AS Level Maths: Statistical Distributions

1 A discrete random variable X has the probability function:

x	0	1	2	3
$P(X = x)$	0.2	a	0.3	0.25

(a) Find the value of a **(1)**

(b) Find $P(X > 1.2)$ **(1)**

(c) Construct a table for the cumulative distribution $F(x)$ **(2)**

(Total for question 1 is 4 marks)

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2 A random variable X has the probability function:

$$P(X = x) = \frac{(2x - 1)}{36} \qquad x = 1, 2, 3, 4, 5, 6$$

(a) Construct a table giving the probability function of X. **(2)**

(b) Find $P(1.4 < X < 3.9)$ **(1)**

(c) Construct a table for the cumulative distribution $F(x)$ **(2)**

(Total for question 2 is 5 marks)

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3 A fair 6 sided die is rolled. The random variable Y represents the score on the die.

(a) Construct a table giving the probability function of Y. **(2)**

(b) Write down the name of this distribution **(1)**

(Total for question 3 is 3 marks)

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4 A discrete random variable X has the probability distribution:

x	0	1	2	3
$P(X = x)$	0.2	a	0.3	b

Where a and b are constants.

The cumulative distribution $F(x)$ of X is given below.

x	0	1	2	3
$F(x)$	c	d	0.78	e

Where c, d and e are constants.

Find the values of a, b, c, d and e.

(Total for question 4 is 3 marks)

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5 The discrete random variable $X \sim B(15, 0.35)$

Find:

(a) $P(X = 5)$
(b) $P(X < 4)$
(c) $P(X \leqslant 10)$

(Total for question 5 is 3 marks)

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6 The probability of Harry being late for school is 0.1. Over a term of 30 days find the probability that Harry is late:

(a) Exactly one time

(b) More than four times

(c) Less than three times

(Total for question 6 is 3 marks)

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7 A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 1)$ for $r = 1, 3$

Given that $P(X = 1) = 0.16$

(a) find the complete probability distribution for X. **(2)**

Mark spins the spinner 50 times.

(b) Find the probability that the spinner lands on the number 4 more than 20 times. **(3)**

The random variable $Y = 8 - 3X$

(c) Find $P(Y + X \geq 1)$ **(3)**

(Total for question 7 is 8 marks)

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8 A fair 4 sided spinner has sides numbered 1, 2, 3 and 4.

(a) Write down the name of the distribution that can be used to model the number the spinner lands on from each spin. **(1)**

The spinner is spun 40 times.

The random variable X represents the number of times the spinner lands on 3.

(b) Find the probability that the spinner lands on 3 exactly 10 times. **(2)**

(c) Find $P(6 \leq X < 10)$ **(3)**

(Total for question 8 is 6 marks)

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9 Mustafa plays a game where he can score 0, 5, 10, 15 or 20 points.

The random variable X, representing the number of points scored, has the following probability distribution, where a and b are constants.

x	0	5	10	15	20
$P(X = x)$	a	0.2	b	0.15	0.3

The probability of scoring more than 5 is three times the probability of scoring 5 or less.

Mustafa plays the game twice and adds the scores together.
Each game is independent of the previous game.

Calculate the probability that the total score is 25 points.

(Total for question 9 is 6 marks)

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10 Richard throws a dart at a target.

He assumes that each throw is independent and that the probability he hits the target is $\frac{1}{5}$

Richard throws 8 darts.

(a) Calculate the probability of at least 3 of these darts hitting the target. **(2)**

Richard throws 8 darts at the target each day for 3 days.

(b) Calculate the probability that at least 3 darts hit the target for exactly 1 of these days. **(2)**

(Total for question 10 is 4 marks)

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11 Katie plays a game where she can score 0, 1, 2, 3 or 4.

The random variable S, represents the Katie's score. She believes that S can be modelled by a uniform distribution.

Write down the probability distribution for S.

(Total for question 11 is 2 marks)

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12 In a game people throw a ball into a bucket. For each person, the random variable X represents the number of times the ball lands in the bucket in the first 5 throws.

Helen models X as B(5, 0.3)

(a) State an assumption Helen makes to use her model. **(2)**

(b) Using Helen's model, find P($X = 2$) **(1)**

For each person the random variable Y represents the throw on which the first ball lands in the bucket.

Using Helen's model,

(c) find P($Y = 5$) **(2)**

(Total for question 12 is 5 marks)

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13 In a game 1, 2, 3, 4 or 5 points can be scored. The random variable S represents the number of points scored.

$$P(S = n) = 0.05 + (n - 1) \times a$$

where a is a constant.

Find the probability distribution for S.

(Total for question 13 is 5 marks)

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14 Erik is investigating whether the number of goals scored by a football team in a game can be modelled using a binomial distribution.

He uses the random variable X to denote the number of goals scored in a game and believes $X \sim B(10, 0.15)$

Using Erik's model,

(a) Find P($X \geq 3$) **(2)**

(b) The team play 38 games, find the expected number of games in which the team will score no goals. **(2)**

Last season the team scored no goals in 7 out of 38 matches and scored 3 or more goals in 6 out of 38 matches.

(c) Explain whether or not your answers to part (a) and (b) support Erik's model. **(1)**

(Total for question 14 is 5 marks)

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15 The random variable X has the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	k	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{4}$	$\frac{k}{5}$

where k is a constant.

(a) Find the value of k.

The random variables X_1 and X_2 are independent and each have the same distribution as X.

(b) Find $P(X_1 + X_2 = 7)$

(Total for question 15 is 6 marks)