AS Level Maths: Quadratics

1 The equation $x^2 + kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

(Total for question 1 is 3 marks)

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2 The equation $kx^2 + 5x + k = 0$, where k is a positive constant has equal roots.

Find the value for k.

(Total for question 2 is 3 marks)

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3 The equation $kx^2 + 6kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

(Total for question 3 is 4 marks)

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4 Find the set of values of a for which the equation $ax^2 + 7x + 4 = 0$ has no real roots.

(Total for question 4 is 3 marks)

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5 Show that the equation $2x^2 + 5 = 6x$ has no real roots.

(Total for question 5 is 3 marks)

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6 The equation $(k + 5)x^2 + 4x + (k + 2) = 0$, where k is a constant has two distinct real solutions for x.

Find the set of possible values for k.

(Total for question 6 is 7 marks)

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7 The equation $x^2 + (n + 1)x + (3 – 3n) = 0$, where n is a constant has two distinct real roots.

Find the set of possible values for n.

(Total for question 7 is 7 marks)

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8 The equation $x^2 + (2k – 3)x + (k + 3) = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

(Total for question 8 is 8 marks)

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9 Show that $x^2 - 6x + 11 > 0$ for all real values of x.

(Total for question 9 is 3 marks)

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10 Prove that, for all values of x,

$$x^2 + 4x + 12 > x + 3$$

(Total for question 10 is 4 marks)

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11 (a) By completing the square, find in terms of the constant p the roots of the equation **(3)**

$$x^2 + px + 4 = 0$$

(b) Hence find the set of values of p for which the equation has no real roots. **(1)**

(Total for question 11 is 4 marks)

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12 The curve C has the equation

$$x^2 + ax + b = 0$$

Where a and b are constants

Given that the minimum point of C has coordinates $(4, -3)$ find the values of a and b.

(Total for question 12 is 4 marks)

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13 $f(x) = 2x^2 + 8x + 1$

Find the values of the constants a, b and c such that

$f(x) = a(x + b)^2 + c$

(Total for question 13 is 4 marks)

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14 (a) Express $x^2 + 9x + 3$ in the form $(x + a)^2 + b$ **(2)**

(b) State the coordinates of the minimum point of the curve $y = x^2 + 9x + 3$ **(2)**

(Total for question 14 is 4 marks)

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15 (a) By completing the square, find in terms of the constant k the roots of the equation **(4)**

$$x^2 + kx - 6 = 0$$

(b) Hence find the exact roots of the equation $x^2 + 4x - 6 = 0$ **(2)**

(Total for question 15 is 6 marks)

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16 The curve ***C*** has the equation $y = \frac{2}{x} + k \qquad x \in \mathbb{R}, x \neq 0$

The line ***L*** has the equation $y = -3x + 2$

(a) Show that the x-coordinate of any point of intersection of ***L*** with ***C*** is given by a solution of the equation

$$3x^2 + (k - 2)x + 2 = 0$$ **(2)**

(b) Hence find the exact values of k for which ***L*** is a tangent of ***C***. **(3)**

(Total for question 16 is 5 marks)

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17 This model can be used to predict the fuel consumption of a car, y miles per gallon, for a car travelling at x miles per hour.

$$y = 55 - 0.015(x - 50)^2$$

Using this model,

(a) Calculate the fuel consumption, in miles per gallon, of a car travelling at 70 miles per hour. **(2)**

(b) Deduce the speed at which the car has the greatest fuel efficiency. **(1)**

(c) State, giving reasons, the limitation on the values of x **(2)**

(Total for question 17 is 5 marks)

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18 A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 18x + 4$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found. **(3)**

The curve has a maximum turning point at P.

(b) Find the coordinates of P. **(2)**

(Total for question 18 is 5 marks)

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19 A curve with equation $y = a(x + b)^2 + c$ has a minimum turning point at (3, 8)

(a) State the values of b and c. **(2)**

When the curve is translated by the vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ the curve passes through the point (-6, 13)

(b) Find the value of a. **(3)**

(Total for question 19 is 5 marks)

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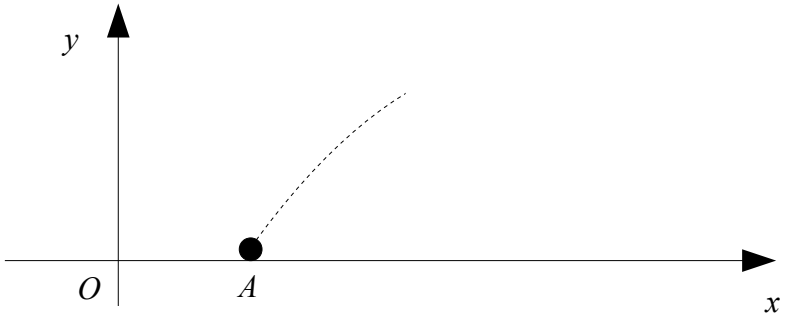
20 (a) Express $2x^2 + 9x + k$ in the form $a(x + b)^2 + c$ **(3)**

(b) Find the values of k for which the curve $y = 2x^2 + 9x + k$ does **not** intersect the line $y = 2$ **(3)**

(Total for question 20 is 6 marks)

21



The path of a projectile, projected from the point A, can be modelled by the equation

$$y = -0.25x^2 + 2.125x - 4.125$$

(a) Find the coordinates of the point A **(3)**

(b) Find the distance **from** ***A*** that the projectile hits the ground. **(1)**

(c) Calculate the maximum vertical height of the projectile **(4)**

(Total for question 21 is 8 marks)

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22 The equation $x^2 - 5x + k = 0$ has repeated roots. Find the value of the constant k.

(Total for question 22 is 2 marks)

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23 (a) Express $4x^2 - 8x + 5$ in the form $a(x + b)^2 + c$ **(3)**

(b) State the number of real roots of the equation $4x^2 - 8x + 5$ **(1)**

(c) Explain fully how the value of r is related to the number of real roots of the equation $p(x + q)^2 + r = 0$ where p, q and r are real constants and $p > 0$ **(2)**

(Total for question 23 is 6 marks)

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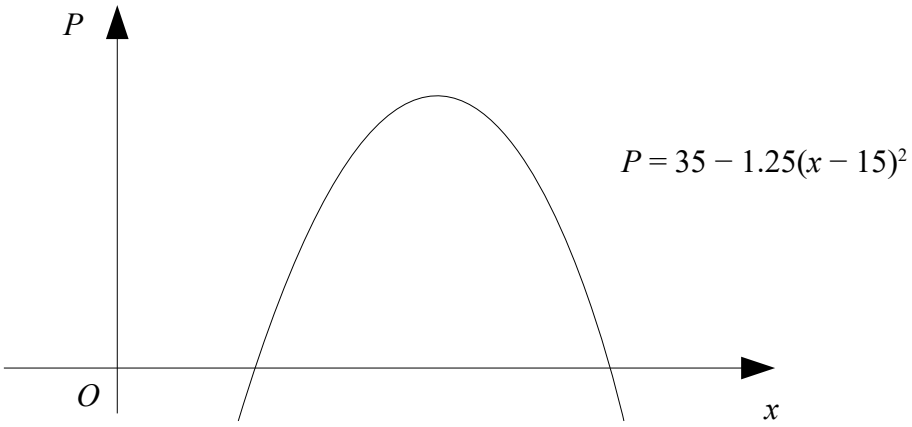
24 (a) Express $3x^2 - 15x + 4$ in the form $a(x + b)^2 + c$ **(3)**

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 15x + 4$ **(2)**

(c) State the equation of the normal to the curve $y = 3x^2 - 15x + 4$ at its minimum point. **(1)**

(Total for question 24 is 6 marks)

25



A football club selling tickets for a football match.

The profit made by the football club is modelled by the equation

$$P = 35 - 1.25(x - 15)^2$$

where P is the profit measured in thousands of pounds and x is the price of each ticket in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £25 is not a sensible price for a ticket. **(2)**

Given that the football club made a profit of more than £20 000

(b) find, according to the model, the least possible price of a ticket. **(3)**

The football club wishes to maximise its profit.

State, according to the model,

(c) (i) the maximum possible profit,

(ii) the ticket price that maximises the profit. **(3)**

(Total for question 25 is 8 marks)