

- 1 The line l passes through the coordinates $(2, 1)$ and $(4, -5)$.

Find an equation for l .

$$x_1 \ y_1 \ x_2 \ y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 1}{4 - 2}$$

$$= \frac{-6}{2} = \underline{\underline{-3}}$$

$$y - y_1 = m(x - x_1)$$

$$\underline{\underline{y - 1 = -3(x - 2)}}$$

$$y - 1 = -3x + 6$$

$$\underline{\underline{y = -3x + 7}}$$

- 2 The line l_1 has the equation $2x + 3y + 5 = 0$
The line l_2 passes through the coordinates $(1, 7)$ and $(5, 1)$.

$$x_1 \ y_1 \ x_2 \ y_2$$

Determine, giving full reasons for your answer, whether l_1 and l_2 are parallel, perpendicular or neither.

$$l_1: 3y = -2x - 5$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

$$m = -\frac{2}{3}$$

$$l_2: m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 7}{5 - 1}$$

$$= \frac{-6}{4} = -\frac{3}{2}$$

Neither

not parallel as not the same gradient
not perpendicular as $-\frac{3}{2} \times -\frac{2}{3} \neq -1$

$x_1 \ y_1 \ x_2 \ y_2$

- 3 (a) Find an equation of the straight line passing through the points $(-2, 5)$ and $(5, -1)$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. (3)

The line crosses the x axis at point A , the y axis at point B and O is the origin.

- (b) Find the area of triangle AOB . (3)

$$a/ \quad m = \frac{-1 - 5}{5 - -2} = \frac{-6}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{-6}{7}(x + 2)$$

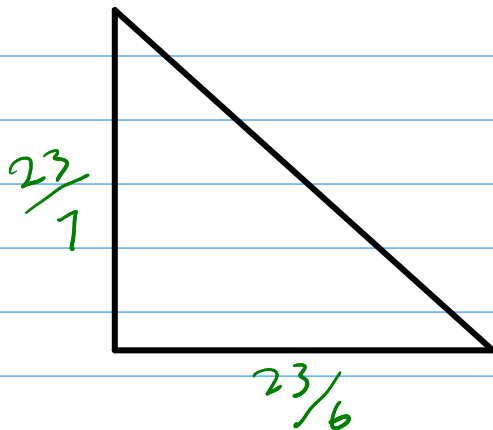
$$7(y - 5) = -6(x + 2)$$

$$7y - 35 = -6x - 12$$

$$\underline{6x + 7y - 23 = 0}$$

b/ crosses x when $y=0$ $6x - 23 = 0$
 $6x = 23$
 $x = \frac{23}{6}$

crosses y when $x=0$ $7y - 23 = 0$
 $7y = 23$
 $y = \frac{23}{7}$



$$\text{Area} = \frac{1}{2} \cdot \frac{23}{6} \cdot \frac{23}{7}$$

$$= \frac{529}{84} \text{ units}^2$$

4 The points A and B have coordinates $(-1, k+2)$ and $(2k-3, 8)$ where k is a constant.

Given the gradient of AB is $\frac{1}{3}$

(a) Show that $k=4$

(2)

(b) Find the equation of the line that passes through A and B .

(3)

(c) Find the equation of the perpendicular bisector of A and B .

(4)

Give your answer in the form $ax + by + c = 0$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{3} = \frac{8 - (k+2)}{2k-3+1}$$

$$\frac{1}{3} = \frac{8 - k - 2}{2k - 2}$$

$$2k - 2 = 3(6 - k)$$

$$2k - 2 = 18 - 3k$$

$$5k = 20$$

$$\underline{\underline{k = 4}}$$

b/ $(-1, 6)$ $m = \frac{1}{3}$
 x_1, y_1

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{1}{3}(x + 1)$$

$$y - 6 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{19}{3}$$

c/ $(-1, 6)$ and $(5, 8)$

midpoint $(2, 7)$
 x_1, y_1

perp $m = -3$

$$y - 7 = -3(x - 2)$$

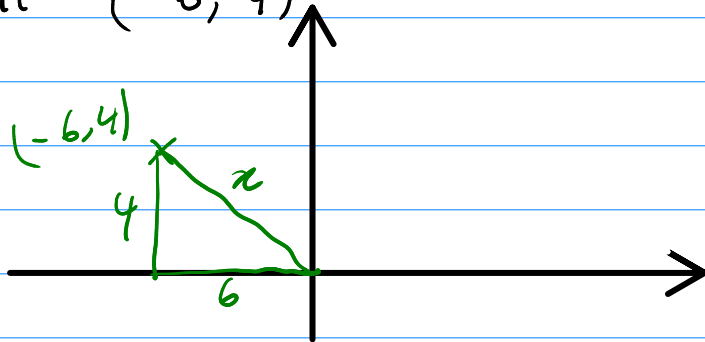
$$y - 7 = -3x + 6$$

$$\underline{\underline{3x + y - 13 = 0}}$$

- 5 The straight line l has equation $2x - 3y + 24 = 0$ and meets the coordinate axis at the points A and B.
Find the distance of the midpoint of AB from the origin.
Give your answer in the form $k\sqrt{13}$

crosses x when $y = 0$ $2x + 24 = 0$
 $x = -12$
 crosses y when $x = 0$ $-3y + 24 = 0$
 $y = 8$
 $(-12, 0)$ and $(0, 8)$

Midpoint $(-6, 4)$



$$x^2 = 4^2 + 6^2$$

$$x^2 = 52$$

$$x = \underline{\underline{2\sqrt{13}}}$$

- 6 The line l_1 has gradient 2 and passes through $(5, 7)$.

(a) Find an equation for l_1 in the form $y = mx + c$

(2)

l_2 is perpendicular to l_1 and passes through $(0, 1)$

(b) Find an equation for l_2 .

(2)

a/ $y - 7 = 2(x - 5)$
 $y - 7 = 2x - 10$
 $y = \underline{\underline{2x - 3}}$

b/ perp $m = -\frac{1}{2}$ $c = 1$
 $y = \underline{\underline{-\frac{1}{2}x + 1}}$

- 7 The line l_1 has the equation $5x + 2y - 4 = 0$
The line l_2 has the equation $x - 4y + 1 = 0$

Find the coordinates of the point where l_1 and l_2 intersect.

$$5x + 2y = 4$$

$$x - 4y = -1 \quad (\text{use calculator})$$

$$\left(\frac{7}{11}, \frac{9}{22} \right)$$

- 8 The line l_1 has the equation $2x - 3y - 4 = 0$
The line l_2 is perpendicular to l_1 and passes through the point $(4, -1)$

Find an equation for l_2 in the form $ax + by + c = 0$

x_1, y_1

$$2x - 4 = 3y$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

$$m = \frac{2}{3} \quad \text{perp } m = -\frac{3}{2}$$

$$y + 1 = -\frac{3}{2}(x - 4)$$

$$2(y + 1) = -3(x - 4)$$

$$2y + 2 = -3x + 12$$

$$\underline{\underline{3x + 2y - 10 = 0}}$$

9 The line l passes through the points $A(1, 4)$ and $B(-2, 13)$.

(a) Find an equation for l . $x_1 \ y_1 \ x_2 \ y_2$

(3)

(b) Find the exact length of AB

(2)

a/

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{13 - 4}{-2 - 1}$$

$$= \frac{9}{-3}$$

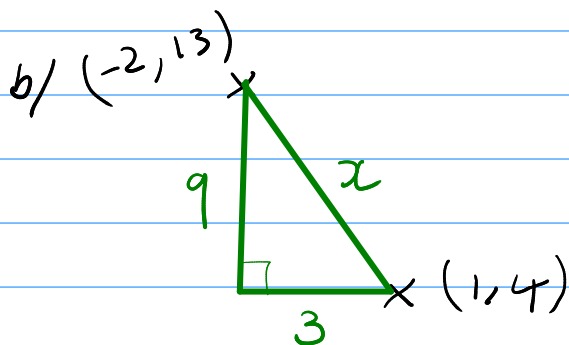
$$= \underline{\underline{-3}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -3(x - 1)$$

$$y - 4 = -3x + 3$$

$$\underline{\underline{y = -3x + 7}}$$



$$3^2 + 9^2 = x^2$$

$$90 = x^2$$

$$x = \sqrt{90}$$

$$= \underline{\underline{3\sqrt{10}}}$$

10 The line l_1 has gradient 3 and passes through $(-2, 5)$.

(a) Find an equation for l_1 in the form $y = mx + c$

(2)

l_2 is perpendicular to l_1 and passes through $(0, 4)$

(b) Find an equation for l_2 .

(2)

(c) Find the coordinates of the point where l_1 and l_2 intersect.

(3)

$$a/ \quad y - 5 = 3(x + 2)$$

$$y - 5 = 3x + 6$$

$$\underline{\underline{y = 3x + 11}}$$

$$b/ \quad \text{perp } m = -\frac{1}{3}$$

$$\underline{\underline{y = -\frac{1}{3}x + 4}}$$

c/ Intersection where:

$$3x + 11 = -\frac{1}{3}x + 4$$

$$\frac{10}{3}x = -7$$

$$x = \frac{-21}{10}$$

$$y = 3\left(\frac{-21}{10}\right) + 11$$

$$= \frac{47}{10}$$

$$\underline{\underline{\left(\frac{-21}{10}, \frac{47}{10}\right)}}$$

- 11 The line l_1 has the equation $5y - 10 = 2x$
 The point P with x coordinate 4 lies on l_1 .
 The line l_2 is perpendicular to l_1 and passes through the point P .

(a) Find an equation for l_2 in the form $ax + by + c = 0$

(4)

The lines l_1 and l_2 cross the x axis at the points Q and R respectively.

(b) Calculate the area of the triangle QPR .

(4)

$$l_1: \begin{aligned} 5y - 10 &= 2x \\ 5y &= 2x + 10 \\ y &= \frac{2}{5}x + 2 \quad \left(m = \frac{2}{5}\right) \end{aligned}$$

$$\begin{aligned} \text{when } x=4 \quad y &= \frac{2}{5}(4) + 2 \\ &= \frac{18}{5} \end{aligned}$$

$$\text{perp } m = -\frac{5}{2} \quad \begin{matrix} (4, 18/5) \\ x_1 \quad y_1 \end{matrix}$$

$$y - \frac{18}{5} = -\frac{5}{2}(x - 4)$$

$$2\left(y - \frac{18}{5}\right) = -5(x - 4)$$

$$2y - \frac{36}{5} = -5x + 20$$

$$10y - 36 = -25x + 100$$

$$\underline{25x + 10y - 136 = 0}$$

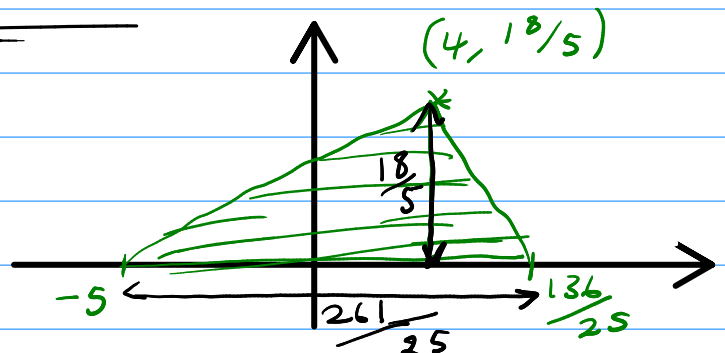
b/ cross x when $y=0$

$$25x - 136 = 0$$

$$x = \frac{136}{25}$$

and $-10 = 2x$

$$x = -5$$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot \frac{261}{25} \cdot \frac{18}{5} \\ &= \frac{2349}{125} = \underline{\underline{18.792}} \end{aligned}$$

12 Three of the following points lie on the same straight line.

Which point does **not** lie on this line?

Tick **one** box.

- (2, -1)
- (-2, 11)
- (-1, 7)
- (1, 2)

$$\begin{matrix} (2, -1) & (-2, 11) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$$m = \frac{11 - (-1)}{-2 - 2} = \frac{12}{-4} = -3$$

$$y + 1 = -3(x - 2)$$

$$y + 1 = -3x + 6$$

$$\underline{y = -3x + 5}$$

$$2 = -3(1) + 5$$

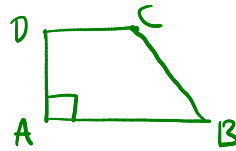
$$2 = 2 \quad \checkmark \quad \checkmark (1, 2)$$

13 ABCD is a trapezium with point A(-2, 5), point B(4, 2) and point C(6, -4).

AB is parallel to DC

AB is perpendicular to AD

$$\begin{matrix} x_1 & y_1 & x_2 & y_2 \end{matrix}$$



(a) Find the equation of CD

(2)

(b) Find the coordinates of D.

(3)

$$AB: \quad m = \frac{2 - 5}{4 - (-2)}$$

$$= \frac{-3}{6} = \underline{-\frac{1}{2}} \quad (\text{gradient of } CD)$$

passes through $(6, -4)$

$$y + 4 = -\frac{1}{2}(x - 6)$$

$$y + 4 = -\frac{1}{2}x + 3$$

$$\underline{y = -\frac{1}{2}x - 1}$$

b/ perp $m = 2$ AD: $y - 5 = 2(x + 2)$ $(-2, 5)$

$$-\frac{1}{2}x - 1 = 2x + 9$$

$$\underline{y = 2x + 9}$$

$$-\frac{5}{2}x = 10$$

$$x = -4$$

when $x = -4$ $y = 2(-4) + 9 = 1$

$$\underline{(-4, 1)}$$

- 14 The point A has the coordinates $(-3, -4)$, point B has the coordinates $(7, 2)$.
 Find the equation the perpendicular bisector of AB

$$\text{Midpoint} = \left(\frac{-3+7}{2}, \frac{-4+2}{2} \right)$$

$$= \left(\underset{x_1}{2}, \underset{y_1}{-1} \right)$$

$$m = \frac{2+4}{7+3} = \frac{6}{10} = \frac{3}{5}$$

$$\text{perp } m = \underline{-\frac{5}{3}}$$

$$y + 1 = -\frac{5}{3}(x - 2)$$

$$3(y + 1) = -5(x - 2)$$

$$3y + 3 = -5x + 10$$

$$3y = -5x + 7$$

$$\underline{\underline{y = -\frac{5}{3}x + \frac{7}{3}}}$$

- 15 The line l_1 has equation $4y - 3x = 11$

The line l_2 passes through the points $(3, 5)$ and $(-5, -1)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

$$l_1: 4y = 3x + 11$$

$$y = \frac{3}{4}x + \frac{11}{4} \quad \underline{m = \frac{3}{4}}$$

$$l_2: m = \frac{-1-5}{-5-3} = \frac{-6}{-8} = \underline{\underline{\frac{3}{4}}}$$

same gradient \therefore parallel

- 16 The point A has the coordinates $(-2, 3)$, point B has the coordinates $(4, -7)$.
 The perpendicular bisector of AB intersects the line $y = 2x + 1$ at the point P .
 Find the coordinates of P .

Midpoint $\left(\frac{-2+4}{2}, \frac{3-7}{2}\right)$
 $(1, -2)$

Gradient of $AB = \frac{-7-3}{4-(-2)} = \frac{-10}{6} = -\frac{5}{3}$

perp $m = \frac{3}{5}$

perpendicular bisector: $y + 2 = \frac{3}{5}(x - 1)$

$5(y+2) = 3(x-1)$

$5y + 10 = 3x - 3$

$5y = 3x - 13$

$y = 2x + 1$

$y = \frac{3}{5}x - \frac{13}{5}$

$2x + 1 = \frac{3}{5}x - \frac{13}{5}$

$10x + 5 = 3x - 13$

$7x = -18$

$x = -\frac{18}{7}$

$y = 2\left(-\frac{18}{7}\right) + 1$
 $= -\frac{29}{7}$

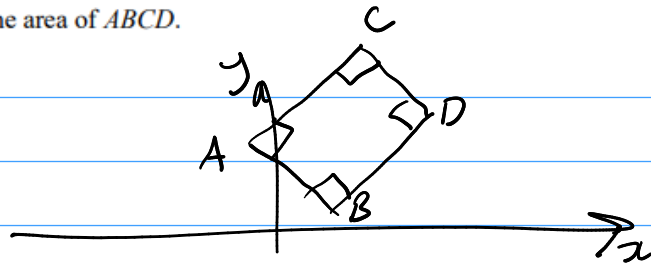
$\left(-\frac{18}{7}, -\frac{29}{7}\right)$

x_1, y_1 x_2, y_2 x_2, y_2 x_2, y_2 17 Points $A(-1, 5)$, $B(1, 1)$, $C(5, 8)$ and $D(7, 4)$ are the vertices of a quadrilateral $ABCD$.(a) Prove that $ABCD$ is a rectangle.

(4)

(b) Find the area of $ABCD$.

(2)



$$AB \text{ gradient} = \frac{1-5}{1+1} = \frac{-4}{2} = -2$$

$$AC \text{ gradient} = \frac{8-5}{5+1} = \frac{3}{6} = \frac{1}{2}$$

$$BD \text{ gradient} = \frac{4-1}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$CD \text{ gradient} = \frac{4-8}{7-5} = \frac{-4}{2} = -2$$

4 sided shape with

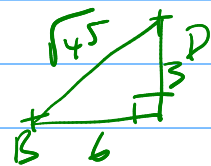
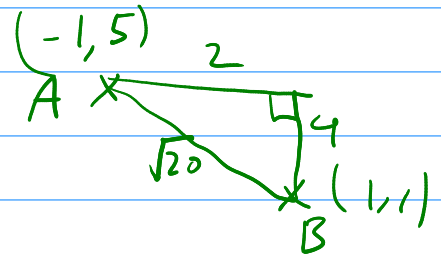
4 right angles AB is perpendicular to AC and BD
 CD is perpendicular to AC and BD

$$\begin{aligned} \text{b/ } AB \text{ length}^2 &= 2^2 + 4^2 \\ &= 20 \\ &= \sqrt{20} \end{aligned}$$

$$BD^2 = 6^2 + 3^2$$

$$BD = \sqrt{45}$$

$$\begin{aligned} \text{Area} &= \sqrt{20} \times \sqrt{45} \\ &= \underline{\underline{30 \text{ units}^2}} \end{aligned}$$



- 18 The line l_1 has equation $2y + 4x + 7 = 0$
The line l_2 has equation $y = mx + 4$, where m is a constant.
Given that l_1 and l_2 are perpendicular.
- (a) Find the value of m . (2)
(b) Find the coordinates of the point where l_1 and l_2 meet. (3)

$$l_1: \quad 2y = -4x - 7$$
$$y = -2x - \frac{7}{2}$$

perpendicular gradient = $\frac{1}{2}$

$$\underline{\underline{m = \frac{1}{2}}}$$

b/ $y = -2x - \frac{7}{2}$ and $y = \frac{1}{2}x + 4$

$$-2x - \frac{7}{2} = \frac{1}{2}x + 4$$

$$-4x - 7 = x + 8$$

$$-15 = 5x$$

$$\underline{\underline{x = -3}}$$

$$y = \frac{1}{2}(-3) + 4$$

$$= \underline{\underline{\frac{5}{2}}}$$

$$\underline{\underline{\left(-3, \frac{5}{2}\right)}}$$

- 19 In 1960 the life expectancy in the UK was 71 years
In 1975 the life expectancy in the UK was 73 years
Given that x years is the life expectancy n years after 1960.

(a) Using a linear model, form an equation linking x with n . (3)

In 2020 the life expectancy in the UK was 81.8 years.

(b) Comment on the suitability of your model in light of this information (3)

$$\begin{array}{cc} (0, 71) & (15, 73) \\ n_1, x_1 & n_2, x_2 \end{array}$$

$$m = \frac{73 - 71}{15} = \frac{2}{15}$$

$$x = \frac{2}{15}n + 71$$

$$\begin{aligned} \text{b/ } n = 60 \quad x &= \frac{2}{15}(60) + 71 \\ &= 79 \end{aligned}$$

The model underestimated the life expectancy
The model may not be suitable.

- 21 Worldwide CO₂ emissions in 1990 were 22.5 billion tonnes.
Worldwide CO₂ emissions in 2010 were 33.6 billion tonnes.

Given that A billion tonnes is the CO₂ emissions n years after 1990.

(a) Using a linear model, form an equation linking A with n .

(3)

In 2016 worldwide CO₂ emissions were 35.7 billion tonnes.

(b) Comment on the suitability of your model in light of this information

(3)

$$\begin{array}{cc} (0, 22.5) & (20, 33.6) \\ n_1, A_1 & n_2, A_2 \end{array}$$

$$m = \frac{33.6 - 22.5}{20} = 0.555$$

$$\underline{\underline{A = 0.555n + 22.5}}$$

b/

$$\begin{aligned} A &= 0.555(26) + 22.5 \\ &= 36.93 \end{aligned}$$

The model overestimated the emissions.

\therefore The model may not be suitable.

- 22 Point C has coordinates $(2, c)$ and point D has coordinates $(d, 8)$
The perpendicular bisector of CD has equation $3y + x = 10$
Find c and d.

$$3y + x = 10$$

$$3y = -x + 10$$

$$y = -\frac{1}{3}x + \frac{10}{3}$$

perpendicular gradient = 3

$$3 = \frac{8 - c}{d - 2}$$

$$3(d - 2) = 8 - c$$

$$3d - 6 = 8 - c$$

$$\underline{c + 3d = 14}$$

$$\text{Midpoint} = \left(\frac{2+d}{2}, \frac{c+8}{2} \right)$$

$$3y + x = 10$$

$$3\left(\frac{c+8}{2}\right) + \frac{2+d}{2} = 10$$

$$3(c+8) + 2+d = 20$$

$$3c + 24 + 2 + d = 20$$

$$\underline{3c + d = -6}$$

$$c + 3d = 14$$

$$3c + d = -6$$

$$\underline{\underline{c = -4}}$$

$$\underline{\underline{d = 6}}$$