1 A discrete random variable X has the probability function:

x	0	1	2	3
$P(X = x)$	0.2	a 0.25	0.3	0.25

(a) Find the value of a

(b) Find $P(X > 1.2)$

(c) Construct a table for the cumulative distribution $F(x)$

a/ $1 - 0.2 - 0.3 - 0.25 = \underline{\underline{0.25}}$

b/ $0.3 + 0.25 = \underline{\underline{0.55}}$

c/

x	0	1	2	3
$F(x)$	0.2	0.45	0.75	1

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2 A random variable X has the probability function:

$$P(X = x) = \frac{(2x - 1)}{36} \qquad x = 1, 2, 3, 4, 5, 6$$

(a) Construct a table giving the probability function of X.

(b) Find $P(1.4 < X < 3.9)$

(c) Construct a table for the cumulative distribution $F(x)$

a/

x	1	2	3	4	5	6
$P(X = x)$	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$

b/ $\frac{3}{36} + \frac{5}{36} = \underline{\underline{\frac{8}{36}}}$

c/

x	1	2	3	4	5	6
$F(x)$	$1/36$	$4/36$	$9/36$	$16/36$	$25/36$	1

3 A fair 6 sided die is rolled. The random variable Y represents the score on the die.

(a) Construct a table giving the probability function of Y.

(b) Write down the name of this distribution

a)

y	1	2	3	4	5	6
$P(Y=y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b) uniform distribution

4 A discrete random variable X has the probability distribution:

x	0	1	2	3
$P(X=x)$	0.2	a 0.28	0.3	b 0.22

Where a and b are constants.

The cumulative distribution $F(x)$ of X is given below.

x	0	1	2	3
$F(x)$	c 0.2	d 0.48	0.78	e 1

Where c, d and e are constants.

Find the values of a, b, c, d and e.

$c = 0.2$, $e = 1$

$0.2 + a + 0.3 = 0.78$

$a = 0.28$

$1 - 0.2 - 0.28 - 0.3 = 0.22 = b$

$d = 0.48$

5 The discrete random variable $X \sim B(15, 0.35)$

Find: $N = 15$ $p = 0.35$

(a) $P(X = 5)$
(b) $P(X < 4)$
(c) $P(X \leqslant 10)$

a/ Binomial PD $P(X=5) = \underline{\underline{0.212}}$

b/ Binomial CD $P(X \leq 3) = \underline{\underline{0.173}}$

c/ $P(X \leq 10) = \underline{\underline{0.997}}$

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6 The probability of Harry being late for school is 0.1. Over a term of 30 days find the probability that Harry is late:

$p = 0.1$ $n = 30$

(a) Exactly one time

(b) More than four times

(c) Less than three times

a/ Binomial PD $P(X=1) = \underline{\underline{0.141}}$

b/ $P(X > 4) = 1 - P(X \leq 4)$
$= 1 - 0.825$
$= \underline{\underline{0.175}}$

c/ $P(X \leq 2) = \underline{\underline{0.411}}$

7 A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 1)$ for $r = 1, 3$

$P(X=1) = P(X=2)$
$P(X=3) = P(X=4)$

Given that $P(X = 1) = 0.16$

(a) find the complete probability distribution for X.

Mark spins the spinner 50 times.

(b) Find the probability that the spinner lands on the number 4 more than 20 times.

The random variable $Y = 8 - 3X$

(c) Find $P(Y + X \geq 1)$

a/

x	1	2	3	4
$P(X=x)$	0.16	0.16	0.34	0.34

b/ $p = 0.34 \quad n = 50$

$P(X > 20) = 1 - P(X \leq 20)$
$= 1 - 0.852$
$= \underline{\underline{0.148}}$

c/ when $X = 1 \quad Y = 5 \quad X + Y = 6$
$X = 2 \quad Y = 2 \quad X + Y = 4$
$X = 3 \quad Y = -1 \quad X + Y = 2$
$X = 4 \quad Y = -4 \quad X + Y = 0$

when $x = 1, 2, 3 \quad Y + X \geq 1$

$0.16 + 0.16 + 0.34 = \underline{\underline{0.66}}$

8 A fair 4 sided spinner has sides numbered 1, 2, 3 and 4.

(a) Write down the name of the distribution that can be used to model the number the spinner lands on from each spin. **(1)**

The spinner is spun 40 times.

The random variable X represents the number of times the spinner lands on 3.

(b) Find the probability that the spinner lands on 3 exactly 10 times. **(2)**

(c) Find $P(6 \leq X < 10)$ **(3)**

a/ uniform distribution

b/ $p = 0.25 \quad n = 40$

Binomial PD $\quad P(X = 10) = \underline{\underline{0.144}}$

c/ $P(6 \leq X < 10) = P(X \leq 9) - P(X \leq 5)$

(Binomial CD) $= 0.440 - 0.0433$

$= \underline{\underline{0.396}}$

9 Mustafa plays a game where he can score 0, 5, 10, 15 or 20 points.

The random variable X, representing the number of points scored, has the following probability distribution, where a and b are constants.

x	0	5	10	15	20
$P(X = x)$	a	0.2	b	0.15	0.3

0.25 0.75

The probability of scoring more than 5 is three times the probability of scoring 5 or less.

Mustafa plays the game twice and adds the scores together.
Each game is independent of the previous game.

Calculate the probability that the total score is 25 points.

$$x + 3x = 1$$
$$x = 0.25$$

$$\underline{a = 0.05} \qquad \underline{b = 0.3}$$

$$P(5, 20) = 0.2 \times 0.3 = 0.06$$
$$P(20, 5) = 0.06$$
$$P(10, 15) = 0.3 \times 0.15 = 0.045$$
$$P(15, 10) = 0.045$$

$$0.06 + 0.06 + 0.045 + 0.045 = \underline{\underline{0.21}}$$

10 Richard throws a dart at a target.

He assumes that each throw is independent and that the probability he hits the target is $\frac{1}{5}$

Richard throws 8 darts.

(a) Calculate the probability of at least 3 of these darts hitting the target.

Richard throws 8 darts at the target each day for 3 days.

(b) Calculate the probability that at least 3 darts hit the target for exactly 1 of these days.

a/ $X \sim B(8, 0.2)$

$P(X \geqslant 3) = 1 - P(X \leqslant 2)$

$= 1 - 0.797$

$= \underline{0.203}$

b/ $0.203 \times 0.797 \times 0.797 = 0.129$

$HH'H' = 0.129$

$H'HH' = 0.129$

$H'H'H = 0.129$

$3(0.129) = \underline{0.387}$

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11 Katie plays a game where she can score 0, 1, 2, 3 or 4.

The random variable S, represents the Katie's score. She believes that S can be modelled by a uniform distribution.

Write down the probability distribution for S.

s	0	1	2	3	4
$P(S=s)$	0.2	0.2	0.2	0.2	0.2

12 In a game people throw a ball into a bucket. For each person, the random variable X represents the number of times the ball lands in the bucket in the first 5 throws.

Helen models X as B(5, 0.3)

(a) State an assumption Helen makes to use her model. **(2)**

(b) Using Helen's model, find $P(X = 2)$ **(1)**

For each person the random variable Y represents the throw on which the first ball lands in the bucket.

Using Helen's model,

(c) find $P(Y = 5)$ **(2)**

a/ The probability of each throw is independent
p is always 0.3.

b/ 0.3087

c/ $0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3 = \underline{\underline{0.07203}}$

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13 In a game 1, 2, 3, 4 or 5 points can be scored. The random variable S represents the number of points scored.

$$P(S = n) = 0.05 + (n - 1) \times a$$

where a is a constant.

Find the probability distribution for S.

s	1	2	3	4	5
$P(S=s)$	0.05	$0.05+a$	$0.05+2a$	$0.05+3a$	$0.05+4a$

$$0.25 + 10a = 1$$
$$10a = 0.75$$
$$a = 0.075$$

s	1	2	3	4	5
$P(S=s)$	0.05	0.125	0.2	0.275	0.35

14 Erik is investigating whether the number of goals scored by a football team in a game can be modelled using a binomial distribution.

He uses the random variable X to denote the number of goals scored in a game and believes $X \sim B(10, 0.15)$

Using Erik's model,

(a) Find $P(X \geq 3)$ **(2)**

(b) The team play 38 games, find the expected number of games in which the team will score no goals. **(2)**

Last season the team scored no goals in 7 out of 38 matches and scored 3 or more goals in 6 out of 38 matches.

(c) Explain whether or not your answers to part (a) and (b) support Erik's model. **(1)**

a) $n=10$ $p=0.15$ $P(X \geqslant 3) = 1 - P(X \leqslant 2)$

$= 1 - 0.820$

$= \underline{\underline{0.180}}$

b) $P(X=0) = 0.197$

$0.197 \times 38 = 7.48$ games

$\underline{7 \text{ games}}$

c) $0.18 \times 38 = 6.84$

6.84 is close to 6 and 7.48 is close to 7

This supports Erik's model.

15 The random variable X has the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	k	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{4}$	$\frac{k}{5}$

where k is a constant.

(a) Find the value of k.

The random variables X_1 and X_2 are independent and each have the same distribution as X.

(b) Find $P(X_1 + X_2 = 7)$

a/ $k + \frac{k}{2} + \frac{k}{3} + \frac{k}{4} + \frac{k}{5} = 1$

$60k + 30k + 20k + 15k + 12k = 60$

$137k = 60$

$k = \frac{60}{137}$

b/ $P(2 \text{ and } 5) = 0.0192$

$P(5 \text{ and } 2) = 0.0192$

$P(3 \text{ and } 4) = 0.0160$

$P(4 \text{ and } 3) = 0.0160$

$2(0.0192) + 2(0.0160) = 0.0703$