

GCE Examinations  
Advanced Subsidiary

# Core Mathematics C4

Paper D

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



*Written by Shaun Armstrong*

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## C4 Paper D – Marking Guide

1. (a)  $= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}$  B1  
 $= \frac{1}{8}[1 + (-3)(-\frac{3}{2}x) + \frac{(-3)(-4)}{2}(-\frac{3}{2}x)^2 + \frac{(-3)(-4)(-5)}{3 \times 2}(-\frac{3}{2}x)^3 + \dots]$  M1  
 $= \frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \dots$  A3
- (b)  $|x| < \frac{2}{3}$  B1 **(6)**
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2. (a)  $2x + 3y + 3x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$  M1 A2  
 $\frac{dy}{dx} = \frac{2x+3y}{4y-3x}$  M1 A1
- (b)  $\text{grad} = \frac{6-6}{-8-9} = 0$  M1  
 $\therefore$  normal parallel to  $y$ -axis  $\therefore x = 3$  M1 A1 **(8)**
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3. (a)  $2x^3 - 5x^2 + 6 \equiv (Ax + B)x(x - 3) + C(x - 3) + Dx$  M1  
 $x = 0 \Rightarrow 6 = -3C \Rightarrow C = -2$   
 $x = 3 \Rightarrow 15 = 3D \Rightarrow D = 5$  A1  
coeffs  $x^3 \Rightarrow A = 2$  B1  
coeffs  $x^2 \Rightarrow -5 = B - 3A \Rightarrow B = 1$  M1 A1
- (b)  $= \int_1^2 (2x + 1 - \frac{2}{x} + \frac{5}{x-3}) dx$   
 $= [x^2 + x - 2 \ln|x| + 5 \ln|x-3|]_1^2$  M1 A2  
 $= (4 + 2 - 2 \ln 2 + 0) - (1 + 1 + 0 + 5 \ln 2)$  M1  
 $= 4 - 7 \ln 2$  A1 **(10)**
- 
4. (a)  $\int x dx = \int k(5 - t) dt$  M1  
 $\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c$  M1 A1  
 $t = 0, x = 0 \Rightarrow c = 0$  B1  
 $t = 2, x = 96 \Rightarrow 4608 = 8k, k = 576$  M1 A1  
 $t = 1 \Rightarrow \frac{1}{2}x^2 = 576 \times \frac{9}{2}, x = \sqrt{5184} = 72$  M1 A1
- (b) 3 hours 5 mins  $\Rightarrow t = 3.0833, x = \sqrt{12284} = 110.83$  M1 A1  
 $\therefore \frac{dx}{dt} = \frac{576(5-3.0833)}{110.83} = 9.96, \frac{dx}{dt} < 10$  so she should have left M1 A1 **(12)**
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5. (a)  $\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ a \\ b \end{pmatrix} = 0 \quad \therefore 3 + 4a + 5b = 0$  M1 A1
- (b)  $4 + s = -3 + 3t \quad (1)$   
 $1 + 4s = 1 + at \quad (2)$   
 $1 + 5s = -6 + bt \quad (3)$  B1  
 $(1) \Rightarrow s = 3t - 7$  M1  
sub. (2)  $\Rightarrow 1 + 4(3t - 7) = 1 + at$   
 $12t - 28 = at, \quad t(12 - a) = 28, \quad t = \frac{28}{12 - a}$  M1 A1  
sub. (3)  $\Rightarrow 1 + 5(3t - 7) = -6 + bt$   
 $15t - 28 = bt, \quad t(15 - b) = 28, \quad t = \frac{28}{15 - b}$  A1  
 $\frac{28}{12 - a} = \frac{28}{15 - b}, \quad 12 - a = 15 - b, \quad b = a + 3$  M1  
sub (a)  $\Rightarrow 3 + 4a + 5(a + 3) = 0, \quad a = -2, \quad b = 1$  M1 A1
- (c)  $t = 2 \quad \therefore \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}, \quad \therefore (3, -3, -4)$  M1 A1 (12)

6. (a)  $u^2 = 1 - x \Rightarrow x = 1 - u^2, \quad \frac{dx}{du} = -2u$  M1  
 $x = 0 \Rightarrow u = 1, \quad x = 1 \Rightarrow u = 0$  B1  
area =  $\int_0^1 x\sqrt{1-x} \, dx = \int_1^0 (1 - u^2) \times u \times (-2u) \, du$  M1  
 $= \int_0^1 (2u^2 - 2u^4) \, du$  A1  
 $= [\frac{2}{3}u^3 - \frac{2}{5}u^5]_0^1$  M1 A1  
 $= (\frac{2}{3} - \frac{2}{5}) - (0) = \frac{4}{15}$  M1 A1
- (b)  $= \pi \int_0^1 x^2(1-x) \, dx$  M1  
 $= \pi \int_0^1 (x^2 - x^3) \, dx$   
 $= \pi [\frac{1}{3}x^3 - \frac{1}{4}x^4]_0^1$  M1 A1  
 $= \pi \{(\frac{1}{3} - \frac{1}{4}) - (0)\} = \frac{1}{12} \pi$  M1 A1 (13)

7. (a)  $\frac{dx}{dt} = 6 \cos t \times (-\sin t), \quad \frac{dy}{dt} = 2 \cos 2t$  M1 A1  
 $\frac{dy}{dx} = \frac{2 \cos 2t}{-6 \cos t \sin t} = \frac{2 \cos 2t}{-3 \sin 2t} = -\frac{2}{3} \cot 2t$  M1 A1
- (b)  $-\frac{2}{3} \cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$  M1 A1  
 $\therefore (\frac{3}{2}, 1), (\frac{3}{2}, -1)$  A1
- (c)  $t = \frac{\pi}{6}, \quad x = \frac{9}{4}, \quad y = \frac{\sqrt{3}}{2}, \quad \text{grad} = -\frac{2}{3\sqrt{3}}$  B1  
 $\therefore y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$  M1  
 $6\sqrt{3}y - 9 = -4x + 9$   
 $2x + 3\sqrt{3}y = 9$  A1
- (d)  $y^2 = \sin^2 2t = 4 \sin^2 t \cos^2 t = 4(1 - \cos^2 t)\cos^2 t$  M2  
 $\cos^2 t = \frac{x}{3} \quad \therefore y^2 = 4(1 - \frac{x}{3})\frac{x}{3}, \quad y^2 = \frac{4}{9}x(3 - x)$  M1 A1 (14)

Total (75)

