FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper J

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong

C3 Paper J – Marking Guide

1. *(a)* $\cos^2 x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ — M1

$\cos 2x = 2\cos^2 x - 1 = 2(4 - 2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$ — M1 A1

(b) $2(\cos y \cos 30 - \sin y \sin 30) = \sqrt{3}(\sin y \cos 30 - \cos y \sin 30)$ — M1 A1

$\sqrt{3}\cos y - \sin y = \frac{3}{2}\sin y - \frac{1}{2}\sqrt{3}\cos y$ — B1

$\frac{3}{2}\sqrt{3}\cos y = \frac{5}{2}\sin y$

$\tan y = \frac{3}{2}\sqrt{3} \div \frac{5}{2} = \frac{3}{5}\sqrt{3}$ — M1 A1 **(8)**

---

2. *(a)* $\text{f}(x) = (x - \frac{3}{2})^2 - \frac{9}{4} + 7 = (x - \frac{3}{2})^2 + \frac{19}{4}$ — M1 A1

$\therefore\ \text{f}(x) \geq \frac{19}{4}$ — A1

(b) $= \text{g}(11) = 21$ — M1 A1

(c) $\text{fg}(x) = \text{f}(2x - 1) = (2x - 1)^2 - 3(2x - 1) + 7$ — M1

$\therefore\ 4x^2 - 4x + 1 - 6x + 3 + 7 = 17$

$2x^2 - 5x - 3 = 0$ — A1

$(2x + 1)(x - 3) = 0$ — M1

$x = -\frac{1}{2}, 3$ — A1 **(9)**

---

3. *(a)*

$$\begin{array}{r|rrrrr} & x^2 & +\,4x & -\,4 & & \\ \hline x^2 - 3x + 3 & x^4 & +\,x^3 & -\,13x^2 & +\,26x & -\,17 \\ & x^4 & -\,3x^3 & +\,3x^2 & & \\ \hline & & 4x^3 & -\,16x^2 & +\,26x & \\ & & 4x^3 & -\,12x^2 & +\,12x & \\ \hline & & & -\,4x^2 & +\,14x & -\,17 \\ & & & -\,4x^2 & +\,12x & -\,12 \\ \hline & & & & 2x & -\,5 \end{array}$$

— M1

$\therefore\ \text{f}(x) = x^2 + 4x - 4 + \frac{2x-5}{x^2-3x+3}$, $A = 4, B = -4, C = 2, D = -5$ — A3

(b) $\text{f}'(x) = 2x + 4 + \frac{2\times(x^2-3x+3)-(2x-5)\times(2x-3)}{(x^2-3x+3)^2}$ — M1 A2

$x = 1 \Rightarrow y = -2$, grad $= 5$

$\therefore$ grad of normal $= -\frac{1}{5}$ — M1

$\therefore\ y + 2 = -\frac{1}{5}(x - 1)$ — M1

$5y + 10 = -x + 1$

$x + 5y + 9 = 0$ — A1 **(10)**

---

4. *(a)* $\frac{\text{d}x}{\text{d}y} = \frac{1}{2}\sec\frac{y}{2}\tan\frac{y}{2}$ — M1

$0 \leq y < \pi\ \therefore\ \tan\frac{y}{2} \geq 0\ \therefore\ \frac{\text{d}x}{\text{d}y} = \frac{1}{2}\sec\frac{y}{2}\sqrt{\sec^2\frac{y}{2} - 1} = \frac{1}{2}x\sqrt{x^2 - 1}$ — M1 A1

$\frac{\text{d}y}{\text{d}x} = 1 \div \frac{\text{d}x}{\text{d}y} = \frac{2}{x\sqrt{x^2-1}}$ — M1 A1

(b) $\frac{\text{d}y}{\text{d}x} = \frac{1}{2}(3 + 2\cos x)^{-\frac{1}{2}} \times (-2\sin x) = -\frac{\sin x}{\sqrt{3+2\cos x}}$ — M1 A1

$x = \frac{\pi}{3}$, $y = 2$, grad $= -\frac{1}{4}\sqrt{3}$ — M1 A1

$\therefore\ y - 2 = -\frac{1}{4}\sqrt{3}(x - \frac{\pi}{3})$ $[\ 3\sqrt{3}x + 12y - 24 - \pi\sqrt{3} = 0\ ]$ — M1 A1 **(11)**

5.	*(a)*	$f(x) > 5$	B1	
	(b)	$y = 5 + e^{2x-3}$		
		$2x - 3 = \ln(y - 5)$	M1	
		$x = \frac{1}{2}[3 + \ln(y - 5)]$	M1	
		$\therefore\ f^{-1}(x) = \frac{1}{2}[3 + \ln(x - 5)],\ x \in \mathbb{R},\ x > 5$	A2	
	(c)	$x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$	M1 A1	
	(d)	$f'(x) = 2e^{2x-3}$	M1	
		grad = 4	A1	
		$\therefore\ y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)]$ $[\,y = 4x + 1 - 2\ln 2\,]$	M1 A1	**(11)**

6.	*(a)*	$\text{LHS} \equiv \frac{2\cos 2x}{\sin 2x} + \frac{\sin x}{\cos x}$	M1	
		$\equiv \frac{\cos 2x}{\sin x \cos x} + \frac{\sin x}{\cos x}$	M1	
		$\equiv \frac{\cos 2x + \sin^2 x}{\sin x \cos x}$	A1	
		$\equiv \frac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$	M1	
		$\equiv \frac{\cos^2 x}{\sin x \cos x}$		
		$\equiv \frac{\cos x}{\sin x}$		
		$\equiv \cot x \equiv \text{RHS}$	A1	
	(b)	$\cot x = \text{cosec}^2 x - 7$		
		$\cot x = 1 + \cot^2 x - 7$	M1	
		$\cot^2 x - \cot x - 6 = 0$		
		$(\cot x + 2)(\cot x - 3) = 0$	M1	
		$\cot x = -2$ or 3	A1	
		$\tan x = -\frac{1}{2}$ or $\frac{1}{3}$	M1	
		$x = \pi - 0.4636$ or 0.32		
		$x = 0.32, 2.68$ (2dp)	A2	**(11)**

7.	*(a)*	$f(x) \geq 0$	B1	
	(b)	$= f(0) = 5$	M1 A1	
	(c)	$fg(x) = f[\ln(x + 3)] = \lvert 2\ln(x + 3) - 5 \rvert$	M1	
		$\therefore\ \lvert 2\ln(x + 3) - 5 \rvert = 3$		
		$2\ln(x + 3) = 2, 8$	M1	
		$\ln(x + 3) = 1, 4$	A1	
		$x = e - 3, e^4 - 3$	M1 A1	
	(d)	let $h(x) = f(x) - g(x)$		
		$h(3) = -0.79,\ f(4) = 1.1$	M1	
		sign change, $h(x)$ continuous $\therefore$ root	A1	
	(e)	$x_1 = 3.396, x_2 = 3.428, x_3 = 3.430, x_4 = 3.431$	M1 A2	
	(f)	$h(3.4305) = -0.000052,\ f(3.4315) = 0.0018$	M1	
		sign change, $h(x)$ continuous $\therefore$ root $\therefore\ \alpha = x_4$ to 4sf	A1	**(15)**

Total **(75)**

Performance Record – C3 Paper J

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	trigonometry	functions	rational expressions, differentiation	differentiation	functions, differentiation	trigonometry	functions, exponentials and logs, numerical methods	
Marks	8	9	10	11	11	11	15	75
Student								