AS Level Maths: Graphs and Transformations

1 $$f(x) = (x + 3)(x + 2)(x - 1)$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x - 3)$ **(2)**

(ii) $y = f(-x)$ **(2)**

(Total for question 1 is 7 marks)

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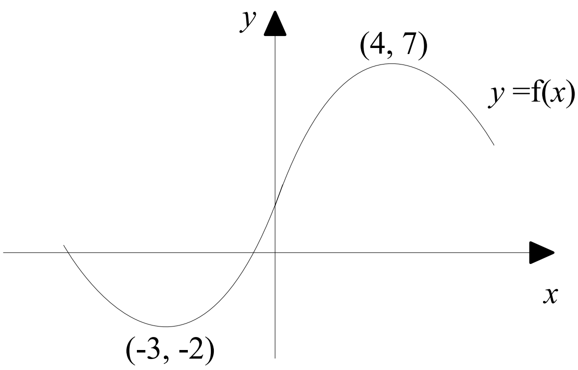
2 (a) Sketch on the same diagram the curves $y = x^2 + 5x$ and $y = -\frac{1}{x}$ **(4)**

(b) State, giving a reason, the number of real solutions to the equation $x^2 + 5x + \frac{1}{x} = 0$ **(2)**

(Total for question 2 is 6 marks)



3 The sketch shows the graph of $y = f(x)$. The curve has a minimum at (-3,-2) and a maximum at (4,7).

Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x) + 2$ **(2)**

(ii) $y = -f(x)$ **(2)**

(Total for question 3 is 4 marks)

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4 $$f(x) = x^2 + 4x + 5$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, and state the coordinates of the minimum point of $y = f(x)$. **(3)**

(b) Sketch the graph of $y = f(x)$ showing the coordinates of intersection with the coordinate axis. **(3)**

(c) Find the minimum points of these curves

(i) $y = 2f(x)$ **(2)**

(ii) $y = f(2x)$ **(2)**

(Total for question 4 is 8 marks)

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5 $$f(x) = x^3 + 4x^2 - 5x$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x + 1)$ **(2)**

(ii) $y = f(2x)$ **(2)**

(Total for question 5 is 7 marks)

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6 Sketch graph of $y = \frac{1}{x} + 2$, showing the points of intersection with the coordinate axis and stating the equations of any asymptotes.

(Total for question 6 is 3 marks)

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7 $$f(x) = (x + 4)(x - 1)(2 - x)$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x + 2)$ **(2)**

(ii) $y = -f(x)$ **(2)**

(Total for question 7 is 7 marks)

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8 $$f(x) = (x + 3)(x - 1)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Find the equation of $y = f(x + 2)$ in the form $y = (x + a)(x + b)^2$ **(2)**

(Total for question 8 is 5 marks)

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9 (a) The curve $y = \frac{2}{x-1}$ is translated by four units in the positive x-direction.

State the equation of the curve after it has been translated. **(2)**

(b) Describe fully the single transformation that transforms the curve $y = \frac{2}{x-1}$ to $y = \frac{3}{x-1}$ **(2)**

(Total for question 9 is 4 marks)

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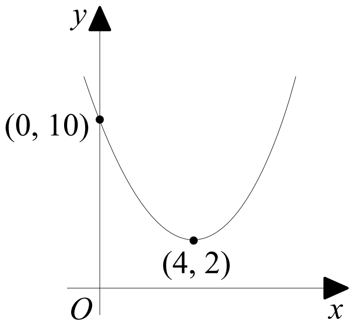
10 Figure 1 shows $y = \mathrm{f}(x)$

Figure 1



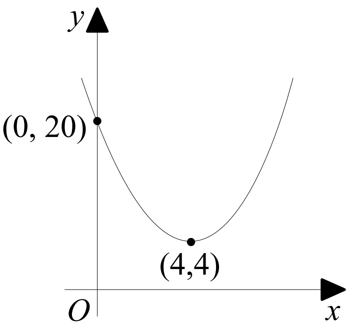


Figure 2

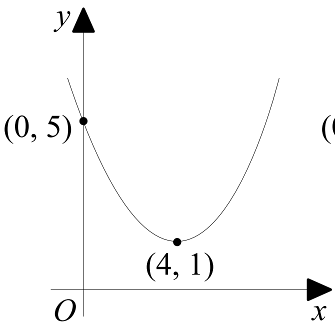


Figure 3

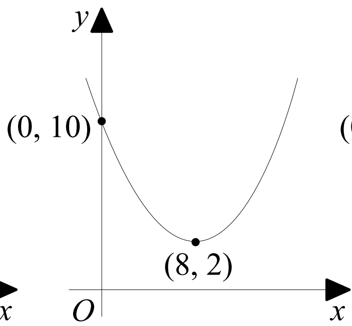


Figure 4

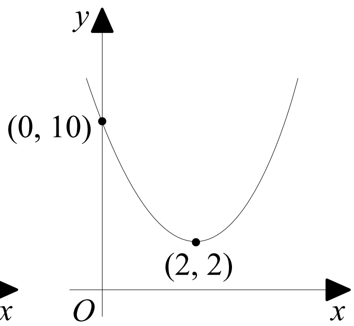


Figure 5

(a) Which figure shows $y = 2\mathrm{f}(x)$? ______________________

(b) Which figure shows $y = \mathrm{f}(2x)$? ______________________

(Total for question 10 is 2 marks)

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11 Given that $\mathrm{f}(x) = 10$ when $x = 4$, which statement must be correct?

Tick (✓) one box.

$\mathrm{f}(2x) = 20$ when $x = 4$ ☐

$\mathrm{f}(2x) = 10$ when $x = 8$ ☐

$\mathrm{f}(2x) = 5$ when $x = 4$ ☐

$\mathrm{f}(2x) = 10$ when $x = 2$ ☐

(Total for question 11 is 1 mark)

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12 Curve C has equation $y = x^2$

C is translated by $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ to give the equation C_1.

Line L has equation $y = x$

L is stretched by scale factor 3 parallel to the x-axis to give the line L_1.

Find the exact distance between the two intersection points of C_1 and L_1

(Total for question 12 is 6 marks)

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13 The graph $y = \dfrac{1}{x}$ is translated by the vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(a) Write down the equation of the transformed graph.

(b) State the equations of the asymptotes of the transformed graph.

(Total for question 13 is 3 marks)

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14 The graph $y = \dfrac{1}{x}$ is translated by the vector $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

(a) Write down the equation of the transformed graph.

(b) State the equations of the asymptotes of the transformed graph.

(Total for question 14 is 3 marks)

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15 (a) Sketch the curve $y = (x - a)(5 - x)^2$ where $0 < a < 5$

indicating the coordinates of the points where the curve and the axes meet.

(b) Hence solve, $(x - a)(5 - x)^2 > 0$ giving your answer in set notation form.

(Total for question 15 is 5 marks)

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16 Sketch the following curves.

(a) $y = \dfrac{3}{x^2}$ **(2)**

(b) $y = x^3 - 8x^2 + 16x$ **(5)**

(Total for question 16 is 7 marks)

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17 (a) Sketch the curve $y = \frac{-2}{x}$ **(1)**

(b) The curve $y = \frac{-2}{x}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated **(2)**

(c) The curve $y = \frac{-2}{x}$ is stretched parallel to the y-axis with scale factor 2 and, as a result, the point (2, -1) on the curve is transformed to the point P.
State the coordinates of P. **(2)**

(Total for question 17 is 5 marks)

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18 $f(x) = (x - a)(x - 3a)(x + b)$ where a and b are positive integers.

(a) Sketch the curve $y = f(x)$ **(2)**

(b) On your sketch mark, in terms of a and b, the points where the curve meets the axes. **(2)**

(Total for question 18 is 4 marks)

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19 The curve $y = (x - 2)^2$ maps onto the curve C_1 following a stretch scale factor $\frac{1}{2}$ in the x-direction

Find the equation of the curve C_1

(Total for question 19 is 2 marks)

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20 (a) Sketch the curve $y = (x + 5)(x + 2)(3 - x)$ **(4)**

(b) The curve $y = (x + 5)(x + 2)(3 - x)$ is translated by the vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.
Write down the equation of the transformed graph. **(2)**

(Total for question 20 is 6 marks)

21

$$f(x) = (x + 1)(x - 2)^2$$

(a) Sketch the curve $y = f(x)$ **(3)**

(b) Hence solve $f(x) \leq 0$ **(2)**

(Total for question 21 is 5 marks)

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22 $$f(x) = (x + 4)(2x - 5)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Deduce the values of x for which

(i) $f(x) \geq 0$

(ii) $f(2x) = 0$ **(3)**

(Total for question 2 is 6 marks)

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23 The curve C has equation

$$y = \frac{k^2}{x} - 2 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C, stating the equation of the horizontal asymptote **(3)**

The line l has equation $y = -3x + 4$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$3x^2 - 6x + k^2 = 0$$ **(2)**

(c) Hence find the exact values of k for which l is a tangent to C. **(3)**

(Total for question 23 is 8 marks)

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24 $$f(x) = (x + 2)(x - 3)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(b) find the two possible values of k. **(2)**

(Total for question 24 is 5 marks)

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25 (a) Using algebra, find all the solutions to the equation $3x^3 - 11x^2 + 6x = 0$ **(3)**

(b) Hence find all the real solutions of $3(y + 2)^6 - 11(y + 2)^4 + 6(y + 2)^2 = 0$ **(3)**

(Total for question 25 is 6 marks)
