$$m = \frac{2+4}{7+3} = \frac{6}{10} = \frac{3}{5}$$

$$\text{perp } m = -\frac{5}{3}$$

$$y+1 = -\frac{5}{3}(x-2)$$

$$3(y+1) = -5(x-2)$$

$$3y+3 = -5x+10$$

$$3y = -5x+7$$

$$y = -\frac{5}{3}x + \frac{7}{3}$$

15 The line l_1 has equation $4y - 3x = 11$

The line l_2 passes through the points (3, 5) and (−5, −1).

x_1 y_1 x_2 y_2

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

$$l_1: \quad 4y = 3x + 11$$

$$y = \frac{3}{4}x + \frac{11}{4} \qquad m = 3/4$$

$$l_2: \quad m = \frac{-1-5}{-5-3} = \frac{-6}{-8} = \frac{3}{4}$$

same gradient $\therefore$ parallel

$x_1 \quad y_1 \qquad\qquad\qquad x_2 \quad y_2$

16 The point A has the coordinates (-2, 3), point B has the coordinates (4, -7).

The perpendicular bisector of AB intersects the line $y = 2x + 1$ at the point P.

Find the coordinates of P.

Midpoint $\left(\frac{-2+4}{2}, \frac{3-7}{2}\right)$

$(1, -2)$
$x_1 \quad y_1$

Gradient of AB $= \frac{7-3}{4--2} = \frac{4}{6} = \frac{2}{3}$

perp $m = -\frac{3}{2}$

perpendicular bisector: $y + 2 = -\frac{3}{2}(x - 1)$

$2(y+2) = -3(x-1)$

$2y + 4 = -3x + 3$

$2y = -3x - 1$

$y = 2x + 1 \qquad\qquad y = -\frac{3}{2}x - \frac{1}{2}$

$2x + 1 = -\frac{3}{2}x - \frac{1}{2}$

$\frac{7}{2}x = -\frac{3}{2}$

$7x = -3$

$x = -\frac{3}{7} \qquad\qquad y = 2\left(-\frac{3}{7}\right) + 1$

$= \frac{1}{7}$

$\left(-\frac{3}{7}, \frac{1}{7}\right)$

$x_1\ y_1$

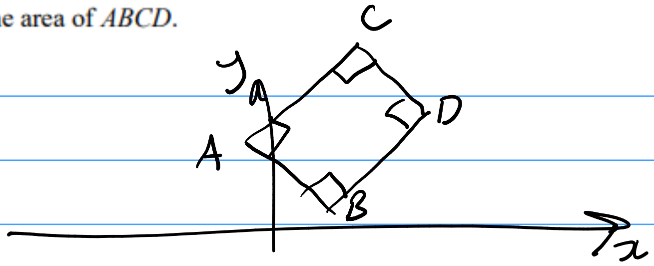
$x_1\ y_1 \quad x_2\ y_2 \quad x_2\ y_2 \quad x_2\ y_2$

17 Points $A(-1, 5)$, $B(1, 1)$, $C(5, 8)$ and $D(7, 4)$ are the vertices of a quadrilateral $ABCD$.

$x_1\ y_1 \quad x_2\ y_2$

(a) Prove that $ABCD$ is a rectangle. **(4)**

(b) Find the area of $ABCD$. **(2)**



$$AB \text{ gradient} = \frac{1-5}{1+1} = \frac{-4}{2} = -2$$

$$AC \text{ gradient} = \frac{8-5}{5+1} = \frac{3}{6} = \frac{1}{2}$$

$$BD \text{ gradient} = \frac{4-1}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$CD \text{ gradient} = \frac{4-8}{7-5} = \frac{-4}{2} = -2$$

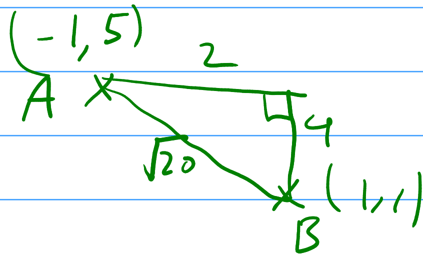
4 sided shape with 4 right angles

AB is perpendicular to AC and BD

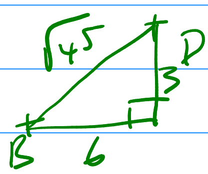
CD is perpendicular to AC and BD

b/

$$AB \text{ length}^2 = 2^2 + 4^2$$
$$= 20$$
$$= \sqrt{20}$$



$$BD^2 = 6^2 + 3^2$$
$$BD = \sqrt{45}$$



$$\text{Area} = \sqrt{20} \times \sqrt{45}$$
$$= 30 \text{ units}^2$$

18 The line l_1 has equation $2y + 4x + 7 = 0$

The line l_2 has equation $y = mx + 4$, where m is a constant.

Given that l_1 and l_2 are perpendicular.

(a) Find the value of m. **(2)**

(b) Find the coordinates of the point where l_1 and l_2 meet. **(3)**

l_1: $2y = -4x - 7$

$y = -2x - 7/2$

perpendicular gradient $= \frac{1}{2}$

$m = \frac{1}{2}$

b/ $y = -2x - \frac{7}{2}$ and $y = \frac{1}{2}x + 4$

$-2x - \frac{7}{2} = \frac{1}{2}x + 4$

$-4x - 7 = x + 8$

$-15 = 5x$

$x = -3$

$y = \frac{1}{2}(-3) + 4$

$= \frac{5}{2}$

$\left(-3, \frac{5}{2}\right)$

19 In 1960 the life expectancy in the UK was 71 years

In 1975 the life expectancy in the UK was 73 years

Given that x years is the life expectancy n years after 1960.

(a) Using a linear model, form an equation linking x with n. **(3)**

In 2020 the life expectancy in the UK was 81.8 years.

(b) Comment on the suitability of your model in light of this information **(3)**

$(0, 71)$ $(15, 73)$

n_1 x_1 n_2 x_2

$$m = \frac{73-71}{15} = \frac{2}{15}$$

$$x = \frac{2}{15}n + 71$$

b) $n = 60$ $x = \frac{2}{15}(60) + 71$

$= 79$

The model underestimated the life expectancy

The model may not be suitable.

21 Worldwide CO_2 emissions in 1990 were 22.5 billion tonnes.
Worldwide CO_2 emissions in 2010 were 33.6 billion tonnes.

Given that A billion tonnes is the CO_2 emissions n years after 1990.

(a) Using a linear model, form an equation linking A with n. **(3)**

In 2016 worldwide CO_2 emissions were 35.7 billion tonnes.

(b) Comment on the suitability of your model in light of this information **(3)**

$(\underset{n_1}{0}, \underset{A_1}{22.5}) \qquad (\underset{n_2}{20}, \underset{A_2}{33.6})$

$$m = \frac{33.6 - 22.5}{20} = 0.555$$

$$\underline{\underline{A = 0.555n + 22.5}}$$

b/ $A = 0.555(26) + 22.5$

$= 36.93$

The model overestimated the emissions.

$\therefore$ The model may not be suitable.

22 Point C has coordinates $(2, c)$ and point D has coordinates $(d, 8)$

$x_1 \; y_1 \qquad x_2 \; y_2$

The perpendicular bisector of CD has equation $3y + x = 10$

Find c and d.

$$3y + x = 10$$
$$3y = -x + 10$$
$$y = -\frac{1}{3}x + \frac{10}{3}$$

perpendicular gradient $= 3$

$$3 = \frac{8 - c}{d - 2}$$
$$3(d-2) = 8 - c$$
$$3d - 6 = 8 - c$$
$$\underline{c + 3d = 14}$$

$$\text{Midpoint} = \left(\frac{2+d}{2}, \frac{c+8}{2}\right) \qquad 3y + x = 10$$

$$3\left(\frac{c+8}{2}\right) + \frac{2+d}{2} = 10$$

$$3(c+8) + 2 + d = 20$$
$$3c + 24 + 2 + d = 20$$
$$\underline{3c + d = -6}$$

$$c + 3d = 14$$
$$3c + d = -6$$

$$\underline{\underline{c = -4}} \qquad \underline{\underline{d = 6}}$$