AS Level Maths: Integration

1 $y = 2x^3 + 5x^2 - 7x + 10$

Find $\int y \, dx$

(Total for question 1 is 4 marks)

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2 Find $\int (3x^2 + 7x - 2) \, dx$

(Total for question 2 is 4 marks)

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3 Find $\int (x + 4)(x - 3) \, dx$

(Total for question 3 is 4 marks)

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4 $f'(x) = 6x^2 - 3x + 8$

Given that the point (1, 8) lies on $y = f(x)$

Find an expression for $f(x)$

(Total for question 4 is 5 marks)

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5 $y = 4\sqrt{x} + \frac{1}{x^2} + 10$

Find $\int y \, dx$

(Total for question 5 is 4 marks)

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6 Find $\int_1^3 (x + 4)(x - 3) \, dx$

(Total for question 6 is 5 marks)

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7 $\frac{dy}{dx} = 10x^4 - 5$

Given that the point (2, 30) lies on the curve

Find an expression for y in terms of x

(Total for question 7 is 5 marks)

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8 Find $\int_1^4 5 + \frac{1}{\sqrt{x}} \, dx$

(Total for question 8 is 5 marks)

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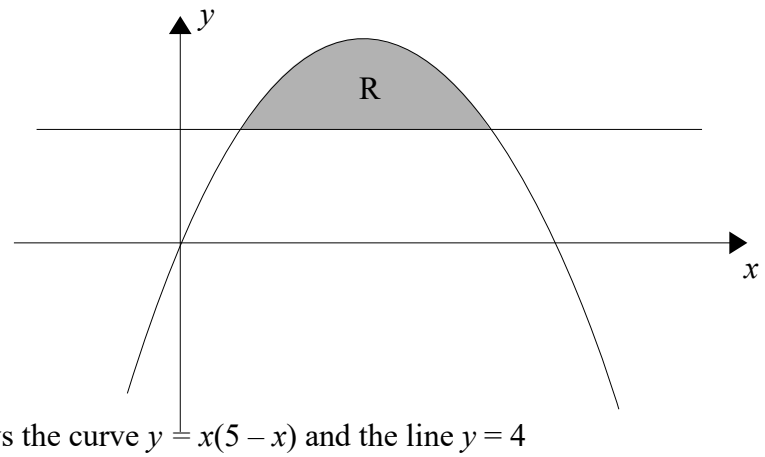
9 The curve with the equation $f(x)$ passes through the point (1, 2)

Given that $f'(x) = 5 + \frac{3x^2 + 2}{x^{\frac{1}{2}}}$

Find $f(x)$ giving your answer in its simplest form.

(Total for question 9 is 7 marks)

10



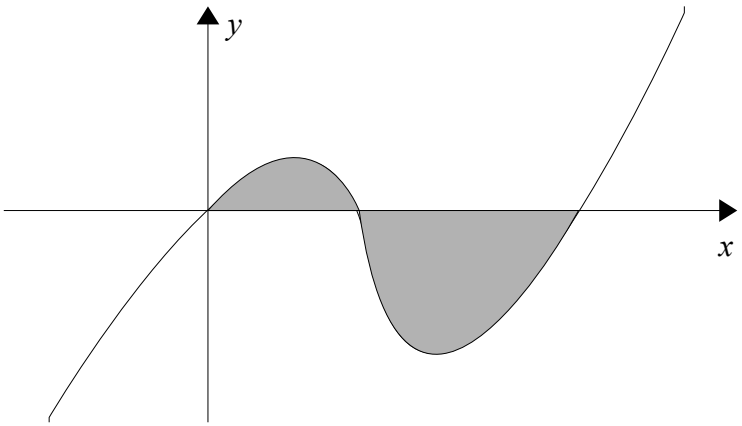
The sketch shows the curve $y = x(5 - x)$ and the line $y = 4$

(a) Find the coordinates of the points where the line intersects the curve. **(2)**

(b) Find the area of the shaded region R. **(6)**

(Total for question 10 is 8 marks)

11



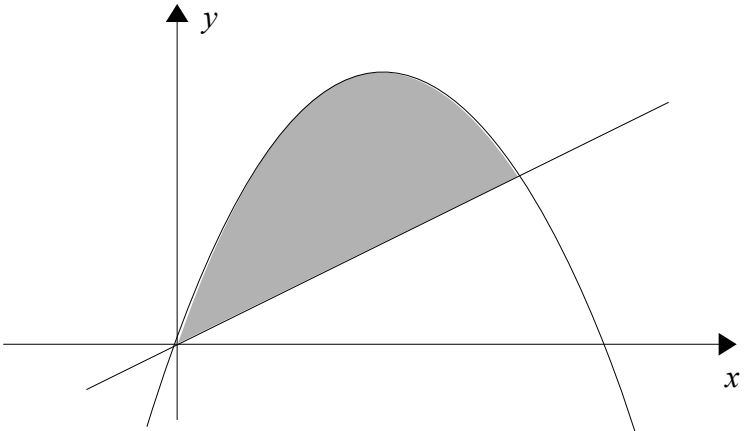
The sketch shows the curve $y = x(x - 2)(x - 5)$

(a) Write down the values of x where the curve crosses the x axis. **(1)**

(b) Find the area of the shaded region. **(8)**

(Total for question 11 is 9 marks)

12



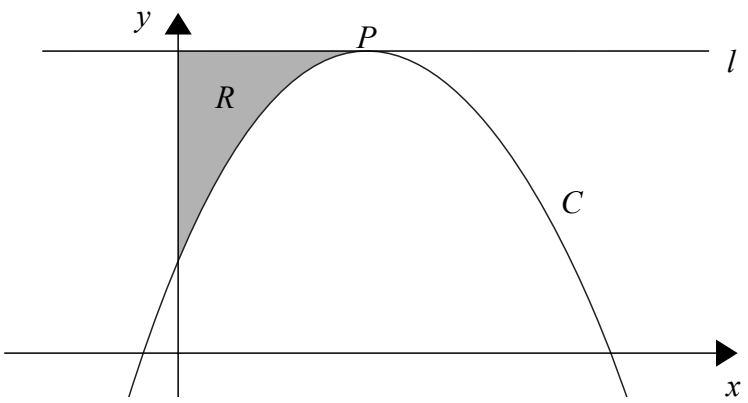
The sketch shows the curve $y = 10x - x^2$ and the straight line $y = 2x$

(a) Find the coordinates of the points where the line intersects the curve. **(2)**

(b) Find the area of the shaded region. **(6)**

(Total for question 12 is 8 marks)

13



The sketch shows the curve C with equation $y = (x + 1)(8 – x)$
The maximum point of curve C is P.
The line l passes through P and is parallel to the x-axis.
The shaded region R is bounded by the curve C, the line l and the y-axis.

Find the area of the shaded region R.

(Total for question 13 is 10 marks)

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14 $\frac{dy}{dx} = \frac{1}{3x^2}$

Find an expression for y in terms of x

(Total for question 14 is 3 marks)

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15 $f'(x) = (2x – 1)^2$ and $f(2) = 5$

Find an expression for $f(x)$

(Total for question 15 is 4 marks)

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16 A curve cuts the x-axis at (2, 0) and has the gradient function $\frac{dy}{dx} = \frac{8}{x^2}$

Find the equation of the curve

(Total for question 16 is 4 marks)

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17 A curve C has equation $y = \frac{3}{2\sqrt{x}}$

The region enclosed between the curve, the x-axis and the lines $x = 1$ and $x = k$
area 12 units.

Given $k > 1$, find the value of k.
Fully justify your answer.

(Total for question 17 is 5 marks)

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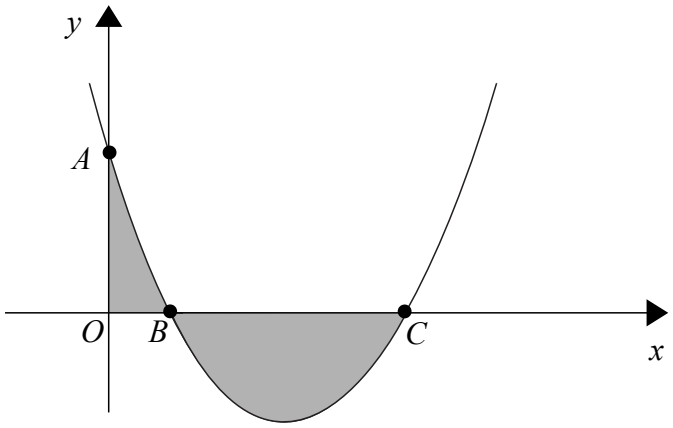
18 $\frac{dy}{dx} = \frac{5}{x^2}$

Find an expression for y

(Total for question 18 is 3 marks)

19



The diagram shows the curve with equation $y = x^2 - 6x + 5$

(a) Write down the coordinates of the points A, B and C.

(b) Find the total area of the two shaded regions.
Fully justify your answer.

(Total for question 19 is 8 marks)

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20 Find $\int \left(4\sqrt{x} - 2\right) \, dx$

writing your answer in its simplest form.

(Total for question 20 is 5 marks)

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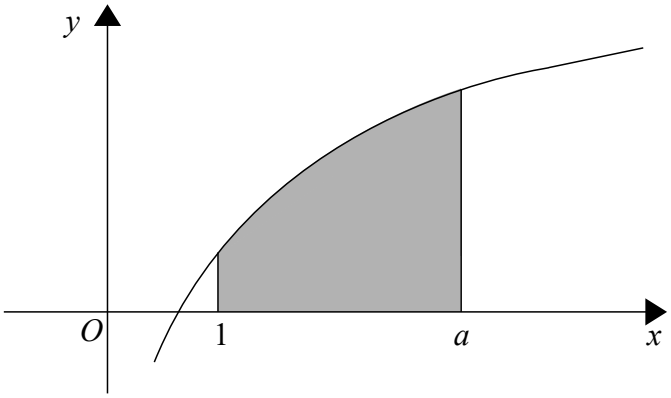
21 The diagram shows part of the graph of $y = 8x^{\frac{1}{3}} - \frac{4}{x^{\frac{1}{3}}}$

The shaded region is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = a$, where $a > 1$



Given that the area of the shaded area is 36 square units, find the exact value of a.

(Total for question 21 is 9 marks)

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22 Find $\int \left(2x^3 + \sqrt{x} + 2\right) dx$

Giving your answer in its simplest form.

(Total for question 22 is 4 marks)

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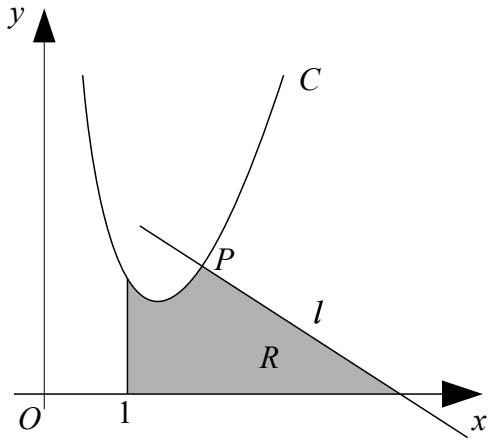
23 The graph shows part of the curve with equation $y = \frac{8}{x^2} + 4x$



$P(2, 10)$ lies on C.
Line l is the normal to C at P.

Region R is bounded by the curve C, the line l and the line with equation $x = 1$.
Show that the area of R is 110.

(Total for question 23 is 10 marks)

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24 (a) Given that k is a constant, find $\int \left(\frac{k}{x^3} + 2x\right) \mathrm{d}x$

Giving your answer in its simplest form. **(3)**

(b) Find the value of k such that $\int_1^2 \left(\frac{k}{x^3} + 2x\right) \mathrm{d}x = 12$ **(3)**

(Total for question 24 is 6 marks)

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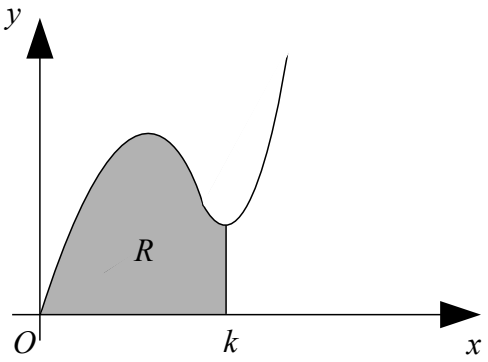
25 The graph shows part of the curve C with equation $y = x^3 - 7x^2 + 15x$



The curve has a minimum turning point at k.

Region R is bounded by the curve C, x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{99}{4}$

(Total for question 25 is 7 marks)

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26 Given that k is a positive constant and $\int_1^k \left(\frac{3}{2\sqrt{x}} + 9 \right) \mathrm{d}x = 8$

(a) Show that $9k + 3\sqrt{k} - 20 = 0$ **(4)**

(b) Hence, using algebra, find any values of k such that $\int_1^k \left(\frac{3}{2\sqrt{x}} + 9 \right) \mathrm{d}x = 8$ **(4)**

(Total for question 26 is 8 marks)

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27 $\mathrm{f}(x) = 2x^3 - x^2 - 8x + 4$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $\mathrm{f}(x)$ **(2)**

(b) Hence, showing all your working, write $\mathrm{f}(x)$ as a product of three linear factors. **(4)**

The finite region R is bounded by the curve with equation $y = \mathrm{f}(x)$ and the x-axis, and lies below the x-axis

(c) Find, using algebraic integration, the exact value of the area of R **(4)**

(Total for question 27 is 10 marks)

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28 Find $\int \left(\frac{4x^3 - 5}{3x^2} \right) \mathrm{d}x$

writing your answer in its simplest form.

(Total for question 28 is 4 marks)

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29 Find the value of k such that $\int_1^8 \left(\frac{k}{\sqrt[3]{x}} \right) \mathrm{d}x = 22.5$

(Total for question 29 is 4 marks)

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30 Find the value of k such that $\int_k^9 \left(\frac{10}{\sqrt{x}} \right) \mathrm{d}x = 20$

(Total for question 30 is 4 marks)

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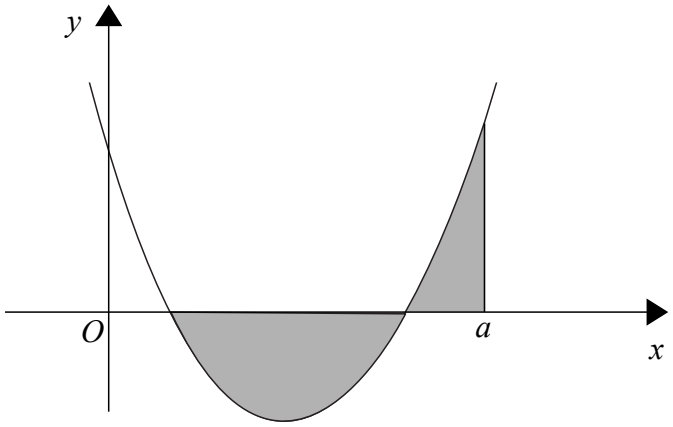
31 (a) Find $\int \left(2x - x^2 \right) \mathrm{d}x$

(b) Evaluate $\int_0^4 \left(2x - x^2 \right) \mathrm{d}x$

(c) Using a sketch, explain why the integral in part (b) does not give the area enclosed between the curve $y = 2x - x^2$ and the x-axis

(Total for question 31 is 5 marks)

32



The diagram shows the curve with equation $y = x^2 - 9x + 14$ and the line $x = a$

Given the total area of the two shaded regions is 29 units^2
Find the exact value of a.

(Total for question 32 is 8 marks)

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33 Find $\int \left(10 - \frac{6}{x^2}\right) \mathrm{d}x$

writing your answer in its simplest form.

(Total for question 33 is 4 marks)

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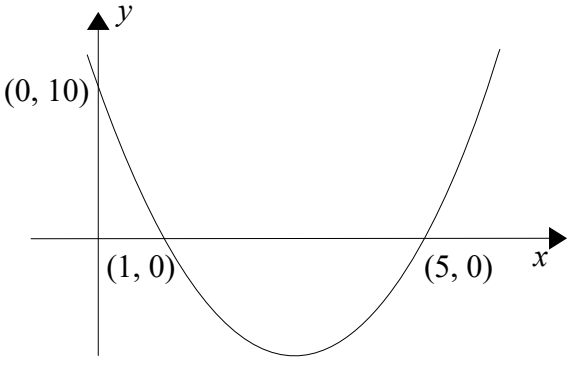
34 $f(x) = (x - 4)(x - 2)(x + 1)$

Find the total area enclosed by $f(x)$ and the x-axis

(Total for question 34 is 5 marks)

35



The sketch shows a quadratic graph that passes through (0, 10), (1, 0) and (5, 0)

Find the area of the finite region bounded by the curve and the x-axis.

(Total for question 35 is 8 marks)

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36 Show that $\int_1^8 \left(\frac{16}{x^2}\right) \mathrm{d}x = 14$

(Total for question 36 is 4 marks)

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37 Show that $\int_1^4 \left(1 + 3\sqrt{x}\right) \mathrm{d}x = 17$

(Total for question 37 is 4 marks)

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38 (a) Find $\int x^2\left(2x + \frac{25}{\sqrt{x}}\right) \mathrm{d}x$ **(5)**

(b) Find $\int_1^9 x^2\left(2x + \frac{25}{\sqrt{x}}\right) \mathrm{d}x$ **(3)**

(Total for question 38 is 8 marks)

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39 Use integration to show that the area enclosed by x-axis and the curve with equation $y = (x + 3)(x - 1)^2$ is $\frac{64}{3}$ square units.

(Total for question 39 is 6 marks)
