

1  $y = 2x^3 + 5x^2 - 7x + 10$

Find  $\int y \, dx$

$$\int y \, dx = \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{7x^2}{2} + 10x + C$$

$$= \frac{1}{2}x^4 + \frac{5}{3}x^3 - \frac{7}{2}x^2 + 10x + C$$

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2 Find  $\int (3x^2 + 7x - 2) \, dx$

$$\frac{3x^3}{3} + \frac{7x^2}{2} - 2x + C$$

$$x^3 + \frac{7}{2}x^2 - 2x + C$$

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3 Find  $\int (x+4)(x-3) \, dx$

$$\int x^2 - 3x + 4x - 12 \, dx$$

$$\int x^2 + x - 12 \, dx$$

$$\frac{x^3}{3} + \frac{x^2}{2} - 12x + C$$

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + C$$

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4  $f'(x) = 6x^2 - 3x + 8$

Given that the point (1, 8) lies on  $y = f(x)$

Find an expression for  $f(x)$

$$f(x) = \frac{6x^3}{3} - \frac{3x^2}{2} + 8x + C$$
$$= 2x^3 - \frac{3}{2}x^2 + 8x + C$$

$$y = 2x^3 - \frac{3}{2}x^2 + 8x + C$$

$$8 = 2(1)^3 - \frac{3}{2}(1)^2 + 8(1) + C$$

$$8 = \frac{11}{2} + C$$

$$C = -\frac{1}{2}$$

$$\underline{\underline{y = 2x^3 - \frac{3}{2}x^2 + 8x - \frac{1}{2}}}$$

5  $y = 4\sqrt{x} + \frac{1}{x^2} + 10$

Find  $\int y \, dx$

$$y = 4x^{\frac{1}{2}} + x^{-2} + 10$$

$$\int y \, dx = \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + 10x + C$$

$$\underline{\underline{= \frac{8}{3}x^{\frac{3}{2}} - x^{-1} + 10x + C}}$$

6 Find  $\int_1^3 (x+4)(x-3) dx$

$$\int_1^3 x^2 + x - 12 dx$$

$$\left[ \frac{x^3}{3} + \frac{x^2}{2} - 12x \right]_1^3$$

$$\left( \frac{(3)^3}{3} + \frac{(3)^2}{2} - 12(3) \right) - \left( \frac{(1)^3}{3} + \frac{(1)^2}{2} - 12(1) \right)$$

$$= \underline{\underline{\frac{-34}{3}}}$$

7  $\frac{dy}{dx} = 10x^4 - 5$

Given that the point (2, 30) lies on the curve

Find an expression for y in terms of x

$$y = \frac{10x^5}{5} - 5x + C$$

$$y = 2x^5 - 5x + C$$

$$30 = 2(2)^5 - 5(2) + C$$

$$30 = 54 + C$$

$$C = -24$$

$$\underline{\underline{y = 2x^5 - 5x - 24}}$$

8 Find  $\int_1^4 5 + \frac{1}{\sqrt{x}} dx$

$$\int_1^4 5 + x^{-\frac{1}{2}} dx$$
$$\left[ 5x + 2x^{\frac{1}{2}} \right]_1^4$$
$$\left( 5(4) + 2(4)^{\frac{1}{2}} \right) - \left( 5(1) + 2(1)^{\frac{1}{2}} \right)$$

$$\underline{\underline{17}}$$

9 The curve with the equation  $f(x)$  passes through the point  $(1, 2)$

Given that  $f'(x) = 5 + \frac{3x^2 + 2}{x^{\frac{1}{2}}}$

Find  $f(x)$  giving your answer in its simplest form.

$$f'(x) = 5 + 3x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

$$f(x) = 5x + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

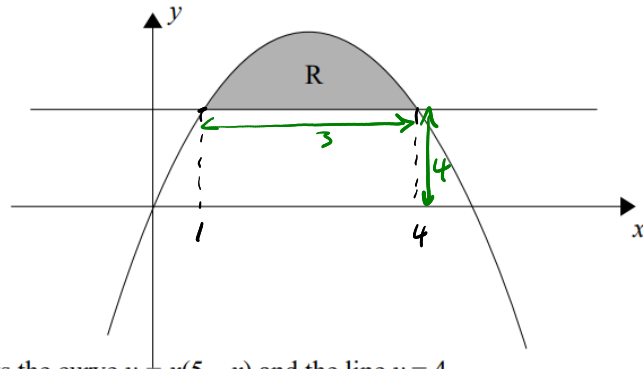
$$= 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$$

$$2 = 5(1) + \frac{6}{5}(1)^{\frac{5}{2}} + 4(1)^{\frac{1}{2}} + C$$

$$C = -\frac{41}{5}$$

$$\underline{\underline{f(x) = 5x + \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - \frac{41}{5}}}$$

10



$$3 \times 4 = 12$$

The sketch shows the curve  $y = x(5 - x)$  and the line  $y = 4$

(a) Find the coordinates of the points where the line intersects the curve. (2)

(b) Find the area of the shaded region R. (6)

a/

$$4 = x(5 - x)$$

$$4 = 5x - x^2$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$\underline{x = 1} \quad \underline{x = 4}$$

$$\underline{\underline{(1, 4)}} \quad \text{and} \quad \underline{\underline{(4, 4)}}$$

b/ Area =  $\int_1^4 (5x - x^2) dx - 12$

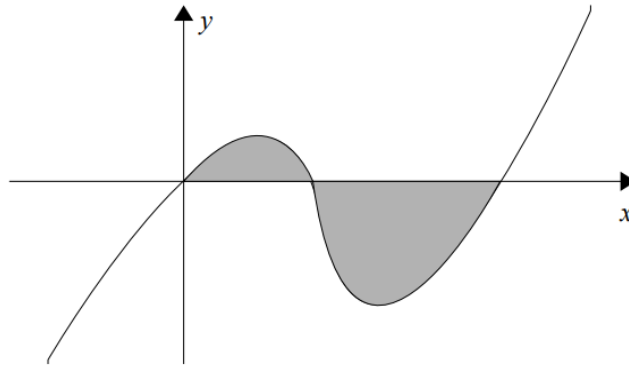
$$= \left[ \frac{5}{2}x^2 - \frac{1}{3}x^3 \right]_1^4 - 12$$

$$= \left( \frac{5}{2}(4)^2 - \frac{1}{3}(4)^3 \right) - \left( \frac{5}{2}(1)^2 - \frac{1}{3}(1)^3 \right) - 12$$

$$= \frac{33}{2} - 12$$

$$= \underline{\underline{\frac{9}{2} \text{ units}^2}}$$

11



The sketch shows the curve  $y = x(x-2)(x-5)$

(a) Write down the values of  $x$  where the curve crosses the  $x$  axis.

(1)

(b) Find the area of the shaded region.

(8)

$$a/ \quad x=0 \quad x=2 \quad x=5$$

$$\int_0^2 x(x-2)(x-5) dx$$

$$= \frac{16}{3}$$

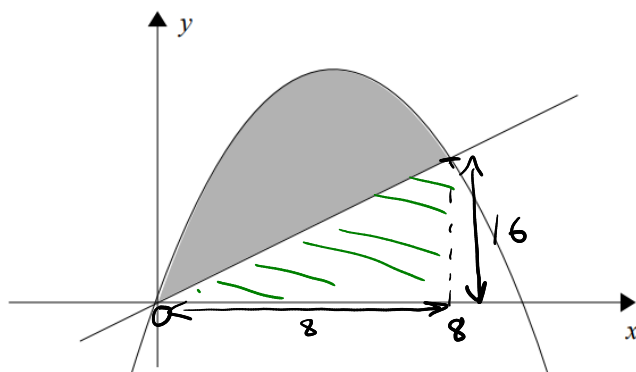
$$\int_2^5 x(x-2)(x-5) dx$$

$$= \frac{63}{4}$$

$$\text{Total Area} = \frac{16}{3} + \frac{63}{4}$$

$$= \frac{253}{12} \text{ units}^2$$

12



The sketch shows the curve  $y = 10x - x^2$  and the straight line  $y = 2x$

(a) Find the coordinates of the points where the line intersects the curve.

(2)

(b) Find the area of the shaded region.

(6)

$$\begin{aligned}
 \text{a/} \quad 10x - x^2 &= 2x \\
 0 &= x^2 - 8x \\
 0 &= x(x - 8) \\
 x &= 0 \quad x = 8 \\
 y &= 0 \quad y = 16 \\
 & (0, 0) \text{ and } (8, 16)
 \end{aligned}$$

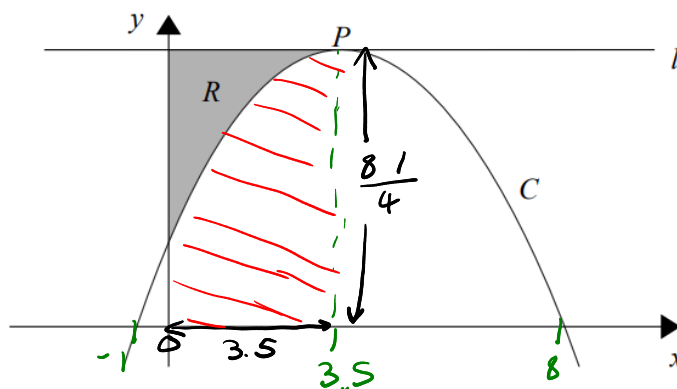
$$\begin{aligned}
 \text{b/} \quad \text{Area of triangle} &= \frac{1}{2} \cdot 8 \cdot 16 \\
 &= 64 \text{ units}^2
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^8 (10x - x^2) dx \\
 & \left[ 5x^2 - \frac{1}{3}x^3 \right]_0^8
 \end{aligned}$$

$$\frac{448}{3} \text{ units}^2$$

$$\begin{aligned}
 \text{Shaded Area} &= \frac{448}{3} - 64 \\
 &= \underline{\underline{\frac{256}{3} \text{ units}^2}}
 \end{aligned}$$

13



The sketch shows the curve  $C$  with equation  $y = (x + 1)(8 - x)$

The maximum point of curve  $C$  is  $P$ .

The line  $l$  passes through  $P$  and is parallel to the  $x$ -axis.

The shaded region  $R$  is bounded by the curve  $C$ , the line  $l$  and the  $y$ -axis.

Find the area of the shaded region  $R$ .

Max point when  $x = 3.5$  (middle of  $-1$  and  $8$ )

$$y = (3.5 + 1)(8 - 3.5)$$

$$= \frac{81}{4}$$

$$P: \left( \frac{7}{2}, \frac{81}{4} \right)$$

$$\text{Area of rectangle} = \frac{7}{2} \cdot \frac{81}{4} = \frac{567}{8} \text{ units}^2$$

$$\int_0^{\frac{7}{2}} (x+1)(8-x) dx = \frac{679}{12} \text{ units}^2$$

$$\text{Shaded Area} = \frac{567}{8} - \frac{679}{12} = \underline{\underline{\frac{343}{24} \text{ units}^2}}$$



14  $\frac{dy}{dx} = \frac{1}{3x^2}$

Find an expression for  $y$  in terms of  $x$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2}$$

$$\underline{y = -\frac{1}{3} x^{-1} + C}$$

15  $f'(x) = (2x - 1)^2$  and  $f(2) = 5$

Find an expression for  $f(x)$

$$f'(x) = 4x^2 - 4x + 1$$

$$f(x) = \frac{4}{3} x^3 - 2x^2 + x + C$$

$$5 = \frac{4}{3}(2)^3 - 2(2)^2 + 2 + C$$

$$5 = \frac{14}{3} + C$$

$$C = \frac{1}{3}$$

$$\underline{f(x) = \frac{4}{3} x^3 - 2x^2 + x + \frac{1}{3}}$$

- 16 A curve cuts the  $x$ -axis at  $(2, 0)$  and has the gradient function  $\frac{dy}{dx} = \frac{8}{x^2}$   
Find the equation of the curve

$$\frac{dy}{dx} = 8x^{-2}$$

$$y = -8x^{-1} + C$$

$$0 = -8(2)^{-1} + C$$

$$0 = -4 + C$$

$$C = 4$$

$$\underline{\underline{y = -8x^{-1} + 4}}$$

- 17 A curve  $C$  has equation  $y = \frac{3}{2\sqrt{x}}$   
The region enclosed between the curve, the  $x$ -axis and the lines  $x = 1$  and  $x = k$  has area 12 units.

Given  $k > 1$ , find the value of  $k$ .  
Fully justify your answer.

$$y = \frac{3}{2} x^{-\frac{1}{2}}$$

$$\int_1^k \frac{3}{2} x^{-\frac{1}{2}} dx = 12$$

$$\left[ 3x^{\frac{1}{2}} \right]_1^k = 12$$

$$3k^{\frac{1}{2}} - 3(1)^{\frac{1}{2}} = 12$$

$$3\sqrt{k} - 3 = 12$$

$$\sqrt{k} = 5$$

$$\underline{\underline{k = 25}}$$

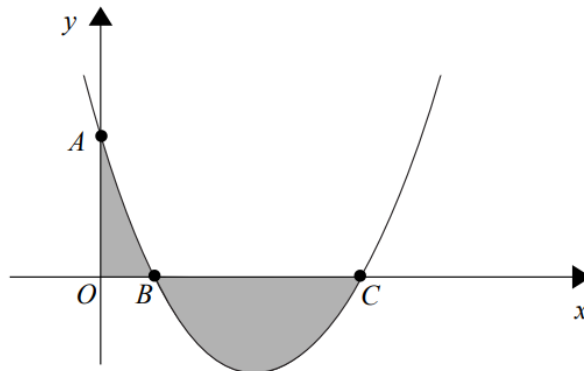
$$18 \quad \frac{dy}{dx} = \frac{5}{x^2}$$

Find an expression for  $y$

$$\frac{dy}{dx} = 5x^{-2}$$

$$\underline{\underline{y = -5x^{-1} + C}}$$

19



The diagram shows the curve with equation  $y = x^2 - 6x + 5$

(a) Write down the coordinates of the points  $A$ ,  $B$  and  $C$ .

(b) Find the total area of the two shaded regions.

Fully justify your answer.

a/  $A: (0, 5)$  (when  $x = 0$ ,  $y = 5$ )

$$0 = x^2 - 6x + 5 \quad (\text{crosses } x \text{ when } y = 0)$$

$$0 = (x - 5)(x - 1)$$

$$x = 5 \quad x = 1$$

$B: (1, 0)$        $C: (5, 0)$

b/  $\int_0^1 x^2 - 6x + 5 \, dx = \frac{7}{3}$

$$\int_1^5 x^2 - 6x + 5 \, dx = -\frac{32}{3}$$

$$\underline{\underline{\frac{7}{3} + \frac{32}{3} = 13 \text{ units}^2}}$$

20 Find  $\int (4\sqrt{x} - 2) dx$

writing your answer in its simplest form.

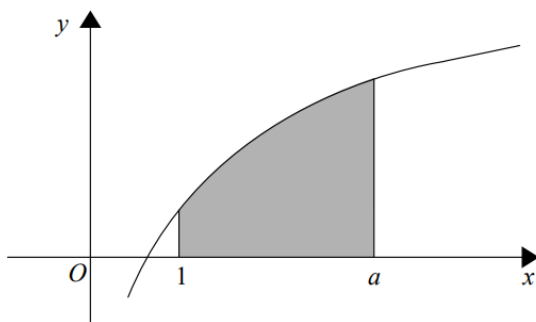
$$\int 4x^{\frac{1}{2}} - 2 dx$$

$$\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + C$$

$$\frac{8}{3}x^{\frac{3}{2}} - 2x + C$$

21 The diagram shows part of the graph of  $y = 8x^{\frac{1}{3}} - \frac{4}{x^{\frac{1}{3}}}$

The shaded region is enclosed by the curve, the x-axis and the lines  $x = 1$  and  $x = a$ , where  $a > 1$



Given that the area of the shaded area is 36 square units, find the exact value of  $a$ .

$$y = 8x^{\frac{1}{3}} - 4x^{-\frac{1}{3}}$$

$$\int_1^a 8x^{\frac{1}{3}} - 4x^{-\frac{1}{3}} dx = 36$$

$$\left[ 6x^{\frac{4}{3}} - 6x^{\frac{2}{3}} \right]_1^a = 36$$

$$\left( 6a^{\frac{4}{3}} - 6a^{\frac{2}{3}} \right) - \left( 6(1)^{\frac{4}{3}} - 6(1)^{\frac{2}{3}} \right) = 36$$

$$6a^{\frac{4}{3}} - 6a^{\frac{2}{3}} = 36$$

$$a^{\frac{4}{3}} - a^{\frac{2}{3}} = 6$$

$$a^{\frac{4}{3}} - a^{\frac{2}{3}} - 6 = 0$$

$$\left( a^{\frac{2}{3}} - 3 \right) \left( a^{\frac{2}{3}} + 2 \right) = 0$$

$$a^{\frac{2}{3}} = 3 \quad \leftarrow$$

$$a^2 = 27$$

$$a = \sqrt{27}$$

$$a^{\frac{2}{3}} = -2$$

No sols.

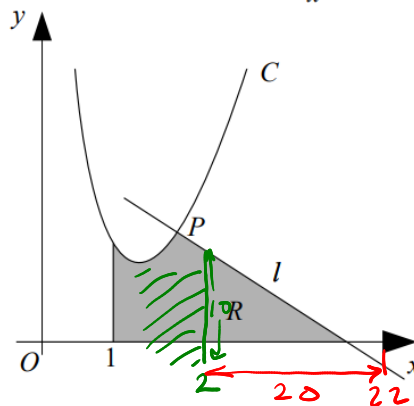
22 Find  $\int (2x^3 + \sqrt{x} + 2) dx$

Giving your answer in its simplest form.

$$\int 2x^3 + x^{\frac{1}{2}} + 2 dx$$

$$\underline{\underline{\frac{1}{2}x^4 + \frac{2}{3}x^{\frac{3}{2}} + 2x + c}}$$

23 The graph shows part of the curve with equation  $y = \frac{8}{x^2} + 4x$



$P(2, 10)$  lies on  $C$ .  
Line  $l$  is the normal to  $C$  at  $P$ .

Region  $R$  is bounded by the curve  $C$ , the line  $l$  and the line with equation  $x = 1$ .  
Show that the area of  $R$  is 110.

$$y = 8x^{-2} + 4x$$

$$\frac{dy}{dx} = -16x^{-3} + 4$$

when  $x = 2$   $\frac{dy}{dx} = 2$   $m = -\frac{1}{2}$  (perp.)

$$l: y - 10 = -\frac{1}{2}(x - 2)$$

$$2y - 20 = -(x - 2)$$

$$2y - 20 = -x + 2$$

$$2y = -x + 22$$

crosses  $x$  when  $y = 0$   $x = 22$

Area of triangle =  $\frac{1}{2} \cdot 20 \cdot 10 = 100 \text{ units}^2$

$$\int_1^2 8x^{-2} + 4x dx = 10 \text{ units}^2$$

$$100 + 10 = \underline{\underline{110 \text{ units}^2}}$$

24 (a) Given that  $k$  is a constant, find  $\int \left( \frac{k}{x^3} + 2x \right) dx$

Giving your answer in its simplest form.

(3)

(b) Find the value of  $k$  such that  $\int_1^2 \left( \frac{k}{x^3} + 2x \right) dx = 12$

(3)

$$a/ \int kx^{-3} + 2x \, dx$$

$$\underline{\underline{-\frac{1}{2}kx^{-2} + x^2 + c}}$$

$$b/ \left[ -\frac{1}{2}kx^{-2} + x^2 \right]_1^2 = 12$$

$$\left( -\frac{1}{2}k(2)^{-2} + (2)^2 \right) - \left( -\frac{1}{2}k(1)^{-2} + (1)^2 \right) = 12$$

$$\left( -\frac{1}{8}k + 4 \right) - \left( -\frac{1}{2}k + 1 \right) = 12$$

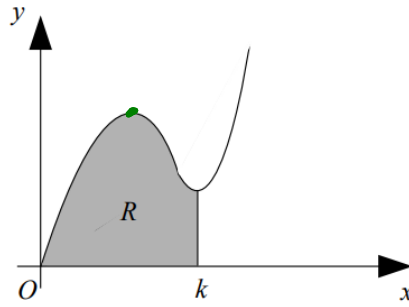
$$-\frac{1}{8}k + 4 + \frac{1}{2}k - 1 = 12$$

$$\frac{3}{8}k + 3 = 12$$

$$\frac{3}{8}k = 9$$

$$\underline{\underline{k = 24}}$$

- 25 The graph shows part of the curve  $C$  with equation  $y = x^3 - 7x^2 + 15x$



The curve has a minimum turning point at  $k$ .

Region  $R$  is bounded by the curve  $C$ ,  $x$ -axis and the line with equation  $x = k$ .

Show that the area of  $R$  is  $\frac{99}{4}$

(Total for question 25 is 7 marks)

turning point where  $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 14x + 15$$

$$3x^2 - 14x + 15 = 0$$

$$\underline{\underline{x = 3}} \quad x = \frac{5}{3} \rightarrow \text{Maximum (seen on graph)}$$

$$\int_0^3 x^3 - 7x^2 + 15x \, dx$$

$$\left[ \frac{1}{4}x^4 - \frac{7}{3}x^3 + \frac{15}{2}x^2 \right]_0^3$$

$$\left( \frac{1}{4}(3)^4 - \frac{7}{3}(3)^3 + \frac{15}{2}(3)^2 \right) - (0)$$

$$\underline{\underline{\frac{99}{4} \text{ units}^2}}$$

26 Given that  $k$  is a positive constant and  $\int_1^k \left( \frac{3}{2\sqrt{x}} + 9 \right) dx = 8$

(a) Show that  $9k + 3\sqrt{k} - 20 = 0$

(b) Hence, using algebra, find any values of  $k$  such that  $\int_1^k \left( \frac{3}{2\sqrt{x}} + 9 \right) dx = 8$

$$\int_1^k \frac{3}{2} x^{-\frac{1}{2}} + 9 dx = 8$$

$$\left[ 3x^{\frac{1}{2}} + 9x \right]_1^k = 8$$

$$(3k^{\frac{1}{2}} + 9k) - (3(1)^{\frac{1}{2}} + 9(1)) = 8$$

$$3k^{\frac{1}{2}} + 9k - 3 - 9 = 8$$

$$3k^{\frac{1}{2}} + 9k - 20 = 0$$

$$9k + 3\sqrt{k} - 20 = 0$$

b/  $(3\sqrt{k} - 4)(3\sqrt{k} + 5) = 0$

$$\sqrt{k} = \frac{4}{3} \quad \sqrt{k} = -\frac{5}{3}$$

$$k = \frac{16}{9} \quad \underline{\underline{\text{no sol}}}$$



$$f(x) = 2x^3 - x^2 - 8x + 4$$

- (a) Use the factor theorem to show that  $(x + 2)$  is a factor of  $f(x)$  (2)
- (b) Hence, showing all your working, write  $f(x)$  as a product of three linear factors. (4)

The finite region  $R$  is bounded by the curve with equation  $y = f(x)$  and the  $x$ -axis, and lies below the  $x$ -axis

- (c) Find, using algebraic integration, the exact value of the area of  $R$  (4)

a/

$$f(-2) = 2(-2)^3 - (-2)^2 - 8(-2) + 4$$

$$= 0$$

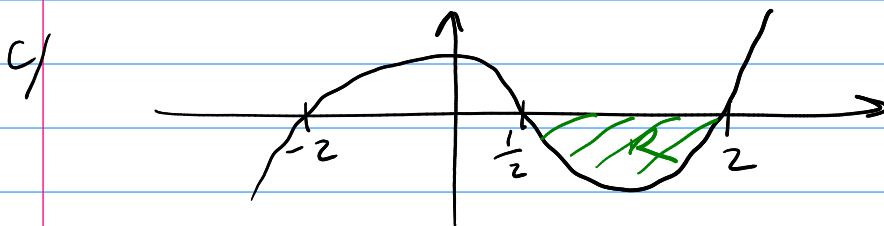
$$f(-2) = 0 \quad \therefore (x + 2) \text{ is a factor}$$

b/

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x + 2 \overline{) 2x^3 - x^2 - 8x + 4} \\ \underline{2x^3 + 4x^2} \phantom{+ 4} \\ -5x^2 - 8x \phantom{+ 4} \\ \underline{-5x^2 - 10x} \phantom{+ 4} \\ 2x + 4 \\ \underline{2x + 4} \\ 0 \end{array}$$

$$(x + 2)(2x^2 - 5x + 2)$$

$$(x + 2)(2x - 1)(x - 2)$$



$$\int_{\frac{1}{2}}^2 (2x^3 - x^2 - 8x + 4) dx$$

$$\left[ \frac{1}{2}x^4 - \frac{1}{3}x^3 - 4x^2 + 4x \right]_{\frac{1}{2}}^2$$

$$\left( \frac{1}{2}(4)^4 - \frac{1}{3}(4)^3 - 4(4)^2 + 4(4) \right) - \left( \frac{1}{2}\left(\frac{1}{2}\right)^4 - \frac{1}{3}\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) \right)$$

$$\frac{117}{32} \text{ units}^2$$

28 Find  $\int \left( \frac{4x^3 - 5}{3x^2} \right) dx$

writing your answer in its simplest form.

$$\int \frac{4x^3}{3x^2} - \frac{5}{3x^2} dx$$

$$\int \frac{4}{3} x^2 - \frac{5}{3} x^{-2} dx$$

$$\underline{\underline{\frac{4}{9} x^3 + \frac{5}{3} x^{-1} + C}}$$

29 Find the value of  $k$  such that  $\int_1^8 \left( \frac{k}{\sqrt[3]{x}} \right) dx = 22.5$

$$\int_1^8 k x^{-\frac{1}{3}} = 22.5$$

$$\left[ \frac{3}{2} k x^{\frac{2}{3}} \right]_1^8 = 22.5$$

$$\frac{3}{2} k (8)^{\frac{2}{3}} - \frac{3}{2} k (1)^{\frac{2}{3}} = 22.5$$

$$6k - \frac{3}{2} k = 22.5$$

$$\frac{9}{2} k = 22.5$$

$$\underline{\underline{k = 5}}$$

30 Find the value of  $k$  such that  $\int_k^9 \left( \frac{10}{\sqrt{x}} \right) dx = 20$

$$\int_k^9 10x^{-\frac{1}{2}} dx = 20$$

$$\left[ 20x^{\frac{1}{2}} \right]_k^9 = 20$$

$$20(9)^{\frac{1}{2}} - 20k^{\frac{1}{2}} = 20$$

$$60 - 20k^{\frac{1}{2}} = 20$$

$$40 = 20k^{\frac{1}{2}}$$

$$2 = k^{\frac{1}{2}}$$

$$\underline{\underline{k = 4}}$$

31 (a) Find  $\int (2x - x^2) dx$

(b) Evaluate  $\int_0^4 (2x - x^2) dx$

(c) Using a sketch, explain why the integral in part (b) does not give the area enclosed between the curve  $y = 2x - x^2$  and the  $x$ -axis

a/

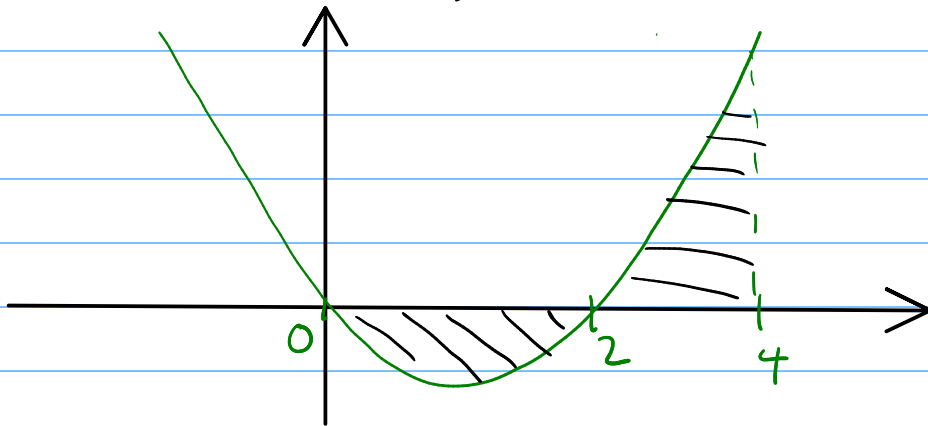
$$\frac{2x^2}{2} - \frac{x^3}{3} + C$$
$$\underline{\underline{x^2 - \frac{1}{3}x^3 + C}}$$

b/

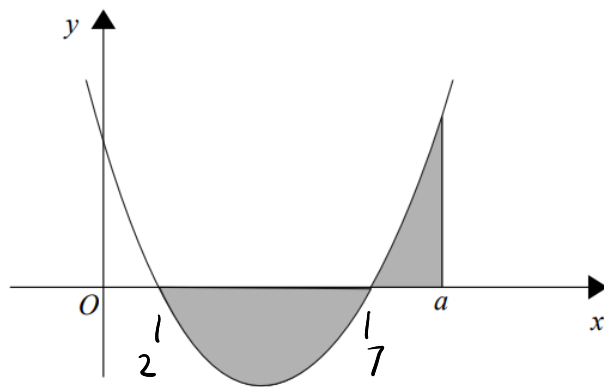
$$\left[ x^2 - \frac{1}{3}x^3 \right]_0^4$$
$$\left( (4)^2 - \frac{1}{3}(4)^3 \right) - (0)$$
$$\underline{\underline{-\frac{16}{3}}}$$

c/

$$y = 2x - x^2$$
$$= x(2 - x)$$



- between 0 and 4 some of the area is above and some below the  $x$  axis.
- the answer will be the difference between the two areas.



The diagram shows the curve with equation  $y = x^2 - 9x + 14$  and the line  $x = a$

Given the total area of the two shaded regions is 29 units<sup>2</sup>

Find the exact value of  $a$ .

Crosses  $x$  when  $y = 0$

$$0 = x^2 - 9x + 14$$

$$0 = (x - 2)(x - 7)$$

$$x = 2 \quad x = 7$$

$$\int_2^7 x^2 - 9x + 14 \, dx = \underline{\underline{\frac{125}{6} \text{ units}^2}}$$

$$29 - \frac{125}{6} = \frac{49}{6} \text{ units}^2$$

$$\int_7^a x^2 - 9x + 14 \, dx = \frac{49}{6}$$

$$\left[ \frac{1}{3}x^3 - \frac{9}{2}x^2 + 14x \right]_7^a = \frac{49}{6}$$

$$\left( \frac{1}{3}a^3 - \frac{9}{2}a^2 + 14a \right) - \left( \frac{1}{3}(7)^3 - \frac{9}{2}(7)^2 + 14(7) \right) = \frac{49}{6}$$

$$\frac{1}{3}a^3 - \frac{9}{2}a^2 + 14a = 0$$

$$a \left( \frac{1}{3}a^2 - \frac{9}{2}a + 14 \right) = 0$$

$$a = 0 \quad a = \frac{27 + \sqrt{57}}{4} \quad a = \frac{27 - \sqrt{57}}{4}$$

$a$  must be  $> 7$   
 $\therefore a = \frac{27 + \sqrt{57}}{4}$

33 Find  $\int \left(10 - \frac{6}{x^2}\right) dx$

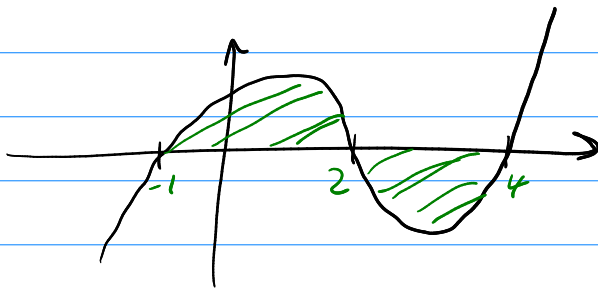
writing your answer in its simplest form.

$$\int 10 - 6x^{-2} dx$$

$$\underline{\underline{10x + 6x^{-1} + C}}$$

34  $f(x) = (x-4)(x-2)(x+1)$

Find the area enclosed by  $f(x)$  and the  $x$ -axis

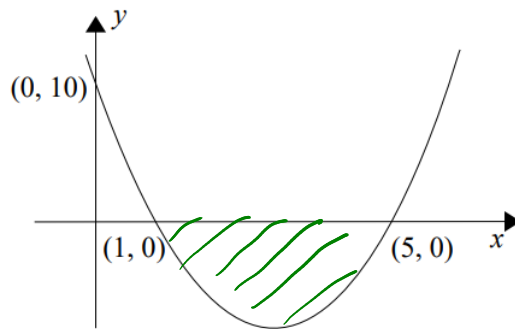


$$\int_2^4 (x-4)(x-2)(x+1) dx = -\frac{16}{3}$$

$$\int_{-1}^2 (x-4)(x-2)(x+1) dx = \frac{63}{4}$$

$$\frac{16}{3} + \frac{63}{4} = \underline{\underline{\frac{253}{12} \text{ units}^2}}$$

35



The sketch shows a quadratic graph that passes through  $(0, 10)$ ,  $(1, 0)$  and  $(5, 0)$

Find the area of the finite region bounded by the curve and the  $x$ -axis.

$$y = k(x-1)(x-5) \quad (0, 10)$$

$$10 = k(-1)(-5)$$

$$\underline{k = 2}$$

$$\int_1^5 2(x-1)(x-5) \, dx$$

$$\underline{\underline{\frac{64}{3} \text{ units}^2}}$$

36 Show that  $\int_1^8 \left(\frac{16}{x^2}\right) dx = 14$

$$\int_1^8 16x^{-2} \, dx$$

$$\left[-16x^{-1}\right]_1^8$$

$$-16(8)^{-1} - -16(1)^{-1}$$

$$-2 + 16$$

$$\underline{\underline{14}}$$

37 Show that  $\int_1^4 (1 + 3\sqrt{x}) dx = 17$

$$\int_1^4 1 + 3x^{\frac{1}{2}} dx$$
$$\left[ x + 2x^{\frac{3}{2}} \right]_1^4$$
$$\left( 4 + 2(4)^{\frac{3}{2}} \right) - \left( 1 + 2(1)^{\frac{3}{2}} \right)$$
$$20 - 3$$
$$\underline{\underline{17}}$$

---

38 (a) Find  $\int x^2 \left( 2x + \frac{25}{\sqrt{x}} \right) dx$

(b) Find  $\int_1^9 x^2 \left( 2x + \frac{25}{\sqrt{x}} \right) dx$

a/

$$\int x^2 \left( 2x + 25x^{-\frac{1}{2}} \right) dx$$
$$\int 2x^3 + 25x^{\frac{3}{2}} dx$$
$$\underline{\underline{\frac{1}{2}x^4 + 10x^{\frac{5}{2}} + C}}$$

b/

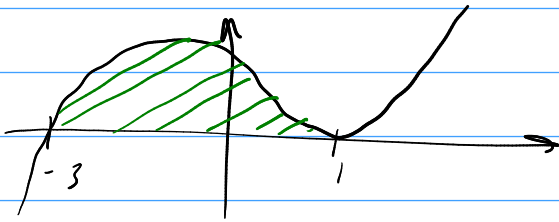
$$\left[ \frac{1}{2}x^4 + 10x^{\frac{5}{2}} \right]_1^9$$
$$\left( \frac{1}{2}(9)^4 + 10(9)^{\frac{5}{2}} \right) - \left( \left( \frac{1}{2}(1)^4 + 10(1)^{\frac{5}{2}} \right) \right)$$

$$\underline{\underline{5700 \text{ units}^2}}$$



39 Use integration to show that the area enclosed by  $x$ -axis and the curve with equation

$y = (x+3)(x-1)^2$  is  $\frac{64}{3}$  square units.



$$\int_{-3}^1 (x+3)(x-1)^2 dx$$

$$\begin{aligned} & (x+3)(x^2-2x+1) \\ & x^3+3x^2-2x^2-6x+x+3 \\ & x^3+x^2-5x+3 \end{aligned}$$

$$\int_{-3}^1 x^3 + x^2 - 5x + 3 dx$$

$$\left[ \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{5}{2}x^2 + 3x \right]_{-3}^1$$

$$\left( \frac{1}{4}(1)^4 + \frac{1}{3}(1)^3 - \frac{5}{2}(1)^2 + 3(1) \right) - \left( \frac{1}{4}(-3)^4 + \frac{1}{3}(-3)^3 - \frac{5}{2}(-3)^2 + 3(-3) \right)$$

$$\underline{\underline{\frac{64}{3} \text{ units}^2}}$$