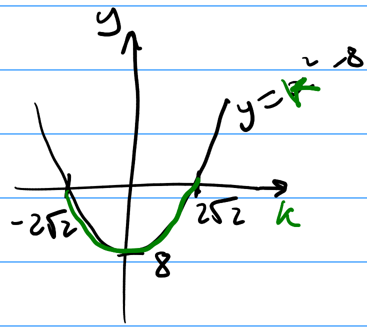
1 The equation $x^2 + kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

$$b^2 - 4ac < 0$$

$$k^2 - 4(1)(2) < 0$$

$$k^2 - 8 < 0$$



$k^2 - 8 = 0 \qquad k = 2\sqrt{2}$ or $k = -2\sqrt{2}$

$$\underline{\underline{2\sqrt{2} < k < 2\sqrt{2}}}$$

---

2 The equation $kx^2 + 5x + k = 0$, where k is a positive constant has equal roots.

Find the value for k.

$$b^2 - 4ac = 0$$

$$(5)^2 - 4(k)(k) = 0$$

$$25 - 4k^2 = 0$$

$$25 = 4k^2$$

$$k^2 = \frac{25}{4}$$

$$k = \pm\frac{5}{2}$$

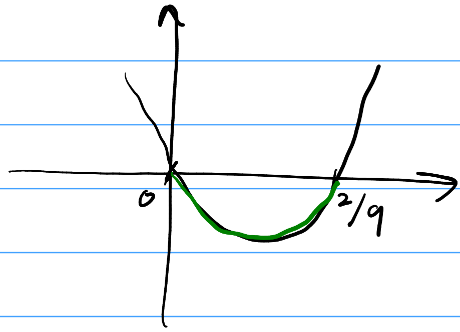
k is positive $\therefore$ $\underline{\underline{k = \frac{5}{2}}}$

3 The equation $kx^2 + 6kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

$$b^2 - 4ac < 0$$

$$(6k)^2 - 4(k)(2) < 0$$
$$36k^2 - 8k < 0$$
$$4k(9k - 2) < 0$$



$k = 0 \quad k = 2/9$

$$0 < k < 2/9$$

---

4 Find the set of values for which the equation $ax^2 + 7x + 4 = 0$ has no real roots.

$$b^2 - 4ac < 0$$

$$(7)^2 - 4a(4) < 0$$
$$49 - 16a < 0$$
$$49 < 16a$$
$$a > \frac{49}{16}$$

---

5 Show that the equation $2x^2 + 5 = 6x$ has no real roots.

$$2x^2 - 6x + 5 = 0$$

$$b^2 - 4ac = (-6)^2 - 4(2)(5)$$
$$= 36 - 40$$
$$= -4$$

As $b^2 - 4ac < 0$ $2x^2 + 5 = 6x$ has no real roots.

6 The equation $(k+5)x^2 + 4x + (k+2) = 0$, where k is a constant has two distinct real solutions for x.

Find the set of possible values for k.

$$b^2 - 4ac > 0$$

$$(4)^2 - 4(k+5)(k+2) > 0$$
$$16 - 4(k^2 + 2k + 5k + 10) > 0$$
$$16 - 4(k^2 + 7k + 10) > 0$$
$$16 - 4k^2 - 28k - 40 > 0$$
$$0 > 4k^2 + 28k + 24$$
$$0 > k^2 + 7k + 6$$
$$0 > (k+6)(k+1)$$
$$k = -6 \qquad k = -1$$

$$\underline{k < -6} \quad \text{or} \quad \underline{k > -1}$$

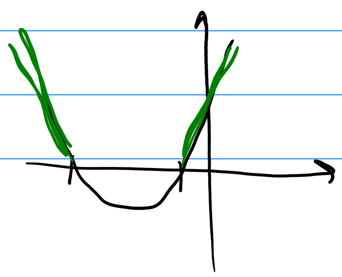
7 The equation $x^2 + (n+1)x + (3-3n) = 0$, where n is a constant has two distinct real roots.

Find the set of possible values for n.

$$b^2 - 4ac > 0$$

$$(n+1)^2 - 4(1)(3-3n) > 0$$
$$(n+1)(n+1) - 12 + 12n > 0$$
$$n^2 + n + n + 1 - 12 + 12n > 0$$
$$n^2 + 14n - 11 > 0$$

$a = 1 \quad b = 14 \quad c = -11 \qquad n = -7 + 2\sqrt{15} \qquad n = -7 - 2\sqrt{15}$



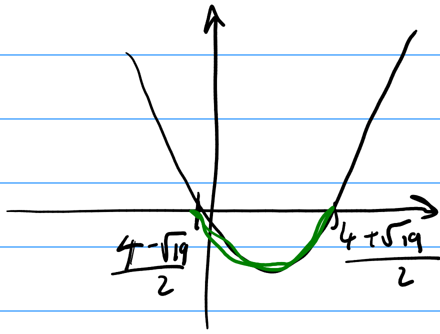
$$\underline{n < -7 - 2\sqrt{15}} \quad \text{or} \quad \underline{n > -7 + 2\sqrt{15}}$$

8 The equation $x^2 + (2k-3)x + (k+3) = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

$$b^2 - 4ac < 0$$

$$(2k-3)^2 - 4(1)(k+3) < 0$$
$$(2k-3)(2k-3) - 4k - 12 < 0$$
$$4k^2 - 6k - 6k + 9 - 4k - 12 < 0$$
$$4k^2 - 16k - 3 < 0$$



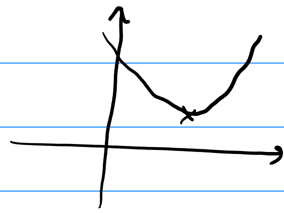
$$k = \frac{4+\sqrt{19}}{2} \qquad k = \frac{4-\sqrt{19}}{2}$$

$$\frac{4-\sqrt{19}}{2} < k < \frac{4+\sqrt{19}}{2}$$

---

9 Show that $x^2 - 6x + 11 > 0$ for all real values of x.

$$(x-3)^2 - 9 + 11$$



$$(x-3)^2 + 2$$

The minimum point is at $(3,2)$. (Positive quadratic with turning point at $(3,2)$). $\therefore x^2 - 6x + 11 > 0$

---

10 Prove that, for all values of x,

$$x^2 + 4x + 12 > x + 3$$

rearrange to give: $x^2 + 3x + 9 > 0$

$$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 9$$

$$\left(x + \frac{3}{2}\right)^2 + \frac{27}{4}$$ Positive quadratic $\therefore$ min point at $\left(\frac{-3}{2}, \frac{27}{4}\right)$

$\therefore x^2 + 3x + 9$ is always greater than 0.

11 (a) By completing the square, find in terms of the constant p the roots of the equation

$$x^2 + px + 4 = 0$$

(b) Hence find the set of values of p for which the equation has no real roots.

a) $\left(x + \frac{p}{2}\right)^2 - \frac{p^2}{4} + 4 = 0$

$$\left(x + \frac{p}{2}\right)^2 = \frac{p^2}{4} - 4$$

$$x + \frac{p}{2} = \sqrt{\frac{p^2}{4} - 4}$$

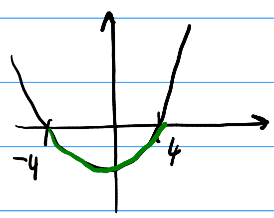
$$x = -\frac{p}{2} \pm \sqrt{\frac{p^2 - 4}{4}}$$

$$\boxed{\begin{aligned} \text{or} \quad x &= \frac{p}{2} \pm \sqrt{\frac{p^2}{4} - \frac{16}{4}} \\ &= \frac{p}{2} \pm \frac{\sqrt{p^2 - 16}}{2} \\ &= \frac{p \pm \sqrt{p^2 - 16}}{2} \end{aligned}}$$

b) No real roots where $\frac{p^2}{4} - 4 < 0$

$$p^2 - 16 < 0$$

$$(p+4)(p-4) < 0$$



$$\underline{\underline{-4 < p < 4}}$$

12 The curve C has the equation

$$x^2 + ax + b = 0$$

Where a and b are constants

Given that the minimum point of C has coordinates (4, –3) find the values of a and b.

$$\left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + b = 0 \qquad (x-4)^2 - 3$$

$$\frac{a}{2} = -4 \qquad -\frac{a^2}{4} + b = -3$$

$$\underline{\underline{a = -8}} \qquad -\frac{64}{4} + b = -3$$

$$-16 + b = -3$$

$$\underline{\underline{b = 13}}$$

---

13 $f(x) = 2x^2 + 8x + 1$

Find the values of the constants a, b and c such that

$f(x) = a(x + b)^2 + c$

$$f(x) = 2(x^2 + 4x) + 1$$

$$= 2[(x+2)^2 - 4] + 1$$

$$= 2(x+2)^2 - 8 + 1$$

$$= 2(x+2)^2 - 7$$

$$\underline{\underline{a = 2}} \qquad \underline{\underline{b = 2}} \qquad \underline{\underline{c = -7}}$$

---

14 (a) Express $x^2 + 9x + 3$ in the form $(x + a)^2 + b$

(b) State the coordinates of the minimum point of the curve $y = x^2 + 9x + 3$

a) $\left(x + \frac{9}{2}\right)^2 - \frac{81}{4} + 3$

$\left(x + \frac{9}{2}\right)^2 - \frac{69}{4}$

b) $\left(-\frac{9}{2}, -\frac{69}{4}\right)$

15 (a) By completing the square, find in terms of the constant k the roots of the equation

$$x^2 + kx - 6 = 0$$

(b) Hence find the exact roots of the equation $x^2 + 4x - 6 = 0$

a) $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} - 6 = 0$

$$\left(x + \frac{k}{2}\right)^2 = \frac{k^2}{4} + 6$$

$$x + \frac{k}{2} = \pm\sqrt{\frac{k^2}{4} + 6}$$

$$x = \frac{-k}{2} \pm \sqrt{\frac{k^2}{4} + 6}$$

b) $k=4$ $\therefore x = \frac{-4}{2} \pm \sqrt{\frac{16}{4} + 6}$

$$= \underline{\underline{-2 \pm \sqrt{10}}}$$

---

16 The curve ***C*** has the equation $y = \frac{2}{x} + k \qquad x \in \mathbb{R}, x \neq 0$

The line ***L*** has the equation $y = -3x + 2$

(a) Show that the x-coordinate of any point of intersection of ***L*** with ***C*** is given by a solution of the equation

$$3x^2 + (k - 2)x + 2 = 0$$ **(2)**

(b) Hence find the exact values of k for which ***L*** is a tangent of ***C***. **(3)**

a) Intersection where $\frac{2}{x} + k = -3x + 2$

$$2 + kx = -3x^2 + 2x$$

$$3x^2 + kx - 2x + 2 = 0$$

$$3x^2 + (k-2)x + 2 = 0$$

b) Tangent where there is one solution $\therefore b^2 - 4ac = 0$

$$(k-2)^2 - 4(3)(2) = 0$$

$$k^2 - 4k + 4 - 24 = 0$$

$$k^2 - 4k - 20 = 0$$

$$\underline{\underline{k = 2 \pm 2\sqrt{6}}}$$

17 This model can be used to predict the fuel consumption of a car, y miles per gallon, for a car travelling at x miles per hour.

$$y = 55 - 0.015(x - 50)^2$$

Using this model,

(a) Calculate the fuel consumption, in miles per gallon, of a car travelling at 70 miles per hour. **(2)**

(b) Deduce the speed at which the car has the greatest fuel efficiency. **(1)**

(c) State, giving reasons, the limitation on the values of x **(2)**

a/ $x = 70$ $\quad y = 55 - 0.015(20)^2$

$= 49$ miles per gallon

b/ 50 mph

c/ The number of miles travelled per gallon cannot be negative.

$0 = 55 - 0.015(x - 50)^2$

$(x - 50)^2 = \frac{55}{0.015}$

$x = 50 \pm \sqrt{\frac{55}{0.015}}$

$x = 111$ and $-10.6 \leftarrow$ cannot be negative.

The model is only suitable for $0 < x < 111$.

18 A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 18x + 4$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

The curve has a maximum turning point at P.

(b) Find the coordinates of P.

a/ $f(x) = -3(x^2 - 6x) + 4$

$= -3[(x - 3)^2 - 9] + 4$

$= -3(x - 3)^2 + 27 + 4$

$= \underline{\underline{-3(x - 3)^2 + 31}}$

b/ $\underline{\underline{(3, 31)}}$

19 A curve with equation $y = a(x+b)^2 + c$ has a minimum turning point at (3, 8)

(a) State the values of b and c.

When the curve is translated by the vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ the curve passes through the point (-6, 13).

(b) Find the value of a.

a/ $b = -3 \quad c = 8$

b/ $y = a(x-3)^2 + 8$

4 to the left $\therefore$ $y = a((x+4)-3)^2 + 8$

$y = a(x+1)^2 + 8$

$(-6, 13)$ $\quad 13 = a(-6+1)^2 + 8$

$5 = 25a$

$\underline{a = \frac{1}{5}}$

---

20 (a) Express $2x^2 + 9x + k$ in the form $a(x+b)^2 + c$

(b) Find the values of k for which the curve $y = 2x^2 + 9x + k$ does **not** intersect the line $y = 2$

a/ $2\left(x^2 + \frac{9}{2}x\right) + k$

$2\left[\left(x + \frac{9}{4}\right)^2 - \frac{81}{16}\right] + k$

$2\left(x + \frac{9}{4}\right)^2 - \frac{81}{8} + k$

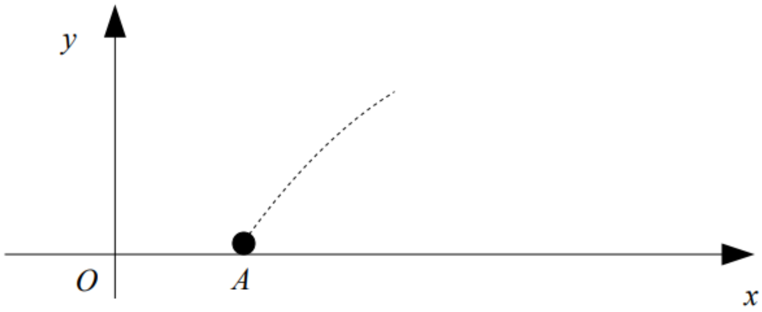
b/ minimum point must be above 2.

Min point $\left(-\frac{9}{4}, k - \frac{81}{8}\right)$

$k - \frac{81}{8} > 2$

$\underline{\underline{k > \frac{97}{8}}}$

21



The path of a projectile, projected from the point A, can be modelled by the equation

$$y = -0.25x^2 + 2.125x - 4.125$$

(a) Find the coordinates of the point A

(b) Find the distance **from** $\boldsymbol{A}$ that the projectile hits the ground.

(c) Calculate the maximum vertical height of the projectile

a/ $-0.25x^2 + 2.125x - 4.125 = 0$

$x = 3$ and $x = 5.5$

$\therefore$ a must be at $\underline{(3, 0)}$

b/ $5.5 - 3 = \underline{2.5}$

c/ $-0.25x^2 + 2.125x - 4.125 = 0$

Max point at midpoint of 3 and 5.5 $\therefore$ $x = 4.25$

$-0.25(4.25)^2 + 2.125(4.25) - 4.125 = \frac{25}{64}$

$\underline{\underline{\frac{25}{64}}}$

22 The equation $x^2 - 5x + k = 0$ has repeated roots. Find the value of the constant k.

$$b^2 - 4ac = 0$$

$$(-5)^2 - 4(1)(k) = 0$$

$$25 - 4k = 0$$

$$25 = 4k$$

$$k = \frac{25}{4}$$

---

23 (a) Express $4x^2 - 8x + 5$ in the form $a(x+b)^2 + c$

(b) State the number of real roots of the equation $4x^2 - 8x + 5$

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2 + r = 0$ where p, q and r are real constants and $p > 0$

a/ $4(x^2 - 2x) + 5$

$4[(x-1)^2 - 1] + 5$

$4(x-1)^2 - 4 + 5$

$4(x-1)^2 + 1$

b/ $(-8)^2 - 4(4)(5)$

$64 - 80$

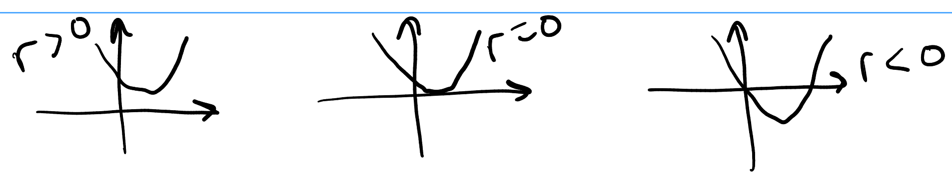
-16

$b^2 - 4ac < 0$ $\therefore$ no real roots

c/ when $r > 0$ There are no real roots as the minimum point is above the x axis

when $r = 0$ One real root (min. point on x-axis)

when $r < 0$ Two real roots



24 (a) Express $3x^2 - 15x + 4$ in the form $a(x+b)^2 + c$

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 15x + 4$

(c) State the equation of the normal to the curve $y = 3x^2 - 15x + 4$ at its minimum point.

a/ $3(x^2 - 5x) + 4$

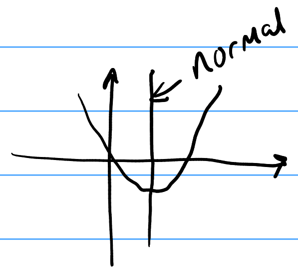
$3\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 4$

$3\left(x - \frac{5}{2}\right)^2 - \frac{75}{4} + 4$

$3\left(x - \frac{5}{2}\right)^2 - \frac{59}{4}$

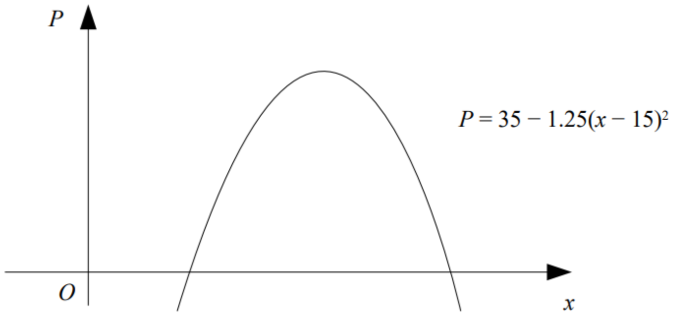
b/ $\left(\frac{5}{2}, -\frac{59}{4}\right)$

c/



$x = \frac{5}{2}$

25



A football club selling tickets for a football match.

The profit made by the football club is modelled by the equation

$$P = 35 - 1.25(x - 15)^2$$

where P is the profit measured in thousands of pounds and x is the price of each ticket in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £25 is not a sensible price for a ticket. **(2)**

Given that the football club made a profit of more than £20 000

(b) find, according to the model, the least possible price of a ticket. **(3)**

The football club wishes to maximise its profit.

State, according to the model,

(c) (i) the maximum possible profit,

(ii) the ticket price that maximises the profit. **(3)**

a/ $P = 35 - 1.25(25-15)^2$

$= 35 - 125$

$= -90$

They will make a loss.

b/ $20 < 35 - 1.25(x-15)^2$

Find the points where $P = 20$

$20 = 35 - 1.25(x-15)^2$

$-15 = -1.25(x-15)^2$

$12 = (x-15)^2$

$\pm\sqrt{12} = x - 15$

$x = 15 \pm \sqrt{12}$

$= 15 \pm 2\sqrt{3}$

Minimum price $= 15 - 2\sqrt{3} = £11.54$

c/ i/ £35000 ii/ £15