1 $\qquad f(x) = (x + 3)(x + 2)(x - 1)$ positive cubic shape

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x - 3)$ **(2)**

(ii) $y = f(-x)$ **(2)**

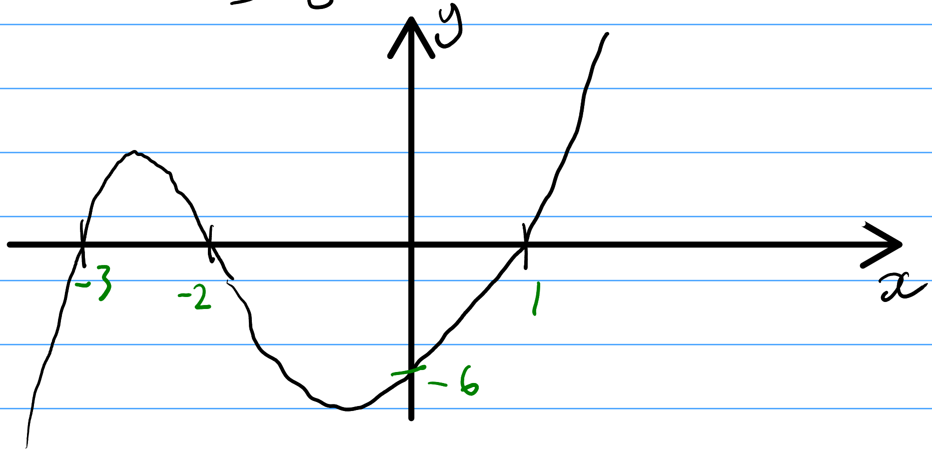
crosses x when $y = 0$

$$0 = (x+3)(x+2)(x-1)$$

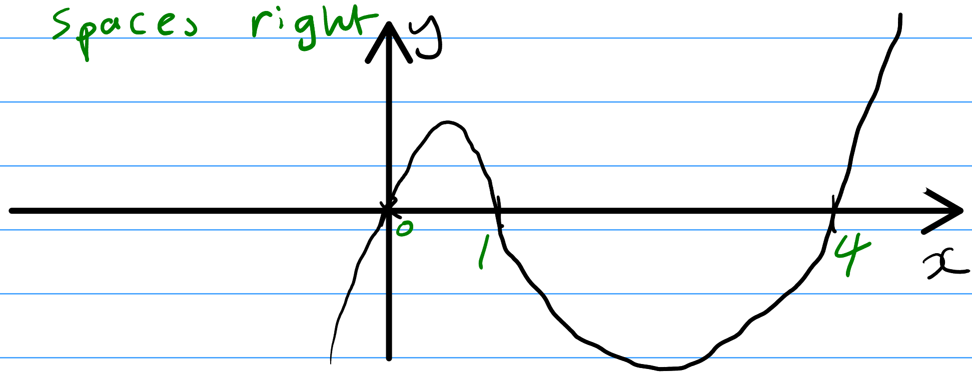
$$x = -3 \quad x = -2 \quad x = 1$$

crosses y when $x = 0$

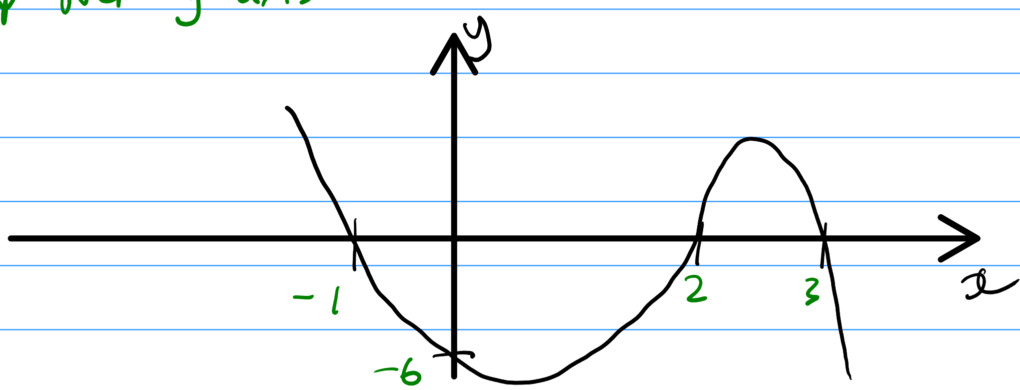
$$y = (3)(2)(-1)$$
$$= -6$$



bi/ 3 spaces right



ii/ flip over y axis



2 (a) Sketch on the same diagram the curves $y = x^2 + 5x$ and $y = -\frac{1}{x}$ (4)

(b) State, giving a reason, the number of real solutions to the equation $x^2 + 5x + \frac{1}{x} = 0$ (2)

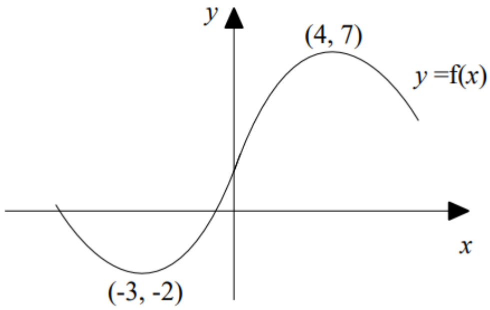
$y = x^2 + 5x$ crosses y at origin

$y = x(x+5)$ crosses x at 0 and -5

$y = -\frac{1}{x}$ negative reciprocal graph

$x^2 + 5x = -\frac{1}{x}$ → one intersection

$x^2 + 5x - \frac{1}{x} = 0$ has one real solution as the graphs intersect once



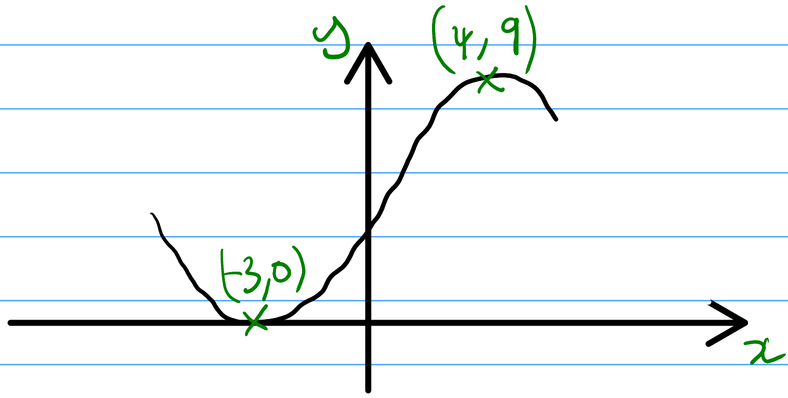
3 The sketch shows the graph of $y = f(x)$. The curve has a minimum at (-3,-2) and a maximum at (4,7).

Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

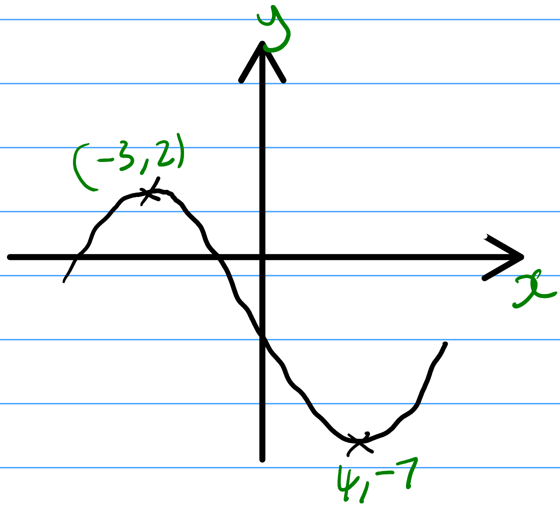
(i) $y = f(x) + 2$ up 2 **(2)**

(ii) $y = -f(x)$ flip over x **(2)**

i/



ii/



4 $f(x) = x^2 + 4x + 5$

(a) Express f(x) in the form $(x + a)^2 + b$, and state the coordinates of the minimum point of $y = f(x)$. **(3)**

(b) Sketch the graph of $y = f(x)$ showing the coordinates of intersection with the coordinate axis. **(3)**

(c) Find the minimum points of these curves

(i) $y = 2f(x)$ **(2)**

(ii) $y = f(2x)$ **(2)**

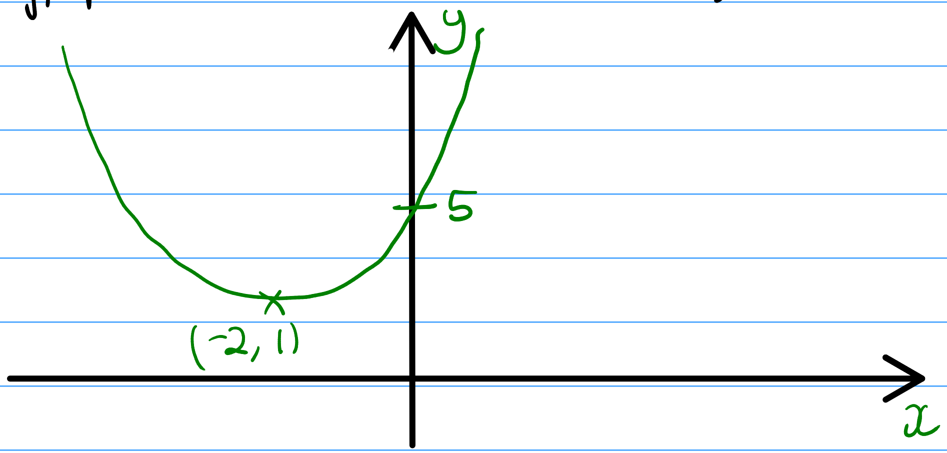
a/ $f(x) = (x+2)^2 - 4 + 5$

$= (x+2)^2 + 1$

Min point at $(-2, 1)$

b/ crosses y when $x=0$ $\underline{y=5}$

The graph does not cross the x axis.



c/ i/ Double y coordinate

$\underline{\underline{(-2, 2)}}$

ii/ Half x coordinate

$\underline{\underline{(-1, 1)}}$

5 $$f(x) = x^3 + 4x^2 - 5x$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

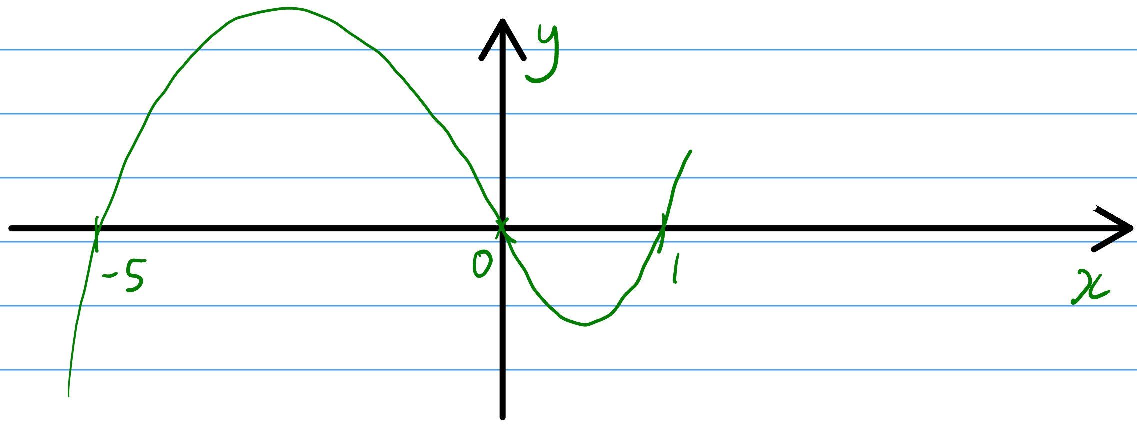
(i) $y = f(x + 1)$ **(2)**

(ii) $y = f(2x)$ **(2)**

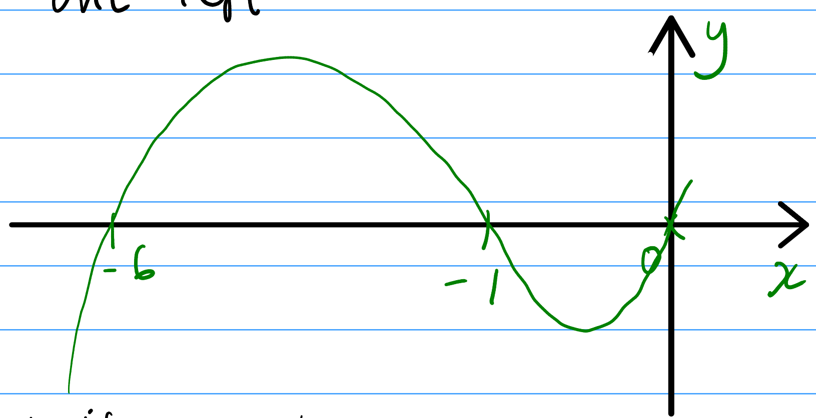
· positive cubic shape

· crosses x when $y = 0$

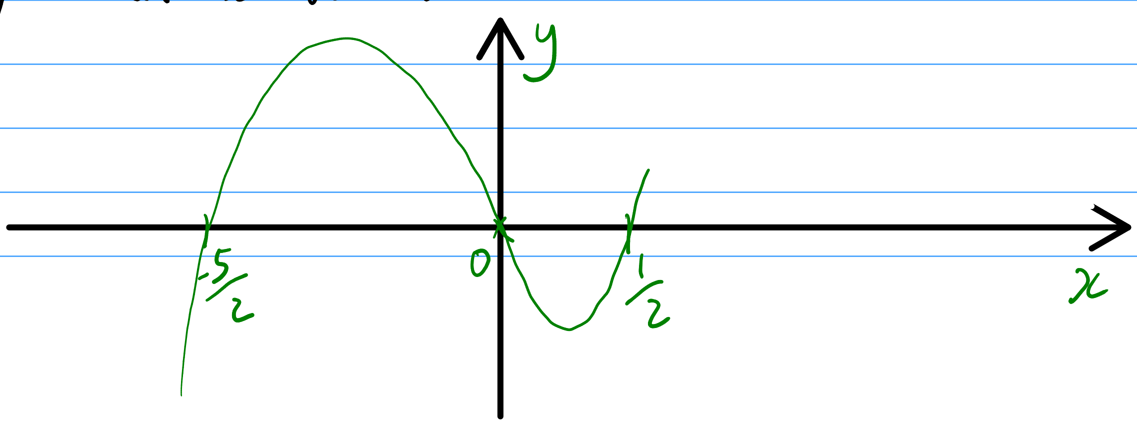
$$0 = x^3 + 4x^2 - 5x$$
$$0 = x(x^2 + 4x - 5)$$
$$0 = x(x+5)(x-1)$$
$$x = 0 \quad x = -5 \quad x = 1$$



bi) one left



bii) half x values



6 Sketch graph of $y = \frac{1}{x} + 2$, showing the points of intersection with the coordinate axis and stating the equations of any asymptotes.

crosses x when $y=0$

$$0 = \frac{1}{x} + 2$$

$$-2 = \frac{1}{x}$$

$$x = -\frac{1}{2}$$

7 $$f(x) = (x+4)(x-1)(2-x)$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x+2)$ 2 left **(2)**

(ii) $y = -f(x)$ Flip over x **(2)**

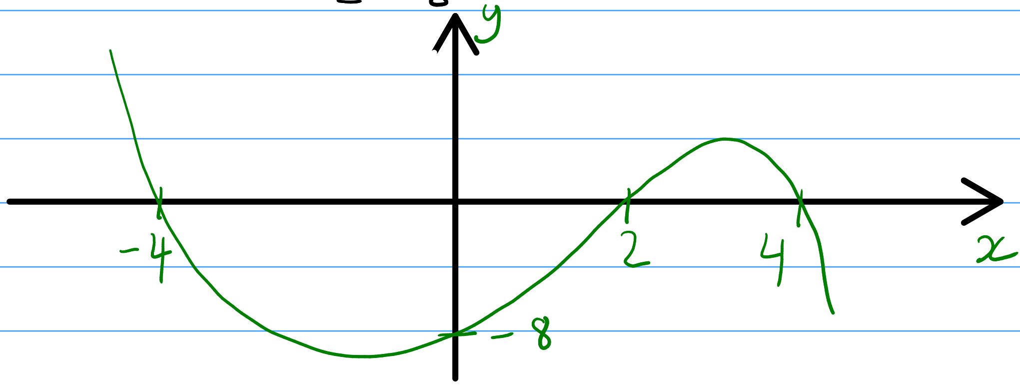
a/
- negative cubic shape
- crosses x when $y=0$

$$0 = (x+4)(x-1)(2-x)$$
$$x=-4 \quad x=1 \quad x=2$$

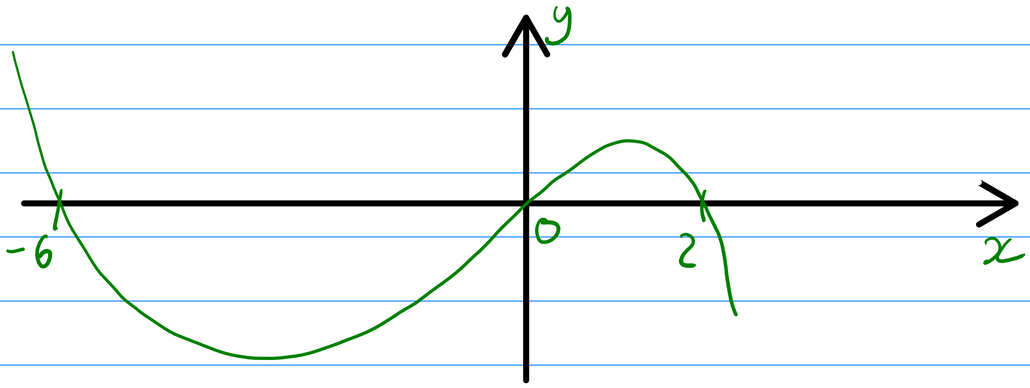
- crosses y when $x=0$

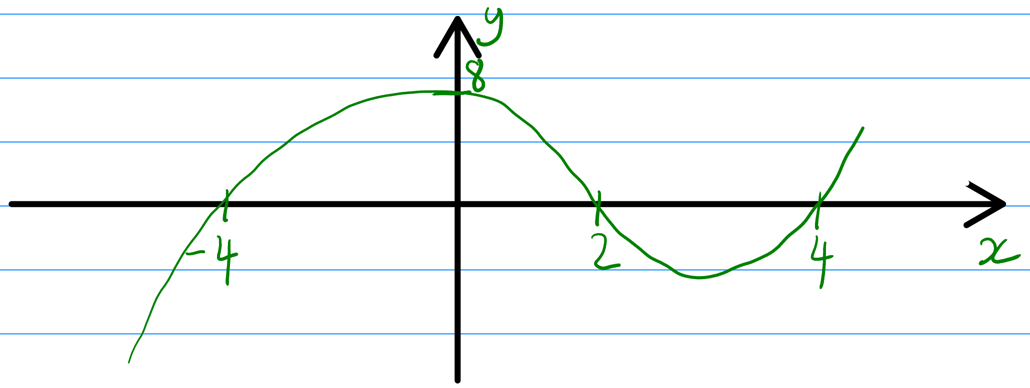
$$y = (4)(-1)(2)$$
$$= -8$$

bi/



ii/





8 $$f(x) = (x + 3)(x - 1)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Find the equation of $y = f(x + 2)$ in the form $y = (x + a)(x + b)^2$ **(2)**

a/ · positive cubic

· crosses x when $y = 0$

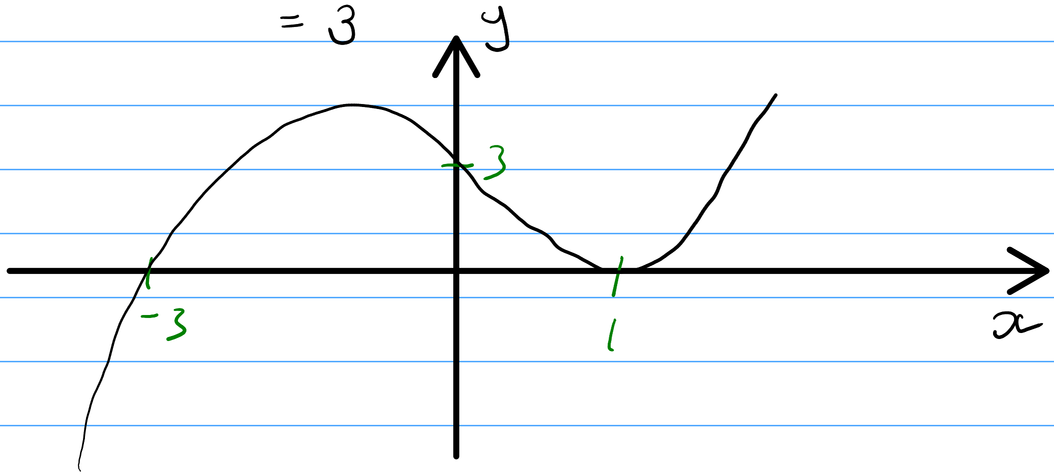
$$0 = (x+3)(x-1)^2$$

$$x = -3 \qquad \underline{x = 1}$$ repeated root

· crosses y when $x = 0$

$$y = (3)(-1)^2$$

$$= 3$$



b/ change x into $x + 2$

$$f(x+2) = (x+2+3)(x+2-1)^2$$

$$= \underline{\underline{(x+5)(x+1)^2}}$$

9 (a) The curve $y = \frac{2}{x-1}$ is translated by four units in the positive x-direction.

State the equation of the curve after it has been translated. **(2)**

(b) Describe fully the single transformation that transforms the curve $y = \frac{2}{x-1}$ to $y = \frac{3}{x-1}$ **(2)**

a/ $f(x-4)$

$$y = \frac{2}{x-4-1}$$

$$y = \frac{2}{x-5}$$

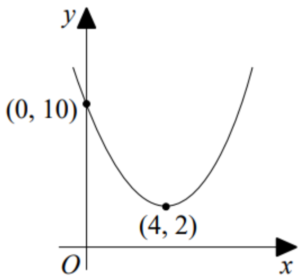
b/ $y = \frac{3}{x-1}$

$$= \frac{3}{2} \cdot \frac{2}{x-1}$$

$y = \frac{3}{2} f(x)$ stretch $\times 1.5$ in y direction

10 Figure 1 shows $y = \mathrm{f}(x)$

Figure 1



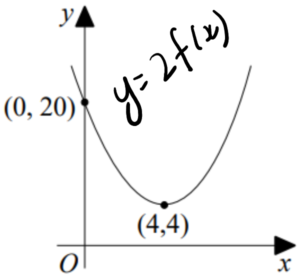


Figure 2

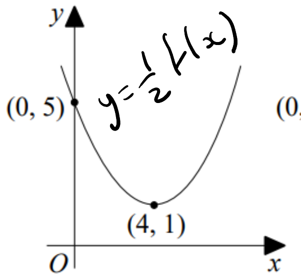


Figure 3

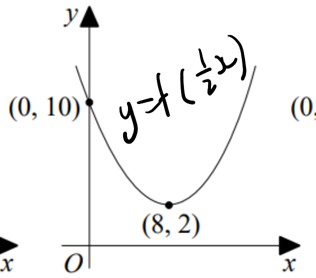


Figure 4

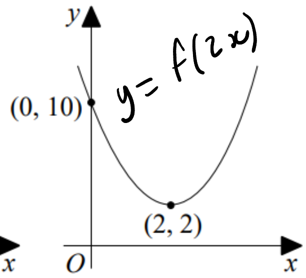


Figure 5

(a) Which figure shows $y = 2\mathrm{f}(x)$? 2

(b) Which figure shows $y = \mathrm{f}(2x)$? 5

---

11 Given that $\mathrm{f}(x) = 10$ when $x = 4$, which statement must be correct?

Tick (✓) one box.

- [ ] $\mathrm{f}(2x) = 20$ when $x = 4$
- [ ] $\mathrm{f}(2x) = 10$ when $x = 8$
- [ ] $\mathrm{f}(2x) = 5$ when $x = 4$
- [x] $\mathrm{f}(2x) = 10$ when $x = 2$

(4, 10)

f(2x) half x value

(2, 10)

12 Curve C has equation $y = x^2$

C is translated by $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ to give the equation C_1. 2 right $f(x-2)$

Line L has equation $y = x$

L is stretched by scale factor 3 parallel to the x-axis to give the line L_1. $f(\frac{1}{3}x)$

Find the exact distance between the two intersection points of C_1 and L_1

$C_1 : \quad y = (x-2)^2$

$L_1 : \quad y = \frac{1}{3}x$

Intersection where $\frac{1}{3}x = (x-2)^2$

$$\frac{1}{3}x = x^2 - 4x + 4$$

$$x = 3x^2 - 12x + 12$$

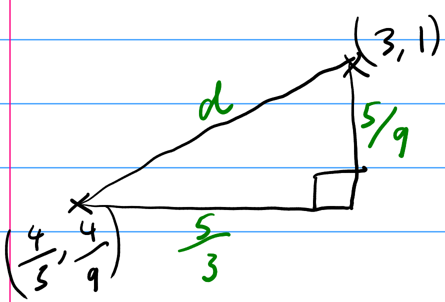
$$0 = 3x^2 - 13x + 12$$

$x = 3 \qquad x = 4/3$

$y = \frac{1}{3}x \qquad y = \frac{1}{3}\left(\frac{4}{3}\right)$

$= \frac{1}{3}(3)$

$(3, 1) \qquad \left(\frac{4}{3}, \frac{4}{9}\right)$



$d^2 = \left(\frac{5}{9}\right)^2 + \left(\frac{5}{3}\right)^2$

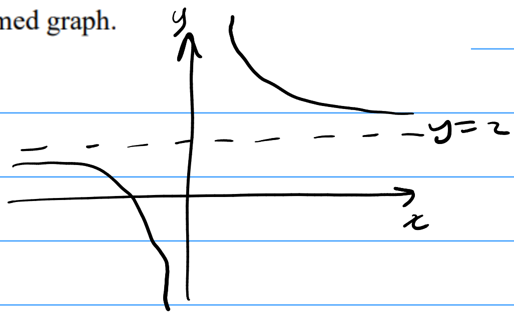
$d^2 = \frac{250}{81}$

$d = \frac{5\sqrt{10}}{9}$

14 The graph $y = \frac{1}{x}$ is translated by the vector $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ 2 up $f(x)+2$

(a) Write down the equation of the transformed graph.

(b) State the equations of the asymptotes of the transformed graph.



a/ $y = \frac{1}{x} + 2$

b/ asymptotes at $y=2$ and $x=0$

15 (a) Sketch the curve $y = (x - a)(5 - x)^2$ where $0 < a < 5$

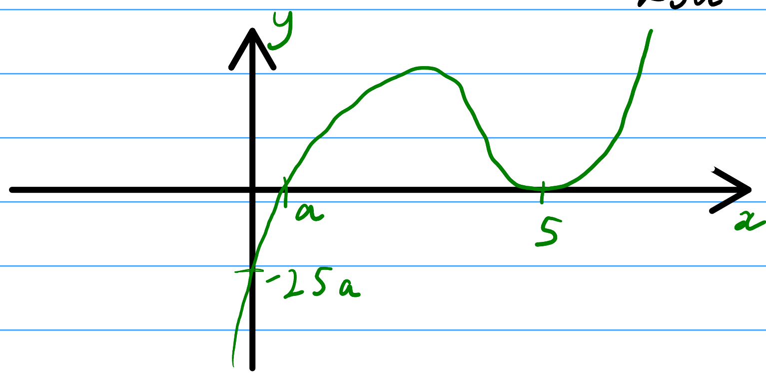
indicating the coordinates of the points where the curve and the axes meet.

(b) Hence solve, $(x - a)(5 - x)^2 > 0$ giving your answer in set notation form.

- positive cubic shape
- crosses x when $y=0$ $\quad 0 = (x-a)(5-x)^2$

  $x=a \quad x=5$ → repeated root
- crosses y when $x=0$ $\quad y = (-a)(5)^2$

  $= -25a$



b/ $a < x < 5$ or $x > 5$

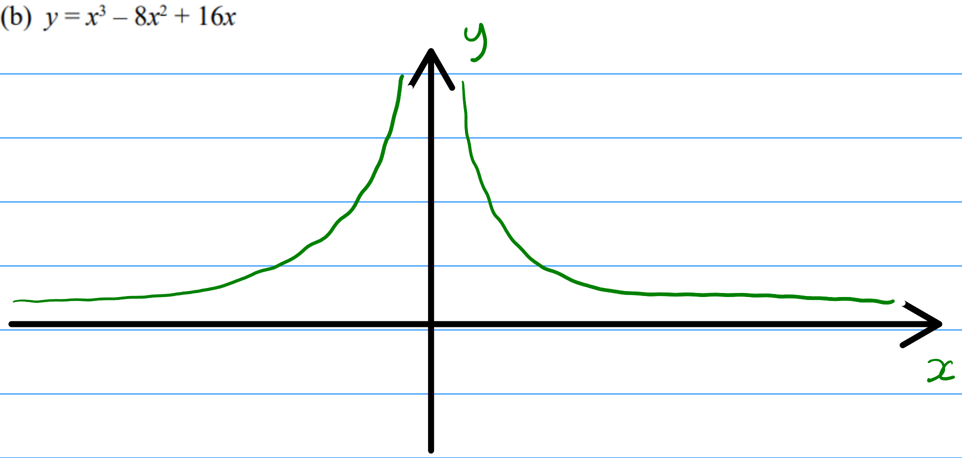
$\{x : a < x < 5\} \cup \{x : x > 5\}$

16 Sketch the following curves.

(a) $y = \dfrac{3}{x^2}$ **(2)**

(b) $y = x^3 - 8x^2 + 16x$ **(5)**

a/

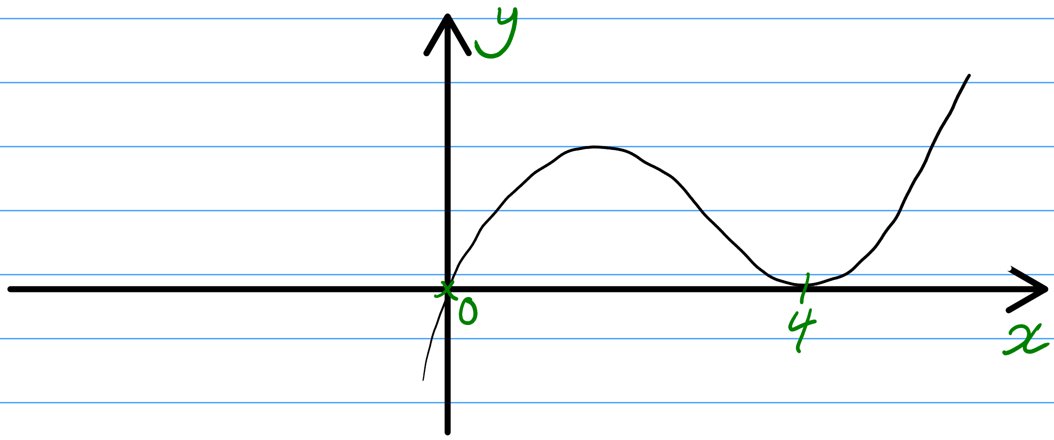


b/ · positive cubic shape

· crosses x when $y = 0$

$$0 = x^3 - 8x^2 + 16x$$
$$0 = x(x^2 - 8x + 16)$$
$$0 = x(x-4)^2$$

$x = 0 \quad x = 4$ (repeated root)



17 (a) Sketch the curve $y=\frac{-2}{x}$ **(1)**

(b) The curve $y=\frac{-2}{x}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated **(2)**

(c) The curve $y=\frac{-2}{x}$ is stretched parallel to the y-axis with scale factor 2 and, as a result, the point (2, -1) on the curve is transformed to the point P.

State the coordinates of P. **(2)**

a/

b/ $f(x-2)$ $\qquad y=\frac{-2}{x-2}$

c/ $2f(x)$ $\qquad (2,-2)$

---

18 $f(x) = (x - a)(x - 3a)(x + b)$ where a and b are positive integers.

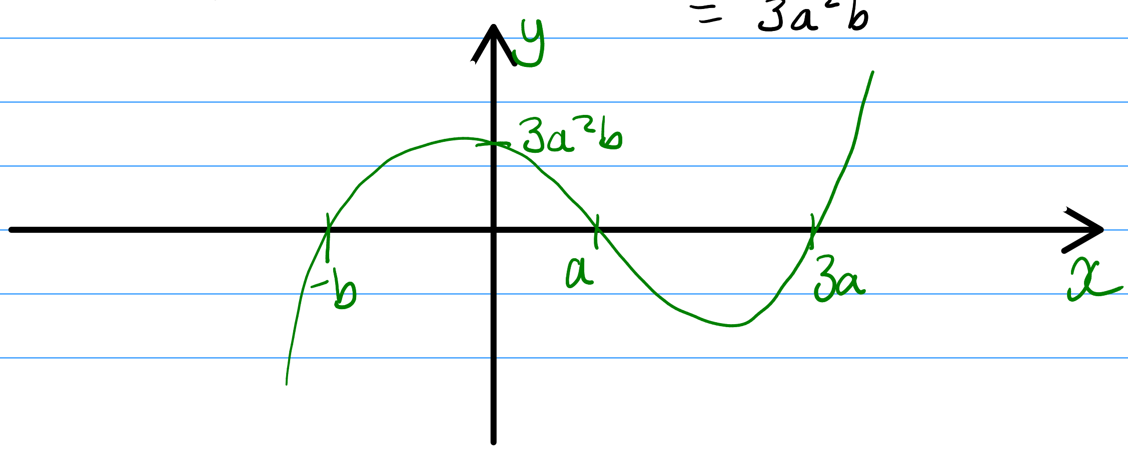
(a) Sketch the curve $y = f(x)$ **(2)**

(b) On your sketch mark, in terms of a and b, the points where the curve meets the axes. **(2)**

- positive cubic shape
- crosses x when $y=0$ $\qquad 0=(x-a)(x-3a)(x+b)$

  $x=a \quad x=3a \quad x=-b$
- crosses y when $x=0$ $\qquad y=(-a)(-3a)(b)$

  $= 3a^2b$



19 The curve $y = (x - 2)^2$ maps onto the curve C_1 following a stretch scale factor $\frac{1}{2}$ in the x-direction

Find the equation of the curve C_1

$f(2x)$

$y = (2x - 2)^2$

---

20 (a) Sketch the curve $y = (x + 5)(x + 2)(3 - x)$ **(4)**

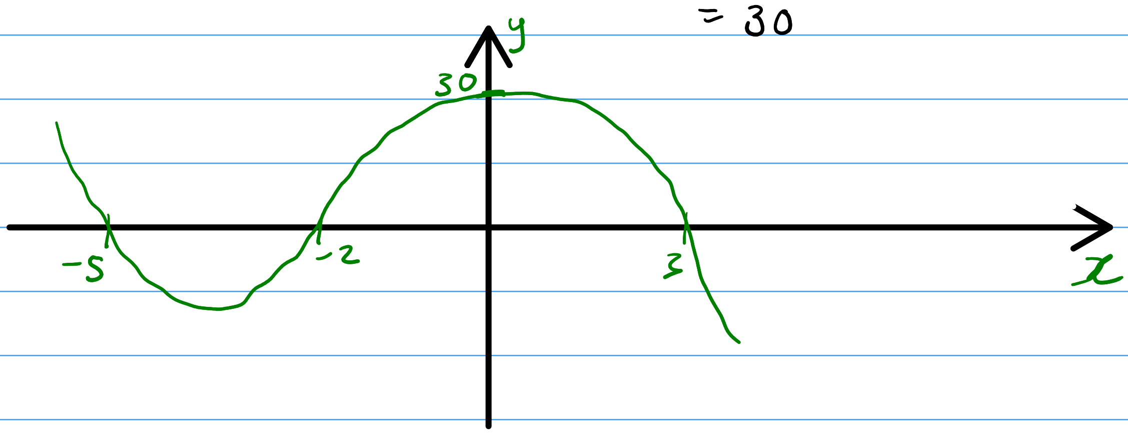
(b) The curve $y = (x + 5)(x + 2)(3 - x)$ is translated by the vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$. $f(x-2)$ **(2)**

Write down the equation of the transformed graph.

a/
- negative cubic shape
- crosses x when $y=0$ $\quad 0 = (x+5)(x+2)(3-x)$

  $x = -5 \quad x = -2 \quad x = 3$
- crosses y when $x=0$ $\quad y = (5)(2)(3)$

  $= 30$



b/ $f(x-2)$

$y = (x - 2 + 5)(x - 2 + 2)(3 - (x - 2))$

$= (x + 3)(x)(5 - x)$

$= \underline{\underline{x(x+3)(5-x)}}$

21 $\quad f(x) = (x+1)(x-2)^2$

(a) Sketch the curve $y = f(x)$ **(3)**

(b) Hence solve $f(x) \leq 0$ **(2)**

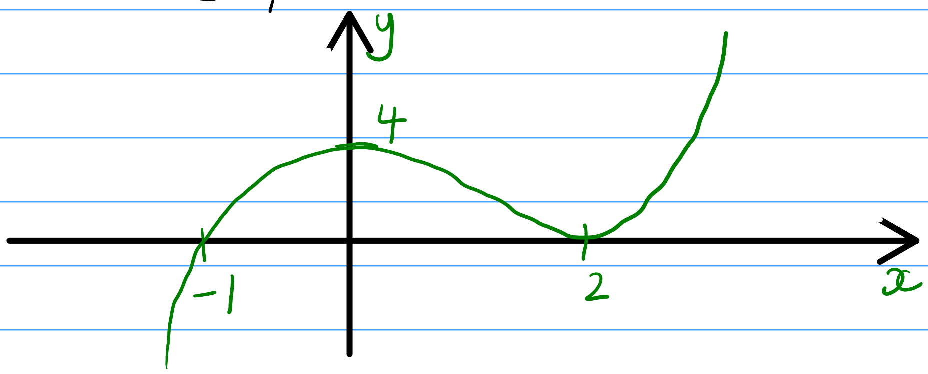
- positive cubic shape
- crosses x when $y = 0$

$$0 = (x+1)(x-2)^2$$

$$x = -1 \qquad x = 2 \quad \text{(repeated root)}$$

- crosses y when $x = 0$

$$y = (1)(-2)^2$$
$$= 4$$



b/ $\quad x \leq -1 \quad$ or $\quad x = 2$

22 $$f(x) = (x + 4)(2x - 5)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Deduce the values of x for which

(i) $f(x) \geq 0$

(ii) $f(2x) = 0$ **(3)**

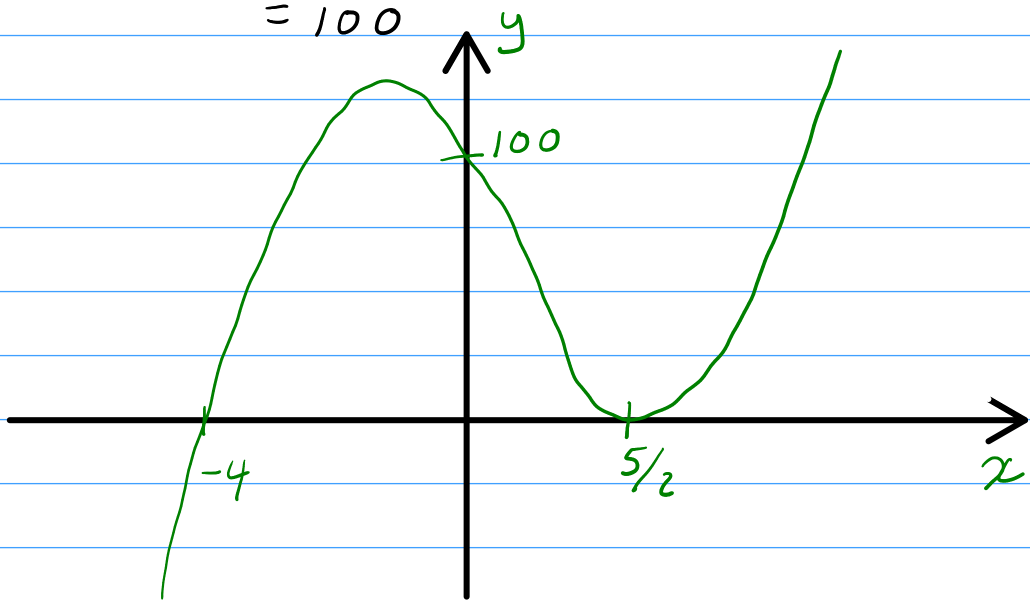
- positive cubic shape
- crosses x when $y = 0$

$$0 = (x+4)(2x-5)^2$$

$x = -4 \quad x = 5/2 \longrightarrow$ repeated root

- crosses y when $x = 0$

$$y = (4)(-5)^2$$
$$= 100$$



b/ i/ $x \geq -4$

ii/ $x = -2$ and $x = 5/4$

23 The curve C has equation

$$y=\frac{k^2}{x}-2 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

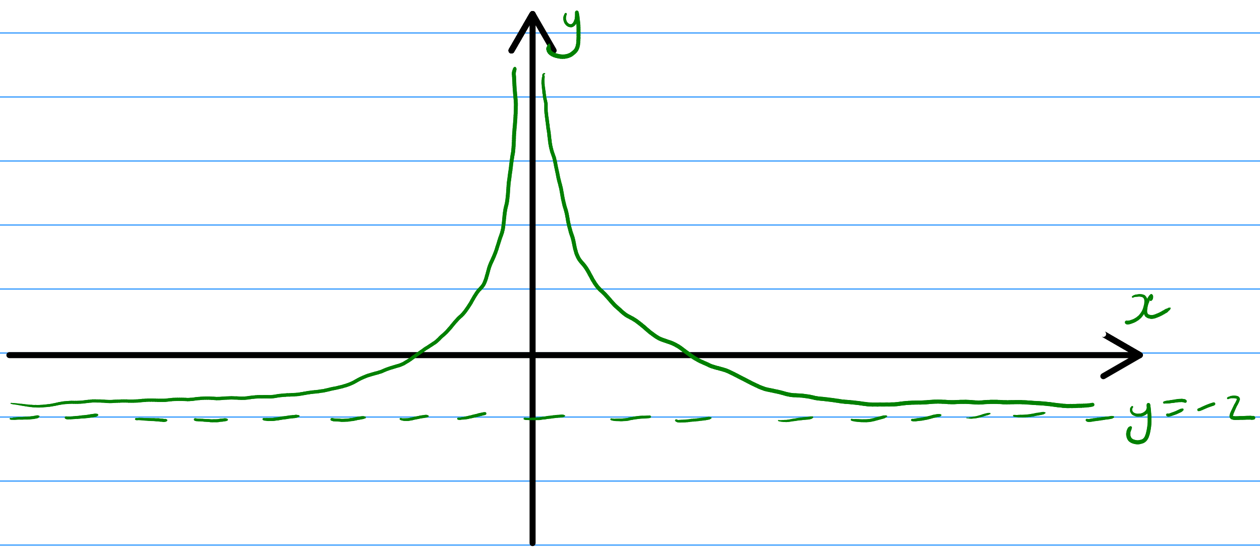
(a) Sketch C, stating the equation of the horizontal asymptote **(3)**

The line l has equation $y = -3x + 4$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$3x^2 - 6x + k^2 = 0$$ **(2)**

(c) Hence find the exact values of k for which l is a tangent to C. **(3)**



b/ $\frac{k^2}{x} - 2 = -3x + 4$

$k^2 - 2x = -3x^2 + 4x$

$3x^2 - 6x + k^2 = 0$

c/ 1 solution where $b^2 - 4ac = 0$

$(-6)^2 - 4(3)(k^2) = 0$

$36 - 12k^2 = 0$

$36 = 12k^2$

$k^2 = 3$

$k = \pm\sqrt{3}$

24 $f(x) = (x + 2)(x - 3)^2$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(b) find the two possible values of k. **(2)**

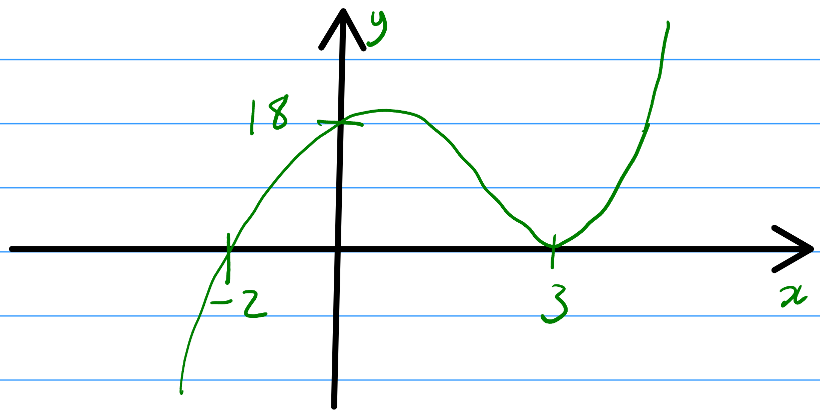
a/
- positive cubic
- crosses x when $y = 0$

$$0 = (x+2)(x-3)^2$$

$x = -2 \quad x = 3 \longrightarrow$ repeated root

- crosses y when $x = 0$

$$y = (2)(-3)^2$$
$$= 18$$



b/ either 2 right or 3 left

$f(x-2) \qquad f(x+3)$

$\underline{k = -2}$ or $\underline{k = 3}$

25 (a) Using algebra, find all the solutions to the equation $3x^3 - 11x^2 + 6x = 0$ **(3)**

(b) Hence find all the real solutions of $3(y+2)^6 - 11(y+2)^4 + 6(y+2)^2 = 0$ **(3)**

a/ $x(3x^2 - 11x + 6) = 0$

$3 \times 6 = 18$

$x(3x^2 - 2x - 9x + 6) = 0$

$x(x - 3)(3x - 2) = 0$

18

1 18

2 9

3 6

$\underline{x = 0}$ $\underline{x = 3}$ $\underline{x = \frac{2}{3}}$

b/ $f(x+2)$ shift 2 left

$y = -2$ $y = 1$ $y = -\frac{4}{3}$